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NUMERICAL PREDICTION OF TURBULENT DISPERSION IN TWO-PHASE JET FLOWS

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ABSTRACT - The paper presents the development and extension of a simple model for the prediction of turbulent dispersion in two-phase flow, and its application to the case of particulate co-axial jets. The model is based on the Eulerian formulation of transport equations for each phase (the two-fluid model) together with a turbulence closure based on an extension of the k-ϵ model. Turbulent dispersion effects are accounted for through phase-weighted averaging of the equations and then relating the resulting velocity fluctuations of each phase to those of the other via a simple particle/eddy interaction model. The model embodied in a numerical procedure is applied to the prediction of dispersion of particles in co-axial air jets, and the results are compared with experiment. It is shown that the turbulent Schmidt number has a decisive influence on the dispersion rates predicted.

1. INTRODUCTION

Dispersion phenomena in two-phase flow, especially of particle laden gases, are attributed to the effects of turbulence. It is therefore imperative that in the modelling of two-phase flow, proper account of the action of turbulence should be taken in the formulation of the governing equations. This paper describes the development of such a model and its application to the prediction of particle dispersion in co-axial air jets. The model is based on the k-ϵ two-equation turbulence closure, especially extended to two-phase flow. This extension entails the formulation of the averaged conservation equations for each phase and modelling the different turbulence related correlations which arise in the averaged equations. Central to the model is the formulation of a relation describing the response of a particle to turbulence eddies; this relation is formulated in a simple manner which enables numerical solutions to be obtained economically, unlike other models which solve differential equations for turbulent fluxes.

The two fluid model employed here involves the formulation of transport equations in an Eulerian frame for each of the phases in terms of dependent variables that are averages in some sense (ensemble, time or volume averages). There are different routes taken to arrive at these equations, involving different averaging procedures. In most of the derivations of the transport equations for the two fluid model, the starting point is the instantaneous Navier-Stokes equations for the individual phases. The equations are then averaged either by time, volume or ensemble averaging processes. The time (Ishii [1]) and ensemble (Drew [2]) averaging procedures usually yield phase weighted equations which embody turbulence fluctuations; thus, the Reynolds stresses appear in the averaged equations automatically. Volume averaging however does not involve turbulence fluctuations and the equations retain their instantaneous form. In this case a second time averaging process is necessary to arrive at the final averaged form. Here there are two routes to take, one using phase weighted fluctuations (in the manner of Favre-averaging that is utilised for variable density flows) as followed by eg. Politis [3], and the second using the raw fluctuating quantities as done by Elghobashi and Abou-Arab [4]. In the last work the resulting equations assume a different form from the rest of the derivations since unlike the others they are in terms of dependent variables that are non-weighted by the phase fraction.

For the present work the equations are derived following the double averaging procedure whereby phase weighted quantities are used along the lines of Politis [3]. However the modelling assumptions made at the final stage of the formulation differ from the ones made by Politis, with the result that the present modelled equations become similar to those obtained by means of a single ensemble average. Those assumptions which mainly affect the momentum interaction term (in this case, the drag force) were introduced by Oliveira [5] and Issa and Oliveira [6], and are here extended to the treatment of the turbulent stresses.

Most of the turbulence models used for the closure of the averaged equations are based on the well established k-ϵ model, although there are several other modelling efforts that employ higher order closures. The main reason behind the preference of the two-equation model is the large demand in computer resources made by the more elaborate models which involve many additional equations that have to be solved. The present effort focuses on the development of a turbulence model based on the extension of the standard k-ϵ model to two-phase flow proposed by Politis [3] and Gosman et al [7]; this model has already been subject to several refinements (Oliveira [5], Issa and Oliveira [6]).

2. EQUATIONS AND TURBULENCE MODELLING

2-1 The α-Weighted Equations of Motion

The phase averaged equations are formulated by carrying out volume averaging first followed by time averaging. The resulting continuity and momentum equations, after simplification, are:

$$\rho_k\Big(\frac{\partial}{\partial t}\overline{\alpha}_k + \nabla\cdot\overline{\alpha}_k\widetilde{\mathbf{u}}_k\Big) = 0 \tag{1}$$

and

$$\rho_k\Big(\frac{\partial}{\partial t}\overline{\alpha}_k\widetilde{\mathbf{u}}_k + \nabla\cdot\overline{\alpha}_k\widetilde{\mathbf{u}}_k\widetilde{\mathbf{u}}_k\Big) = -\overline{\alpha}_k\nabla\widetilde{\mathrm{p}} + \overline{\alpha}_k\nabla\cdot\widetilde{\tau}_k + \nabla\cdot\overline{\alpha}_k\widetilde{\tau}^t_k + \rho_k\overline{\alpha}_k\widetilde{\mathbf{g}} + \overline{\mathbf{F}_{\mathrm{D}_k}} \tag{2}$$

where α_k is the volume fraction of phase k, τ and τ^t are respectively the molecular and turbulent stress tensors and $\mathbf{F}_{\mathrm{D}}$ is the drag force per unit volume, the only interphase momentum transfer term relevant to the present application (since $\rho_d/\rho_c \gg 1$, Hinze [8]). Densities of each phase, ρ_k (k=c continuous phase; k=d dispersed phase), are taken as constant and the same averaged pressure $\widetilde{\mathrm{p}}$ acts on both phases. In Eq. (2) it has been assumed that pressure and molecular stresses averaged over the interface are equal to the respective averaged bulk values.

The velocity vector $\widetilde{\mathbf{u}}$ is a phase-weighted average quantity, defined by:

$$\mathbf{u} = \widetilde{\mathbf{u}} + \mathbf{u}'' = \overline{\mathbf{u}} + \mathbf{u}' \tag{3}$$

where $\mathbf{u}$ is the instantaneous (volume-averaged) velocity and $\mathbf{u}''$ is the fluctuating component. The phase-weighted-averaging process used is based on the following relations:

$$\overline{\alpha\mathbf{u}''} = 0, \tag{4}$$

and

$$\overline{\alpha\mathbf{u}} = \overline{\alpha}\,\widetilde{\mathbf{u}} \tag{5}$$

where the overbar denotes time averages. Time and phase averaged velocity, and fluctuations, are related by the expressions:

$$\overline{\mathbf{u}''} = -\overline{(\alpha\mathbf{u}')}\ /\ \overline{\alpha} \tag{6}$$

and

$$\overline{\mathbf{u}} = \widetilde{\mathbf{u}} + \overline{\mathbf{u}''}, \tag{7}$$

which will be used below. As an outcome of the derivation, the expression for the turbulent stress in the momentum equation is:

$$\widetilde{\tau}^t_k = -\frac{\rho_k\overline{\alpha_k(\mathbf{u}''\mathbf{u}'')_k}}{\overline{\alpha_k}}, \tag{8}$$

thus $\widetilde{\tau}^t$ is the phase-weighted average of the tensor $\mathbf{u}''\mathbf{u}''$.

2.2 Main Modelling Assumptions

In order to obtain a closed set of equations in terms of the dependent variables $\widetilde{\mathbf{u}}_k$ and $\widetilde{\mathrm{p}}$, assumptions have to be made regarding the different correlations in the phase momentum equations (2) and, as will be seen later, in the k and ϵ equations.

The turbulence stress tensor in Eq. (8) needs to be modelled. Within the Boussinesq assumption, other works formulate a relationship between $\widetilde{\tau}^t$ and the rate of strain tensor based on the phase averaged velocity. However, it is not clear whether that approach which is a simple generalisation of the single-phase relationship is applicable here since the correlation in velocity fluctuation now contains the phase fraction α. In the present work an alternative route is taken based on the arguments of Oliveira [5] who showed that

$$\widetilde{\tau}^t = -\frac{\rho\overline{\alpha(\mathbf{u}''\mathbf{u}'')}}{\overline{\alpha}} \doteq \overline{\tau}^t = -\rho\overline{(\mathbf{u}'\mathbf{u}')}$$

This suggests that the turbulent stresses ought to be related to the time-averaged velocity gradients and not the phase-averaged ones since the correlation now contain only $\mathbf{u}'\mathbf{u}'$. Thus, $\widetilde{\tau}^t$ is approximated by:

$$\widetilde{\tau}^t_k = \mu^t_k(\nabla\,\overline{\mathbf{u}}_k + \nabla\,\overline{\mathbf{u}}^{\mathrm{T}}_k) - \tfrac{2}{3}(\mu^t_k\nabla\cdot\overline{\mathbf{u}}_k + \rho_k\overline{\mathrm{k}}_k)\boldsymbol{\delta}, \tag{9}$$

where $\boldsymbol{\delta}$ is the identity tensor. The time-average velocity is related to the phase-average value by Eq. (7), which must be introduced into (9) to obtain the stress in terms of the dependent variable $\widetilde{\mathbf{u}}$ and the phase weighted turbulence kinetic energy $\widetilde{\mathrm{k}}_k$. The details of this operation are left for section 2.4.

The turbulence kinetic energy of the continuous phase $\widetilde{\mathrm{k}}_c$ is obtained from its own transport equation and the turbulent viscosity μ^t_c is given by the k-ϵ model, as explained in section 2.5. The dispersed phase turbulent viscosity and kinetic energy need to be specified as functions of the respective continuous phase values, as explained below.

Similar to the transport of momentum by turbulence, correlations involving volume fraction and velocity fluctuations, which represent transport of α by turbulence, are modelled assuming the gradient eddy

diffusion hypothesis:

$$\overline{\alpha_c \mathbf{u}'_c} = -\eta_c \nabla \overline{\alpha}_c, \tag{10}$$

$$\overline{\alpha_d \mathbf{u}'_d} = -\eta_d \nabla \overline{\alpha}_d. \tag{11}$$

In these equations η_k is the turbulent diffusivity of α_k which for the continuous phase will be obtained from $\eta_c=\nu_c^t/\sigma_\alpha$, with an appropriate eddy Schmidt number, σ_α. A similar expression based on the same on the same value of σ_α is used for η_d. Since $\alpha_c+\alpha_d=1$ and $\nabla\alpha_c=-\nabla\alpha_d$, Eq. (10) can be re-written as:

$$\overline{\alpha_d \mathbf{u}'_c} = -\eta_c \nabla\, \overline{\alpha_d}. \tag{12}$$

Equation (11) shows that η_d is the diffusivity of α_d transported by turbulent fluctuations of the dispersed-phase velocity, whereas Eq. (12) shows that the diffusivity η_c is related to the transport of α_d by continuous-phase velocity fluctuations. In previous work (Issa and Oliveira [9] and [6]) a value of unity was assigned to the eddy Schmidt number. However a review of the literature reveals that σ_α is typically below 1. Hinze [8] reports from data of Forstall and Shapiro a value of σ_α=0.71 for the spread of helium in a round free jet of air. For a plane jet, Hinze reports from the data of Van der Hegge Zijnen considerable variation of σ_α across the jet with local values as low as 0.42. Subramanian and Ganesh [10] measured an eddy Schmidt number, based on diffusivities averaged over the cross section of a jet, of 0.47 and their analysis shows local values as low as 0.125. Values used in predictions are also typically around or below 0.7 (Lee & Chung [11]); Simonin [12] used a value of 0.67, as well as 0.5 which he recommends (Simonin [13]) for large heavy particles. McTigue [14] used values as low as 0.15 to match measured data of solid sedimentation in water, and remarked that the turbulent Schmidt number tends to decrease substantially away from the wall. From this evidence it appears that the eddy Schmdit number may vary significantly and this justifies the parametric study of its influence on particle dispersion presented in section 3.

In the model of Gosman *et al.* [7], all the remaining correlations are calculated from a particle response function C_t, which links the instantaneous velocity fluctuations of the dispersed phase to the velocity fluctuations of the continuous phase as:

$$\mathbf{u}'_d = C_t \mathbf{u}'_c\,. \tag{13}$$

This is a key assumption of the model since it enables any correlation to be easily worked out; for example $\overline{\mathbf{u}'_d\mathbf{u}'_c}$ would become $\overline{C_t\mathbf{u}'_c\mathbf{u}'_c} = C_t\overline{\mathbf{u}'_c\mathbf{u}'_c}$. From the above it follows that:

$$k_d = C_t^2\, k_c$$

where k is the turbulence kinetic energy, and that:

$$\nu_d^t = C_t^2\, \nu_c^t$$

However the work of Issa and Oliveira [6] reveals that such a modelling approach is not completely satisfactory. This is because when C_t is small as is the case in the present study, very low dispersed phase kinetic energy and eddy viscosity are predicted from the above relations contrary to experimental evidence. The standard model therefore underestimates the rate of dispersion of a particulate jet considerably.

A study (to be published elsewhere) in which the model of C_t is compared with the theory developed by Tchen (cited by Hinze [8]) reveals that it is more appropriate to consider C_t as the ratio of the root mean square values (hence $C_t \equiv u'_d/u'_c$, where the r.m.s is $u' = \sqrt{\overline{\mathbf{u}'\cdot\mathbf{u}'}}$), and to consider different response functions for the various correlations. To this aim, the following definitions are introduced:

$$C_k = \frac{\overline{\mathbf{u}'_d\cdot\mathbf{u}'_d}}{\overline{\mathbf{u}'_c\cdot\mathbf{u}'_c}} \tag{14}$$

$$C_\nu = \frac{\nu_d^t}{\nu_c^t} \tag{15}$$

$$C_i = \frac{\overline{\mathbf{u}'_d\cdot\mathbf{u}'_c}}{\overline{\mathbf{u}'_c\cdot\mathbf{u}'_c}} \tag{16}$$

which will suffice for the present turbulence model. The response function C_t, relating the phase r.m.s. velocities, can be viewed as $\sqrt{C_k}$ and is formulated in section 2.5.

2.3 Modelling the Drag Force

Drag is modelled assuming that the dispersed phase is a cloud of small spherical particles. It can generally be written in a linearised form as:

$$\mathbf{F}_{D_c} = C_f \alpha_d \alpha_c (\mathbf{u}_d - \mathbf{u}_c) \tag{17}$$

with

$$C_f = (\tfrac{3}{4}\rho_c u_r C_D)/d_p = 18\mu_c f(Re_p)/d_p^2 \tag{18}$$

where the drag coefficient is given as a function of the particle Reynolds number ($Re_p=\overline{u}_r d_p/\nu_c$, $\nu_c=\mu_c/\rho_c$) by the standard formula:

$$C_D = \frac{24}{Re_p} f(Re_p) = \frac{24}{Re_p}(1+0.15 Re_p^{0.687}). \tag{19}$$

In the expressions above, u_r is the relative velocity ($u_r=\|\mathbf{u}_d-\mathbf{u}_c\|$) and d_p is the particle diameter. After averaging and making use of the gradient diffusion assumption for α (Oliveira [5]), the drag force becomes:

$$\overline{\mathbf{F}_{D_c}} = C_f\Big(\,\overline{\alpha}_d\overline{\alpha}_c(\widetilde{\mathbf{u}}_d-\widetilde{\mathbf{u}}_c) + (\,\overline{\alpha}_c\eta_c+\overline{\alpha}_d\eta_d)\,\nabla\overline{\alpha}_d - \overline{\alpha'_d\alpha'_d\mathbf{u}_r}\,\Big). \tag{20}$$

2.4 Modelling the Phase-Weighted Turbulent Stresses

The stress for the continuous phase is defined in Eq. (8) and is modelled using the Boussinesq eddy

viscosity assumption given by Eq. (9). With the help of Eqs. (6) and (7), and the eddy diffusivity (10), the Reynolds stress tensor becomes:

$$\widetilde{\tau}_c^t = \mu_c^t(\nabla \widetilde{\mathbf{u}}_c + \nabla \widetilde{\mathbf{u}}_c^T) - \tfrac{2}{3}(\mu_c^t \nabla\cdot\widetilde{\mathbf{u}}_c + \rho_c \widetilde{k}_c)\delta +$$
$$+ \mu_c^t \eta_c\Big(-2\,(\nabla\overline{\alpha}_d)(\nabla\overline{\alpha}_d) + \tfrac{2}{3}(\nabla\overline{\alpha}_d)\cdot(\nabla\overline{\alpha}_d)\delta\Big)/\ \overline{\alpha}_c^2 \quad (21)$$

An equation like (21) will hold for the dispersed phase as well, after substituting dispersed phase quantities for the continuous phase ones.

2.5 The k and ϵ Equations and the Modelling of the Additional Terms

The α-weighted equations for the transport of turbulence kinetic energy (k) and its rate of dissipation (ϵ), for the continuous phase, are written as:

$$\rho_c\Big(\frac{\partial}{\partial t}\overline{\alpha}_c\,\widetilde{k}_c + \nabla\cdot\overline{\alpha}_c\widetilde{\mathbf{u}}_c\widetilde{k}_c\Big) =$$
$$= \nabla\cdot(\overline{\alpha}_c\frac{\mu_c^t}{\sigma_k}\nabla\widetilde{k}_c) + \overline{\alpha}_c(G - \rho_c\widetilde{\epsilon}_c) + S_d^k \quad (22)$$

$$\rho_c\Big(\frac{\partial}{\partial t}\overline{\alpha}_c\,\widetilde{\epsilon}_c + \nabla\cdot\overline{\alpha}_c\widetilde{\mathbf{u}}_c\widetilde{\epsilon}_c\Big) =$$
$$= \nabla\cdot(\overline{\alpha}_c\frac{\mu_c^t}{\sigma_\epsilon}\nabla\widetilde{\epsilon}_c) + \overline{\alpha}_c\frac{\widetilde{\epsilon}_c}{\widetilde{k}_c}(C_1 G - C_2\rho_c\widetilde{\epsilon}_c) + S_d^\epsilon. \quad (23)$$

These equations represent a generalisation of the single-phase k-ϵ model (Jones & Launder [15]) applied to the continuous phase except for the additional terms S_d which account for the interaction between dispersed particles with the continuous phase turbulence. The turbulent viscosity and the generation of $\widetilde{k}$ are computed from:

$$\nu_c^t = C_\mu\frac{\widetilde{k}_c^2}{\widetilde{\epsilon}_c}, \quad (24)$$

$$G = \mu_c^t\nabla\widetilde{\mathbf{u}}_c\!:\!(\nabla\widetilde{\mathbf{u}}_c + \nabla\widetilde{\mathbf{u}}_c^T) \quad (25)$$

and the constants used in the present work are the standard ones (C_1=1.44, C_2=1.92, C_μ=0.09, σ_k=1.0, σ_ϵ= 1.22).

The additional source term in the k equation arises from the time average of the inner product between the instantaneous external forces and the fluctuating continuous-phase velocity (Favre [16]). The external forces are drag and gravity (see Eq. 2) and the additional term is:

$$S_d^k = \overline{(\mathbf{F}_{D_c} + \rho_c\alpha_c\mathbf{g})\cdot\mathbf{u}_c''} =$$
$$= \overline{C_f\alpha_c\alpha_d(\mathbf{u}_d - \mathbf{u}_c)\cdot\mathbf{u}_c''} + \rho_c\mathbf{g}\cdot\overline{\alpha_c\mathbf{u}_c''} =$$
$$\simeq C_f\,\overline{\alpha}_c\Big(\overline{\alpha_d\mathbf{u}_c''}\cdot(\widetilde{\mathbf{u}}_d - \widetilde{\mathbf{u}}_c) + \overline{\alpha_d(\mathbf{u}_d'' - \mathbf{u}_c'')\cdot\mathbf{u}_c''}\Big)$$

which after the introduction of the model assumptions in section 2.2 becomes:

$$S_d^k = -C_f\Big(\eta_c(\widetilde{\mathbf{u}}_d - \widetilde{\mathbf{u}}_c)\cdot\nabla\,\overline{\alpha}_d + 2\overline{\alpha}_c\overline{\alpha}_d\widetilde{k}_c(1-C_i)\Big). \quad (26)$$

The main contribution for S_d^k is given by the last term in (26), which constitutes a sink of turbulence energy because for particle flow $C_i \le 1$. It will induce a dissipation equal to the term divided by the turbulence time scale (k/ϵ). Hence the additional source in the ϵ-equation is modelled as:

$$S_d^\epsilon = -C_3\frac{\widetilde{\epsilon}_c}{\widetilde{k}_c}\,2C_f\overline{\alpha}_c\overline{\alpha}_d\widetilde{k}_c(1-C_i), \quad (27)$$

where the model constant C_3 is taken as unity in the absence of any knowledge of its actual magnitude.

2.6 New C_t Formulation

A model for the response function C_t is here derived from a simplified analysis of the particle equation of motion which is written in terms of a disturbed velocity field as:

$$\rho_d\mathcal{V}_d\frac{d\mathbf{u}_d'}{dt} = \rho_c\mathcal{V}_d\frac{D\mathbf{u}_c'}{Dt} + \mathfrak{D}(\mathbf{u}_c' - \mathbf{u}_d') +$$
$$+ \tfrac{1}{2}\rho_c\mathcal{V}_d(\frac{D\mathbf{u}_c'}{Dt} - \frac{d\mathbf{u}_d'}{dt}) - \rho_c\mathcal{V}_d C_L\overline{\mathbf{u}}_r\times(\nabla\times\mathbf{u}_c'), \quad (28)$$

where D/Dt denotes material derivative with respect to the fluid, C_L is the lift coefficient and the drag parameter $\mathfrak{D}$ is to be determined from whatever drag model used. The pressure gradient term has already been eliminated by making use of the approximate fluid momentum equation, $\rho_c D\mathbf{u}_c'/Dt = -\nabla p$. For simplicity, the mean flow is assumed to be in the x-y plane, aligned with the x-axis. The vorticity vector is then along the z-axis, $\nabla\times\mathbf{u}_c' = \xi\,\mathbf{k}$ ($\xi = \partial v_c'/\partial x - \partial u_c'/\partial y$), and the lift term in (28) is perpendicular to the mean flow direction, becoming

$$\overline{\mathbf{u}}_r\times(\nabla\times\mathbf{u}_c') = -\overline{u}_r\xi\,\mathbf{j}.$$

where **i**, **j** and **k** are base vectors and u, v and w the velocity components, along the 3 directions. Now, from the definition of C_t given by Eq. (13) and with the assumption that $d\mathbf{u}_d'/dt \approx D\mathbf{u}_d'/Dt$, the particle equation (28) becomes

$$\rho_d\mathcal{V}_d C_t\frac{D\mathbf{u}_c'}{Dt} = \rho_c\mathcal{V}_d\frac{D\mathbf{u}_c'}{Dt} + \mathfrak{D}\mathbf{u}_c'(1-C_t) +$$
$$+ \tfrac{1}{2}\rho_c\mathcal{V}_d\frac{D\mathbf{u}_c'}{Dt}(1-C_t) + \rho_c\mathcal{V}_d C_L\overline{u}_r\xi\,\mathbf{j}, \quad (29)$$

where it is clear from the presence of the last term that one needs to consider two C_t values: for the main flow direction, C_{tx}, and for the cross stream direction, C_{ty}. Finally, in order to obtain a closed expression for C_t one needs to introduce an approximation for the eddy acceleration, $D\mathbf{u}_c'/Dt$. In the range of the energy-containing eddies, which are assumed to give the main contribution to the dispersion of the particles, the theory of homogeneous turbulence gives the approximate equation $Du'^2/Dt = -Au'^3/l_e$ (Batchelor [17]) hence the acceleration may be scaled as $|D\mathbf{u}_c'/Dt| \approx u_c'^2/l_e$. With this approximation, Eq. (29) can be rearranged into:

$$C_{tx} = \frac{3+\beta}{1+\beta+2\rho_d/\rho_c} \quad (30)$$

for the main flow direction, and

$$C_{t_y} = \frac{3 + \beta + 2C_L C_2 R}{1 + \beta + 2\rho_d/\rho_c} \tag{31}$$

for the cross stream direction. The additional term in the numerator for the cross stream direction was obtained from the approximation $\xi = C_2 v'_c / l_e$, and the factor R relates the mean relative velocity to the lateral r.m.s. fluctuation, $R \equiv \overline{u}_r / v'_c$. In the above equation $\beta = 2\mathfrak{D} l_e / \rho_c \mathcal{V}_d u'_c$ which, for spherical particles, becomes:

$$\beta = \frac{12\,\mathfrak{D}}{\pi d_d \mu_c} \left(\frac{l_e}{d_d}\right)^2 \frac{1}{Re_t},$$

where $Re_t = u'_c l_e / \nu_c$ is the turbulence Reynolds number. For particle flow the density ratios are of the order $\rho_d/\rho_c \sim 2 \times 10^3$ and therefore C_t is always smaller than 1. The response in the crosstream direction is always greater than in the main flow direction, increasing with the ratio R, however, this effect is small for particle flow.

2.7 Numerical Solution of the Model Equations

The averaged transport equations for momentum and mass of each phase, and for the turbulence kinetic energy and turbulence dissipation of the continuous phase, are solved by a finite-volume numerical methodology. The equations are discretised on a non-staggered mesh and the sets of discretised equations are solved iteratively in a sequential manner. The algorithm developed utilises the pressure-correction technique extended to two-phase flow and applied in a time-marching fashion whereby the velocity, pressure and scalars at a new time level are computed from their values at the previous time level.

3. RESULTS

The model is applied to the prediction of dispersion in a particle laden jet. Experimental data were obtained by Hishida and Maeda [18] who give a full description of the experimental set up which is shown in Fig. 1. It consists of a jet of air laden with solid particles issuing vertically downwards from an inner pipe of 13 mm diameter (D); the latter is enclosed by a larger pipe of 30 mm diameter carrying a confining particle-free air stream.

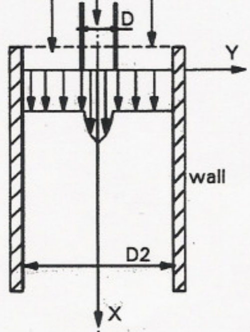


Figure 1 Sketch of the experimental geometry.

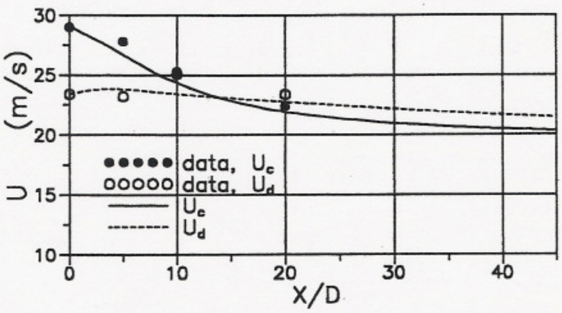
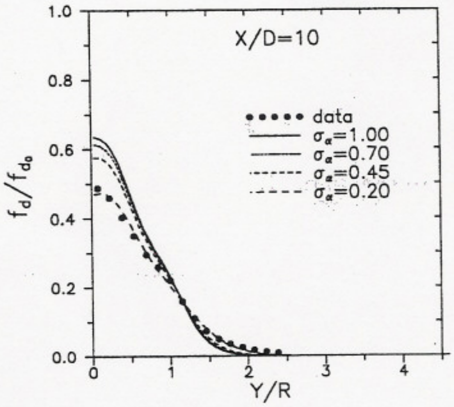


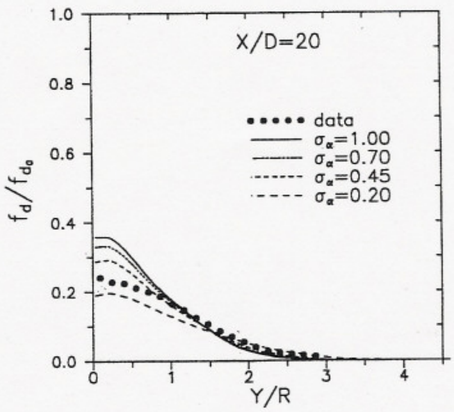
Figure 2 Centre-line decay of axial mean velocity for the continuous ($\widetilde{u}_c$) and dispersed ($\widetilde{u}_d$) phases. Comparison with the data.

The particles have a mean diameter of 64.4 μ and a density of 2590 kg/m^3 which corresponds to a Stokes relaxation time of 33 ms for a free fall velocity of 28 cm/s. Air velocities at inlet are 29 m/s, for the primary jet at the axis, and an approximately uniform value of 15.6 m/s for the secondary stream. The measured inlet particle velocity is smaller than the air velocity with a value of 23 m/s at the axis for a concentration of $\overline{\alpha}_{d_0} = 2.5\ 10^{-4}$; hence, due to their weight, particles accelerate until their velocity eventually surpasses that of the air stream. From that point, the velocity of the particles relative to the fluid will first increase due to their large inertia; subsequently, due to drag, it will approach the free fall velocity. This behaviour is illustrated by Fig. 2, which shows the axial variation of the predicted mean streamwise velocity for the fluid and the particles, where data corresponding to the 3 axial measuring stations are plotted (x/D=5, 10 and 20).

It has already been shown [9] that predictions of the mean axial velocity agreed fairly well with the experimental data. The main focus here is in the prediction of particle dispersion which is better assessed by the variation of the particle flux ($f_d = \rho_d \overline{\alpha}_d \widetilde{u}_d$). Figure 3 shows the radial variation of the measured and predicted particle flux at two stations, x/D=10 and 20. These predictions were obtained with the new C_t formulation developed in 2.6, which was used for C_k and C_i. According to the findings of the previous work [6] the eddy viscosity of the dispersed phase was here set equal to the continuous phase one, hence $C_\nu = 1$. This is in agreement with Tchen's theory for long dispersion times (Hinze [8]) which is valid when there is no crossing-trajectories effect (Csanady [19]). The parameter which is varied in Fig. 3 is the turbulent Schmidt number (σ_α) and it is seen that the predictions become closer to the data as σ_α is decreased (for a value of $\sigma_\alpha = 0.2$ the agreement is very good). A reduction of the Schmidt number corresponds to an increased rate of transport of particles by turbulent fluctuations, signified by the correlations $\overline{\alpha_d u'_c}$ and $\alpha_d u'_d$. Hence the spreading rate of the particles is directly linked with the turbulent drag term given in 2.3, similar to what McTigue [14] found for the sedimentation of solid particles in water. It is not clear, at present, whether $\sigma_\alpha = 0.2$ is too low; Simonin [12], for



a) Radial profile at x/D = 10



b) Radial profile at x/D = 20

Figure 3 Comparison of measured and predicted axial particle-flux using several eddy Schmidt numbers (f_{d0}: inlet value at centre-line).

example, used a Schmidt number of 0.67 for the same problem. However, as mentioned in section 2.2, experimental evidence reveals a large variation of Schmidt/Prandt numbers together with the fact that it is not constant across the flow. This point will require further investigation.

For the same parameters, Fig. 4 shows the variation of particle flux along the centre-line, where the data clearly fall between the predictions using σ_α=0.45 and σ_α=0.2 respectively.

According to Csanady [19] a mean relative velocity between particles and fluid brings about a reduction of the particle autocorrelation and hence of the particle diffusion coefficient. If this coefficient is identified with the eddy viscosity of the dispersed phase, then Csanady's crossing-trajectories effect is quantified as $\nu_d^t = \nu_{d0}^t (1 + C_\beta (\overline{u}_r/u_c')^2)^{-1/2}$(see also Picart *et al.*, [20]), where the eddy viscosity for zero relative velocity (ν_{d0}^t) is here set equal to the fluid one. Csanady expression has been approximately confirmed by experimental measurements, for example Wells and Stock [21] whose data was matched by Simonin [12] with a constant C_β of 0.45 . In terms of the C_ν formulation of 2.2, this is:

$$C_\nu = \left(1 + 0.45\ (\overline{u}_r/u_c')^2\right)^{-1/2} \quad . \tag{32}$$

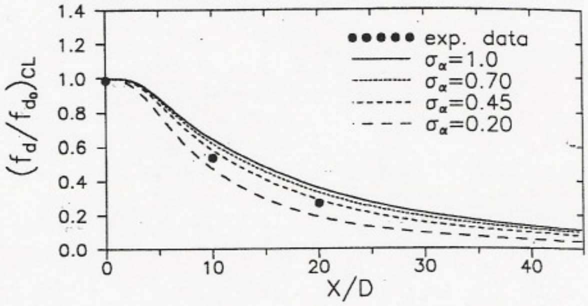


Figure 4 Effect of the eddy Schmidt number on the decay of the axial particle-flux along the centre-line.

However in Csanady's study the r.m.s. particle velocity was considered to be equal to the r.m.s. fluid velocity; when this is not so, as it is the case here, then u_c' in the equation above must be replaced by u_d', leading to the alternate C_ν function:

$$C_\nu = \left(1 + 0.45\ (\frac{\overline{u}_r}{C_t\ u_c'})^2\right)^{-1/2} \quad , \tag{33}$$

This expression includes both effects: crossing-trajectories (measured by $\overline{u}_r$) and inertia. Figure 5 shows the centre-line variation of particle flux for 3 C_ν formulations (with C_k=C_i=C_t and σ_α=0.70). There is almost no difference between Csanady's equation without correction for inertia and Tchen's finding ($C_\nu = 1$). The value of C_ν affects mainly the region close to the jet exit (up to x/D ≈ 5), with the length over which the particle flux maintains its inlet value (the potential core for particle concentration) tending to increase as C_ν decreases. This effect of C_ν on the initial dispersion rate cannot be properly assessed with the data obtained by Hishida et al [18] for the present case since the first measured profile of f_d is only at x/D=10.

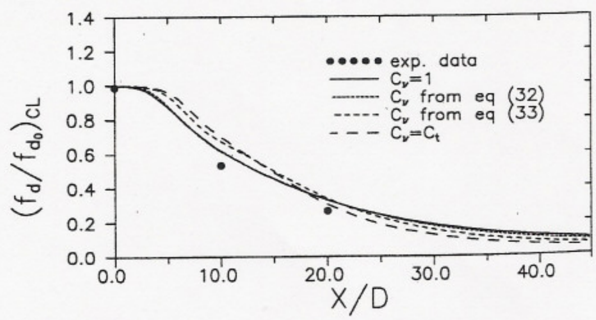


Figure 5 Effect of the C_ν-function ($\nu_d^t = C_\nu \nu_c^t$) on the predicted axial decay of particle-flux along the axis.

4. CONCLUSIONS

Predictions of solid-particle dispersion in a two-phase confined air jet are made using the two-fluid model. The governing equations are expressed in terms of phase averaged velocities and are solved by a finite-

volume procedure. Turbulence of the continuous and the dispersed phases, which is the driving force for dispersion, is accounted for by an extension of the k-ϵ model.

The continuous phase turbulent stresses are modelled by the eddy viscosity assumption in terms of gradients of the un-weighted velocities, while turbulence of the dispersed phase is accounted for by appropriate response functions. A new particle response function is derived to relate the turbulence kinetic energy of the dispersed phase to that of the continuous phase. For particle flows ($\rho_d/\rho_c \gg 1$) this function where the r.m.s velocity fluctuations are used in the formulation becomes identical to the result derived by Tchen, who followed the statistical theory of homogeneous turbulence.

The eddy viscosities of the dispersed and continuous phases are considered either equal to each other, when the averaged relative velocity is small compared with the turbulence velocity scale, or related by a factor similar to the one proposed by Csanady when crossing-trajectory effects are important. It is shown that when this factor is small there is a tendency for the jet to penetrate a longer distance without reduction in particle concentration.

It is shown that there is good agreement between predicted and measured particle dispersion if the eddy Schmidt number is reduced from 1 to between 0.45 to 0.2. Evidence from the literature suggests that this number is not constant but varies across the flow. Since its effect on the rate of dispersion is shown here to be quite considerable, there is a need to further study the problem of the variation of σ_α across the field. It should be recalled that in the model used there are two Schmidt numbers for the transport of α_d, one relates to the fluid velocity fluctuations and the other to the particle fluctuations. The question of whether the same value should be used for both, and also of the definition of the diffusion coefficient for each, must be addressed in future work.

NOMENCLATURE

C_1,C_2	turbulence model constants
C_3, C_μ	turbulence model constants
C_2	constant in C_t formulation
C_t	particle response function, dimensionless
C_k	ratio of turbulence kinetic energy, dimensionless
C_ν	ratio of eddy viscosities, dimensionless
C_i	interaction response function, dimensionless
C_β	constant in Csanady expression
C_f,C_D	drag coefficients, dimensionless
C_L	lift coefficient, dimensionless
d_p	particle diameter, m
D, D_2	diameter of inner and outer pipes, m
f(Re)	correction for non-Stokes drag, dimensionless
f_d	particle mass flux per unit area, kg/m^2 s
$\mathbf{F}_D$	drag force, kg/m^2 s^2
$\mathbf{g}$	gravity acceleration, m/s^2
G	generation of turbulence kinetic energy, kg/s^{3}m
$\mathbf{i,j,k}$	Cartesian base vectors
k	turbulence kinetic energy, m^2/s^2
l_e	characteristic scale of energy-containing eddies, m
p	pressure, N/m^2
R	ratio of mean relative velocity to lateral r.m.s, dimensionless
Re	Reynolds number, dimensionless
S_d^k, S_d^ϵ	source terms due to dispersed phase in the k and ϵ Eqs., kg/m s^3 and kg/m s^4, respectively
t	time, s
$\mathbf{u}$	velocity vector, m/s
u', v'	velocity r.m.s. , m/s
u,v,w	Cartesian velocity components, m/s
x,y	streamwise and cross-stream coordinates, m

Greek and other symbols

α	volume fraction, dimensionless
β	parameter in the formulation of C_t, dimensionless
$\boldsymbol{\delta}$	identity tensor
ϵ	rate of turbulence dissipation, m^2/s^3
η	phase eddy-diffusivity, m^2/s
μ, ν	dynamic and kinematic viscosity, kg/m.s and m^2/s
ρ	density, kg/m^3
σ_α	turbulent Schmidt number, dimensionless
τ	stress tensor, kg/m s^2
ξ	vorticity component, s^{-1}
$\mathfrak{D}$	general drag parameter, kg/s
$\mathcal{V}_d$	volume of a particle, m^3
∇	nabla operator $\equiv \partial/\partial x_i \cdot \mathbf{i}_i$, 1/m

Superscripts

t	turbulent or eddy
T	transpose of a matrix, or tensor
~	phase average (tilde)
—	time average (overbar)
′	fluctuation relative to time average (prime)
″	fluctuation relative to phase average (double prime)

Subscripts

c, d	continuous and dispersed phase
k	phase indicator (k=c or d)
0	at inlet centerline
p	particle
r	relative to continuous phase

REFERENCES

1. M. Ishii, *Thermo-Fluid Dynamic Theory of Two-Phase Flow*, Eyrolles, Paris, 1975.
2. D. A. Drew, Mathematical Modelling of Two-Phase Flow, *Annual Review of Fluid Mechanics*, vol. 15, pp. 261-291, 1983.
3. S. Politis, Prediction of Two-Phase Solid-Liquid Turbulent Flow in Stirred Vessels, Ph.D. thesis,

Imperial College, University of London, 1989.

4. S. E. Elghobashi and T. W. Abou-Arab, A Two-Equation Turbulence Model for Two-Phase Flows, *Phys. Fluids*, vol 26, pp. 931-938, 1983.
5. P. J. Oliveira, Computer Modelling of Multidimensional Multiphase Flow and Application to T-Junctions, Ph.D. thesis, Imperial College, Univ. of London, 1992.
6. R. I. Issa and P. J. Oliveira, Modelling of Turbulent Dispersion in Two Phase Flow Jets, *Proc. 2nd Conf. on Turbulence Modelling and Experiment*, Elsevier, pp. 947-957, 1993.
7. A. D. Gosman, C. Lekakou, S. Politis, R. I. Issa, and M. K. Looney, Multidimensional Modelling of Turbulent Two-Phase Flows in Stirred Vessels, *AIChE J.*, vol. 38, p. 1946, 1992.
8. J. O. Hinze, *Turbulence*, 2^{nd} edition, MacGraw-Hill, 1975.
9. R. I. Issa and P. J. Oliveira, Method for Prediction of Particulate Jets. In *Proc. 5th Workshop on Two Phase Flow Predictions*, Erlangen, pp. 39 and 41, 1990.
10. V. Subramanian and R. Ganesh, Particle–Gas Dispersion Effects in a Round Jet, *Can. J. Chem Eng.*, vol 62, pp. 161-164, 1984.
11. K. B. Lee and M. K. Chung, Refinement of the Mixing-Length Model for Prediction of Gas-Particle Flow in a Pipe, *Int. J. Multiphase Flow*, vol. 13, pp. 275-282, 1987.
12. O. Simonin, An Eulerian Approach for Turbulent Two-Phase Flows Loaded with Discrete Particles, *Proc. 5th Workshop on Two Phase Flow Predictions*, Erlangen, p. 40 and pp. 156-166, 1990.
13. O. Simonin, Second-Moment Prediction of Dispersed Phase Turbulence in Particle-Laden Flows, *8th Symp. on Turbulent Shear Flows*, Munich, pp. 7-4-1 – 7-4-6, 1991.
14. D. F. McTigue, Mixture Theory of Turbulent Diffusion of Heavy Particles, in R. Meyer (ed.), *Theory of Dispersed Multiphase Flow*, Academic Press, pp. 227-250, 1983.
15. W. P. Jones and B. E. Launder, The Prediction of Laminarisation with a Two-Equation Model of Turbulence, *Int. J. Heat Mass Transf.*, vol. 15, pp. 301-314, 1972.
16. A. Favre, Equations des Gazs Turbulents Compressibles, *J. de Mecanique*, vol. 4, pp. 361-421, 1965.
17. G. K. Batchelor, *The Theory of Homogeneous Turbulence*, Cambridge Science Classics, p. 103, 1953.
18. K. Hishida and M. Maeda, Turbulence Characteristics of Gas-Solids Two-Phase Confined Jet (Effect of Particle Density). In *Proc. 5th Workshop on Two Phase Flow Predictions*, Erlangen, March 19-22, pp. 3-14, 1990.
19. G. T. Csanady, Turbulent Diffusion of Heavy Particles in the Atmosphere, *J. Atm. Sci.*, vol. 20, pp. 201-208, 1963.
20. A. Picart, A. Berlemont and G. Gouesbet, Modelling and Predicting Turbulence Fields and the Dispersion of Discrete Particles Transported by Turbulent Flows, *Int. J. Multiphase Flow*, vol. 12, pp. 237-261, 1986.
21. M. R. Wells and D. E. Stock, The Effects of Crossing Trajectories on the Dispersion of Particles in a Turbulent Duct, *J. Fluid Mechanics*, vol. 136, pp. 31-62, 1983.