Third International Conference on Multiphase Flow, ICMF'98
Lyon, France, June 8-12, 1998

ACCOUNTING FOR NON-EQUILIBRIUM TURBULENT FLUCTUATIONS IN THE EULERIAN TWO-FLUID MODEL BY MEANS OF THE NOTION OF INDUCTION PERIOD

Issa, R. I.,[a] **and Oliveira, P. J.**[b]
a Department of Mechanical Engineering
Imperial College of Science, Technology and Medicine, London SW7 2BX, UK.
b Departamento de Engenharia Electromecânica
Universidade da Beira Interior, 6200 Covilhã, Portugal.

Abstract

The application of two-fluid models with a turbulence closure based on suitably adapted two-equation models together with response functions leads to reasonable prediction of all mean quantities, including the lateral velocity rms fluctuations. However, since those response functions are based on the assumption of local equilibrium between the fluid turbulence and the particle response, it turns out that the streamwise velocity rms fluctuations of the particulate phase are grossly underpredicted, a feature which causes no concern in boundary-layer type flows but may be important in more complex flows. The purpose of this work is to address this question. It is shown for the case of a confined particle-laden air jet that the particles remain in the section under study for a short time, compared with their relaxation time, and thus the elapsed time cannot ensure stationary conditions for the particles turbulence motion. Inclusion of a corrective factor accounting for the non-steady behaviour during that induction period, following a proposal by Friedlander (1957), gives axial rms velocity fluctuations in agreement with the experimental data showing that non-equilibrium and inlet-conditions effects were indeed responsible for the aforementioned disagreement.

1. Introduction

In recent years considerable effort has been expended in the development of turbulence closures for particle laden two-phase flows at a level of the second moments (see eg. Simonin et al 1993, and Reeks 1993). The need for such higher order closures arises mainly from the absence of local equilibrium between fluctuations of the two phases which would enable a simple correlation between the two. These closure models are however still at the research stage; they also have limited scope for application to practical engineering problems due to their complexity and high demands on computing resources. Thus the simpler two-equation models in two-phase flow continue to be attractive from a practical point of view and therefore offer a scope for development if due account is taken of all effects when formulating the local response of the particles to the surrounding fluid turbulence and also if non-equilibrium conditions can be included in the model. The purpose of the present paper is therefore to introduce a simple model accounting for non-equilibrium into a general two-fluid formulation.

When applied to particle-laden jet flows, or other boundary-layer type flow, the two-fluid model incorporating response functions based on local equilibrium (see eg. Issa and Oliveira 1997), gives good predictions for the lateral velocity fluctuations for both phases. However, it grossly underpredicts the axial velocity fluctuations of the particles. This occurs because the particles travel along the jet for a very short time compared with their relaxation time (t_p), and thus are unable to reach equilibrium with the fluid turbulence. The particle fluctuating velocity is therefore only a fraction of the imposed inlet value, and in general this value is much higher than that resulting from local equilibrium response function. This situation is not problematic in boundary-layer flows because the axial velocity fluctuations will not affect the momentum balance, since the axial gradients are negligible. However, in some flow situations it may influence the mean flow; it may also be that its value in itself may be of interest, as would happen if for example one is interested in calculating the axial turbulent heat flux due to the particulate phase in a pipe flow. A simplified form of this problem was analysed in 1957 by Friedlander (see Hinze 1975, page 469), who devised a formula to account for such an "induction period" effect in homogeneous turbulence. In that work, Friedlander integrated the particle instantaneous equation of motion in which drag was the only interaction considered. The resulting equation relates the time-varying particle fluctuating velocity to that of the stationary case, and has the particle Lagrangian auto-correlation function as a parameter which is obtained from the standard response functions (as in Issa and Oliveira 1997). The equation to be derived has the property that for long times, when the induction period is over, the particle fluctuation must tend to that of the stationary value.

Friedlander's analysis shows that the particle rms velocity obtained from an equilibrium response function (such as in Issa and Oliveira, 1997) is smoothed by an exponential function decaying with t/t_p

during the so called induction period. For large times, this function tends to unity and the equilibrium velocity rms is recovered; for short times, the smoothing function tends to zero and the imposed initial rms dominates.

The purpose of the present paper is thus to adapt this "Induction Period" notion of Friedlander into a general two-fluid Eulerian formulation and to see how well it can predict the axial particle velocity fluctuations in a confined jet for which detailed experimental data are available. In Friedlander's analysis, the particle fluctuating velocity was assumed to be zero at t_0 (particle starting from rest); this needs to be modified for the present case in which the particle fluctuating velocity is specified at inlet. Also, in the present steady-state analysis with the Eulerian two-fluid model, the elapsed time gap "t" appearing in the function which characterises the induction period needs to be replaced by some appropriate value based on the mean particle velocity and the flying distance from inlet. The work will also show the possible weaknesses of the approach which is unable, for example, to predict particle fluctuating velocities higher than the imposed inlet values.

2. Governing Equations

The phase averaged equations for each phase, the continuous fluid phase (index $k=f$) and the particulate dispersed phase ($k=p$) taken as separated continua seen from an Eulerian framework, are the continuity equations:

$$\nabla \cdot (\overline{\alpha}_k \rho_k \widetilde{\mathbf{u}}_k) = 0 \tag{1}$$

and the momentum equations:

$$\frac{\partial}{\partial \mathrm{t}}(\overline{\alpha}_k \rho_k \widetilde{\mathbf{u}}_k) + \nabla \cdot (\overline{\alpha}_k \rho_k \widetilde{\mathbf{u}}_k \widetilde{\mathbf{u}}_k) = -\overline{\alpha}_k \nabla \widetilde{\mathrm{p}} + \overline{\alpha}_k \nabla \cdot \widetilde{\tau}_k + \nabla \cdot \overline{\alpha}_k \widetilde{\tau}_k^t + \rho_k \overline{\alpha}_k \mathbf{g} + \overline{\mathbf{F}_{\mathrm{D}_k}} \tag{2}$$

The dependent variables are the average pressure $\widetilde{\mathrm{p}}$, the time averaged volume fractions $\overline{\alpha}_k$, and the phase averaged velocities $\widetilde{\mathbf{u}}_k \equiv \overline{\alpha_k \mathbf{u}_k} / \overline{\alpha}_k$. Here, the overbar denotes simple time-averaging, fluctuations with respect to $\overline{\mathbf{u}}$ are denoted $\mathbf{u}'$ and those with respect to $\widetilde{\mathbf{u}}$ are denoted $\mathbf{u}''$. The averaged quantities $\overline{\mathbf{u}}$ and $\widetilde{\mathbf{u}}$ and their respective fluctuating values are connected by simple relations which can be found in previous work.

In the momentum equations (2), $\overline{\mathbf{F}_{\mathrm{D}}}$ is an interaction term which is assumed to be due solely to aerodynamic drag, owing to the large density ratio in the present air/solid particles flow ($\rho_p/\rho_f \approx 10^3$), and is thus modelled as proportional to the time averaged relative velocity:

$$\overline{\mathbf{F}}_{\mathrm{D}_f} = \mathrm{C_f} \overline{\alpha}_p \overline{\alpha}_f (\overline{\mathbf{u}}_p - \overline{\mathbf{u}}_f) \tag{3}$$

If the particles forming the dispersed phase are spherical and have a constant diameter d_p, then a standard drag law can be used, giving $\mathrm{C_f} \equiv (18\mu_f \mathrm{f}(\mathrm{Re}_p))/\mathrm{d}_p^2$ where $\mathrm{f}(\mathrm{Re}) = (1 + 0.15\mathrm{Re}_p^{0.687})$ is a correction function of the particle Reynolds number, $\mathrm{Re}_p = \rho_f \overline{u}_r \mathrm{d}_p/\mu_f$.

Following Issa and Oliveira (1997), and in order to use a 2-equation turbulence model, the turbulent stress tensors in (2) are assumed to be given by:

$$\widetilde{\tau}_k^t = \mu_k^t (\nabla \overline{\mathbf{u}}_k + \nabla \overline{\mathbf{u}}_k^{\mathrm{T}}) - \tfrac{2}{3}(\mu_k^t \nabla \cdot \overline{\mathbf{u}}_k + \rho_k \mathrm{k}_k)\delta, \tag{4}$$

the Boussinesq approximation, linear on the time-averaged velocity gradients. This velocity needs to be related to the main dependent variable, $\widetilde{\mathbf{u}}$, so that (4) becomes an explicit relation between $\widetilde{\tau}$ and $\widetilde{\mathbf{u}}$.

Transport equations for the turbulence kinetic energy of the continuous phase (k_f) and for its dissipation rate (ϵ_f) are formulated and solved in order to obtain the eddy viscosity for that phase, $\mu_f^t = \rho_f \mathrm{C}_\mu \mathrm{k}_f/\epsilon_f$, in a way similar to that in the single-phase k-ϵ model (Jones and Launder, 1972). Some additional terms arise in these equations as compared with the single-phase case, due to the slip existing between the two phases, but details of these can be found in the previous work and will not be repeated here.

Finally, in the development of the present two-fluid model, correlations between volume-fractions and velocity fluctuations often arise. These are modelled with the gradient diffusion hypothesis,

$$\overline{\alpha_p \mathbf{u}'_f} = -\eta_f \nabla \overline{\alpha}_p \quad \text{or} \quad \overline{\alpha_p \mathbf{u}'_p} = -\eta_p \nabla \overline{\alpha}_p. \tag{5}$$

and the turbulent diffusivities η_f and η_p need to be related to the diffusivity of a tracer in the fluid, $\eta_0 = (u'_f)^2 \mathrm{T_L}$ where $\mathrm{T_L}$ is the Lagrangian time scale of the fluid motion.

3. Dispersed Phase Turbulent Quantities

Closure of the previous equations can be achieved through the introduction of response functions (Issa and Oliveira, 1997):

$$(\mu_p^t/\rho_p) = \mathrm{C}_\nu \, (\mu_f^t/\rho_f) \, , \quad \eta_p = \eta_f = \mathrm{C}_\eta \, \eta_0 \tag{6}$$

$$\mathrm{k}_p = \mathrm{C}_k \, \mathrm{k}_f \, , \; \overline{\mathbf{u}'_f \cdot \mathbf{u}'_p} = \mathrm{C}_i \, \overline{\mathbf{u}'_f \cdot \mathbf{u}'_f} \, . \tag{7}$$

which need to be obtained from some simplified theory. Based on the comprehensive analysis of Wang and Stock (1993), valid for homogeneous isotropic turbulence and accounting for both effects of particle inertia and crossing trajectories, Issa and Oliveira derived the response functions appearing in (6) and (7). It was also possible to demonstrate that $\eta_p = \eta_f$, as already indicated in the second equation in (6).
The C_k-function, relating the turbulent kinetic energies or the square of the velocity rms, turns out to be equal the C_i-function defined above, and are given by:

$$\mathrm{C}_k = \mathrm{C}_i = \frac{1}{1+\mathrm{St_T}\Gamma} \quad \text{(in the direction of the gravity)} \tag{8a}$$

$$= \frac{1+\mathrm{St_T}(\Gamma-0.5\gamma \mathrm{m_T})}{(1+\mathrm{St_T}\Gamma)^2} \quad \text{(in the direction perpendicular to gravity)} \tag{8b}$$

where $\Gamma \equiv (1+(\mathrm{m_T}\gamma)^2)^{1/2}$, the Stokes number is defined as $\mathrm{St_T} \equiv \mathrm{t}_p/\mathrm{T}$ (inertia parameter) where T is the fluid integral time scale seen by a particle in the absence of relative velocity, $\gamma \equiv \overline{u}_r/u'_f$ (crossing-trajectories parameter), and $\mathrm{m_T} \equiv \mathrm{T}/\mathrm{T_E}$ (turbulence structure parameter). T was correlated to the standard Stokes number $\mathrm{St} = \mathrm{t}_p/\mathrm{T_E}$ by Wang and Stock and was seen to vary between the Lagrangian time scale $\mathrm{T_L}$ and the Eulerian time scale $\mathrm{T_E}$. In this work, we use $\mathrm{T_L} = 0.41\, \mathrm{k}_f/\epsilon_f$ (Calabrese and Middleman, 1979) and the ratio $\beta \equiv \mathrm{T_L}/\mathrm{T_E}$ was fixed at 0.356. The particle relaxation time follows from the usual definition, yielding $\mathrm{t}_p = \rho_p \mathrm{d}_p^2/18\mu_f \mathrm{f(Re}_p)$.

The diffusivity and eddy viscosity response functions equal each other and also have different values along the axial (aligned with the gravity) and the radial directions, due to the continuity effect (see Wang and Stock, 1993):

$$\mathrm{C}_\nu = \mathrm{C}_\eta = \frac{1}{\Gamma} \, \frac{\mathrm{T}}{\mathrm{T_E}} \, \frac{1}{\beta} \quad \text{(gravity direction)} \tag{9a}$$

$$= \frac{1-0.5\mathrm{m_T}\gamma/\Gamma}{\Gamma} \, \frac{\mathrm{T}}{\mathrm{T_E}} \, \frac{1}{\beta} \quad \text{(perpendicular to gravity)} \tag{9b}$$

In order to relate diffusivities to turbulent viscosities, the usual Schmidt number for a passive scalar is introduced, $\eta_0 = \nu_f^t/\sigma_0$, with σ_0 taking the value 0.7 in the calculations presented here.

4. Effect of the Induction Period

It is well known (e.g. Sommerfeld and Wennerberg, 1991; Issa and Oliveira, 1993) that response functions, as those used above or others based on simpler theories (where typically the C_k is based on the results of Tchen (see Hinze, 1975) and C_ν is based on those of Csanady, 1963), lead to predicted values of the axial particle velocity fluctuations which are substantially below measured values. There are two causes for that underprediction: the particle turbulent fluctuations in a jet (or other type of shear flow) cannot be considered in equilibrium with the fluid turbulence (which itself varies from point to point and is not therefore homogeneous, as assumed in the theories on which the response functions are based); and particle velocity fluctuations at inlet do influence the turbulence at some distance from inlet. Methods devised to take into account the first cause will also require the velocity fluctuations of the particulate phase at inlet, so the two causes are in fact coupled. In these methods, additional transport equations are solved for the turbulent kinetic energy of the particles, for its dissipation rate, and in some instances for the particle/fluid velocity correlation (examples in the works of Simonin et al, 1993, or in Tu and Fletcher, 1996). The introduction of more equations, which have many terms requiring closure, is still at an incipient stage, and justifies a study where a simpler approach is applied. This is precisely the objective of the present paper, in which the

induction period notion proposed back in 1957 by Friedlander (see Hinze 1975) is applied to account for non-equilibrium in the particle turbulence.

For a particle with density much higher than the fluid density, as the ones considered here, the particle dynamic equation reduces to:

$$\frac{d\mathfrak{v}_p}{dt} + \frac{\mathfrak{v}_p}{t_p} = \frac{\mathfrak{v}_f}{t_p}$$

where $\mathfrak{v}_p$ is the particle Lagrangian fluctuating velocity and $\mathfrak{v}_f$ is the fluid fluctuating velocity at the particle position. The induction period (see Hinze, 1975) occurs when, at the initial instant, the condition of the particles is different from the stationary case. In the present application the particles are injected with a given inlet velocity ($\mathfrak{v}_{p0}$) and are then convected downstream by the fluid and disperse due to fluid turbulence. If $\mathfrak{v}_{p\infty}(t)$ denotes the solution of the above equation for the stationary case (i.e. $\overline{\mathfrak{v}_{p\infty}(t)^2} = \overline{\mathfrak{v}^2_{p\infty}}$ independent of t) then the solution that satisfies the initial condition is:

$$\mathfrak{v}_p(t) = \mathfrak{v}_{p\infty}(t) - (\mathfrak{v}_{p\infty}(0) - \mathfrak{v}_{p0})\exp(-t/t_p).$$

After multiplying $\mathfrak{v}_p(t')$ by $\mathfrak{v}_p(t'+t)$, where t is a time gap, and time averaging, we obtain:

$$\frac{\overline{\mathfrak{v}^2_p}(t)}{\overline{\mathfrak{v}^2_f}} = \left(\frac{\overline{\mathfrak{v}^2_{p\infty}} - \overline{\mathfrak{v}^2_{p0}}}{\overline{\mathfrak{v}^2_f}}\right)\Big(1 - 2\exp(-t/t_p)\,R_{pL}(t) + \exp(-2t/t_p)\Big) + \frac{\overline{\mathfrak{v}^2_{p0}}}{\overline{\mathfrak{v}^2_f}} \tag{10}$$

In this equation, $R_{pL}(t)$ is the lagrangian auto-correlation coefficient for the particle motion, defined as $R_{pL}(t) = \overline{\mathfrak{v}_{p\infty}(t')\mathfrak{v}_{p\infty}(t'+t)}/\overline{\mathfrak{v}^2_{p\infty}}$. It can be obtained directly from the results of Wang and Stock (1993), on which the response function are based. There are in fact two values, one in the direction of the gravity, denoted $R_{p,33}$ and given by:

$$R_{p,33}(t) = \frac{St_T\Gamma\exp(-t/t_p) - \exp(-\Gamma\, t/T)}{St_T\Gamma - 1} \tag{11a}$$

and the other in the perpendicular direction:

$$R_{p,11}(t) = \frac{St_T(\Gamma - 0.5\gamma m_T A)\exp(-t/t_p) - (1 - B - 0.5 m_T\gamma\, t/T)\exp(-\Gamma\, t/T)}{St_T\Gamma - 1} \times C \tag{11b}$$

where Γ, T and the modified Stockes number St_T have been defined before. The definitions $A \equiv (St_T^2\Gamma^2 + 1)/(St_T^2\Gamma^2 - 1)$, $B \equiv m_T\gamma St_T^2\Gamma/(St_T^2\Gamma^2 - 1)$ and $C \equiv (St_T\Gamma + 1)/(1 + St_T\Gamma - 0.5 m_T\gamma St_T)$ have been used to keep the expression for the lateral correlation coefficient compact.

In conclusion, and as discussed by Hinze, it is seen from equation (10) that the induction period is determined by $\exp(-t/t_p)$. If the time gap "t" is not large compared with the particles relaxation time (say, if $t/t_p \le 3$), Eq. (10) shows that $\overline{\mathfrak{v}^2_p}(t)$ is influenced by the inlet value $\mathfrak{v}_{p0}$ and also by the shape of the correlation function. Since this is affected by inertia, crossing-trajectories and continuity effects, $\overline{\mathfrak{v}^2_p}(t)$ is also indirectly influenced by those effects. On the other hand, for large elapsed times Eq. (10) yields $\overline{\mathfrak{v}^2_p}(t)$ tending to the stationary solution, $\overline{\mathfrak{v}^2_{p\infty}}$, which is obtained without accounting for the induction period.

It is remarked that the influence of this induction period model on the mean flow and k-ϵ equations is made through the turbulent kinetic energy of the dispersed phase; this is now given by Eq. (10) instead of Eqs. (7) and (8).

5. Results

An air jet laden with solid glass particles flows downwards after issuing from a 13 mm diameter ($D \equiv R/2$) inner tube placed co-axially in a larger 30 mm outer tube (D_2). Fig. 1 gives the geometry and the co-ordinate system used in the computations. Measurements in this confined two-phase jet have been taken by Hishida and Maeda (1991) at several stations ($x/D = 5$, 10 and 20) and served as a study case in the V Workshop on Two-Phase Flow Predictions (Sommerfeld et al 1991).

The transport equations given in Section 2 have been solved by a general elliptic finite-volume method which has been described in detail in a previous reference (Issa and Oliveira, 1994). The boundary

conditions are as indicated in Fig. 1: there is a no-slip wall at $y = R$ (where the usual log-law and the condition $\partial\overline{\alpha}_p/\partial y = 0$ are imposed), a symmetry line at $y = 0$, and an outlet at $x = L = 600$ mm (where vanishing streamwise gradients are imposed except for pressure which is linearly extrapolated). Finally, the inlet conditions at $x = 0$ were directly based on the experimental measurements, with k evaluated as $k = \frac{1}{2}(u'^2 + 2v'^2)$ and ϵ estimated from $\epsilon = C_\mu k^{1.5}/0.03Y$ ($Y = D$ for $r < D/2$, and $Y = (D_2 - D)/2$ for $r > D/2$). The particles volume-fraction at inlet was $\approx 2.5\ 10^{-4}$, thus justifying the absence of inter-particle collisions in the model; the mass loading is however not negligible ($\dot{m}_p/\dot{m}_f \approx 0.30$) indicating that two-way coupling is actually present and is indeed catered for by the numerical method in all situations.

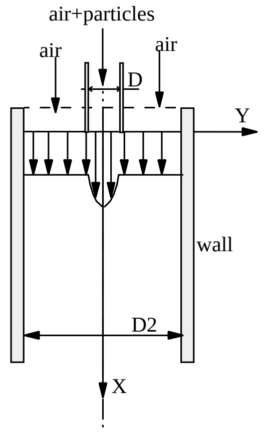

Fig. 1 Flow configuration

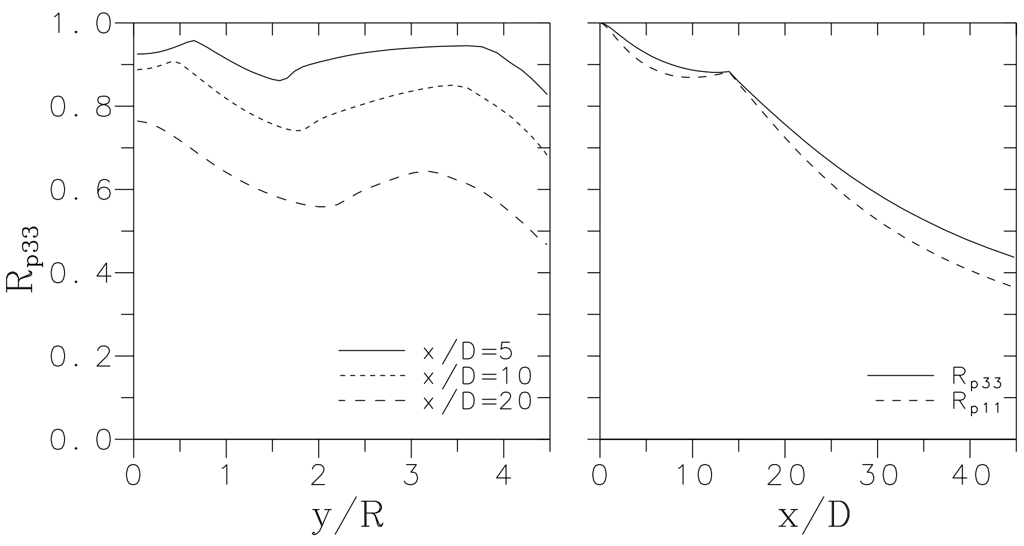

Fig. 2 Variation of the particle auto-correlation coefficient (a)lateral profile (b) along axis

The variation of the particle Lagrangian velocity correlations, as given by equation (11a) (streamwise component, aligned with gravity) and (11b) (lateral component, perpendicular to gravity) are plotted in Fig. 2. It is seen that even for a position near the outlet of the computational domain, at $x/D \approx 45$, there is still considerable correlation with the inlet values ($R_{pL} \approx 0.4$) and therefore, for this flow, the induction period lasts longer than the flying time of a particle in the domain under study. Inlet and non-equilibrium effects are thus to be expected. A simple evaluation can be made based on the simplified exponential correlation function $R_{pL} \sim \exp(-t/t_p)$; the averaged particle velocity in the domain under study is $\overline{u}_p \sim 20$ m/s giving a flying time of $t \sim L/\overline{u}_p = 0.6/20{=}30$, a value not large compared with the relaxation time of the present particles, $t_{p\infty} \sim 28$ ms; thus $R_{pL} \sim 0.34$.

The discontinuity in the slope of the R_{pL} vs. x curve, seen at $x/D \simeq 14$ is due to the zero mean slip velocity occurring at that point ($\Rightarrow \gamma = 0$; thus $R_{p,33} = R_{p,11}$, see equations 11a and 11b); this happens because the inlet particle velocity is less than the inlet air velocity and the particles are initially accelerated,

passing through the situation where $\overline{u}_p = \overline{u}_f$, and it is only much further downstream that they acquire the limiting drift due to gravity. For a decreasing crossing-trajectories effect ($\gamma \to 0$), the particle auto-correlation tends to increase (no slip, more correlation).

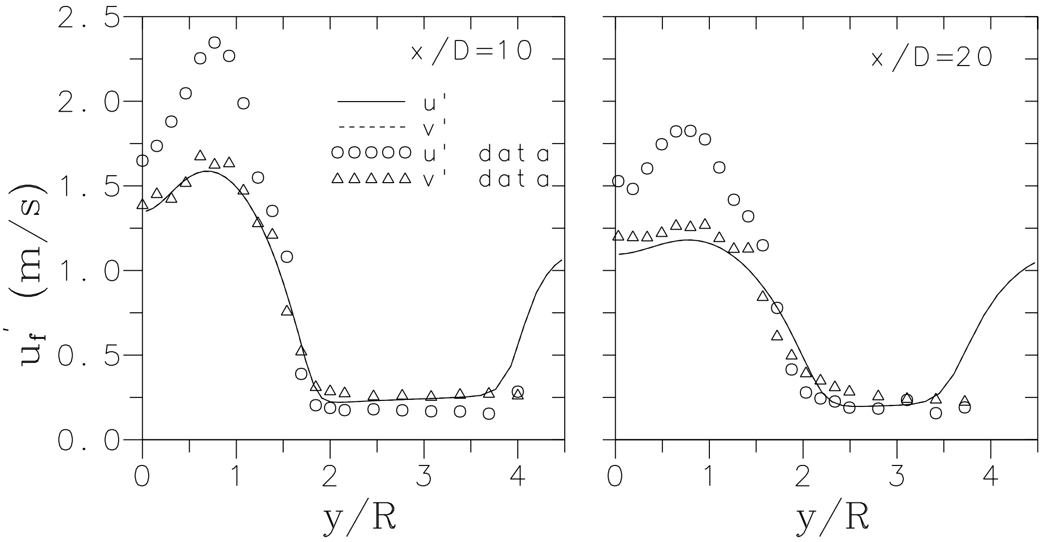

Fig. 3 Lateral profiles of the fluid phase velocity rms fluctuations

In Fig. 3 the air rms velocity fluctuations (lines) are compared with the measured data (marks) at two stations, x/D=10 and 20, thus indicating the best agreement possible which should be kept in mind in the comparisons with the dispersed phase turbulent velocities, to be given later. In this boundary-layer flow, the k-ϵ model cannot distinguish u'_f from v'_f, both are given by $\surd 2k/3$, and so the solid and dashed lines in the figure are superimposed. There is better agreement with the measured lateral velocity fluctuations, those which drive the jet growth due to lateral momentum transfer. It is recalled that the actual magnitude of u'_f is not relevant in this problem, since $\partial u'_f/\partial x = 0$ owing to the boundary-layer character of this flow.

The turbulent rms velocity fluctuation of the particulate phase are compared with the data in Fig. 4 at the three stations for which experimental data are available. These are represented by marks, with the axial value (u'_p) distinguished from the lateral (v'_p) particle velocity rms value, while the predicted profiles are plotted with lines. The 2 lines at the bottom of the figures give u'_p and v'_p resulting from the standard response functions ($u'_p = (\surd C_k)u'_f$ with C_k from Eq. (8a), and $v'_p = (\surd C_k)v'_f$ with C_k from Eq. (8b); $u'_p \equiv (\overline{u'^2_p})^{1/2}$) without accounting for non-equilibrium during the induction period. The difference seen between these two curves is due to the continuity effect (Csanady 1963) embedded into the response functions, which makes the longitudinal velocity fluctuations to be slightly higher than the transversal fluctuations, but this difference is only marginal compared with that seen in the data. Otherwise the agreement between predicted and measured *lateral* rms velocity fluctuations of the particles is satisfactory. It is also stressed that for each station there is a value of y/R, at the edge of the inner particulate jet, beyond which there are no particles and the measurements have no meaning there.

Turning attention now to the main results of this work, the dashed line denoted “new” in Fig. 4 represents the predictions of the axial rms velocity fluctuations of the particles obtained with expression (10), which accounts for non-equilibrium during the induction period. The approximation $\overline{\mathfrak{v}^2_p} \simeq \overline{\mathbf{u}^2_p}$ has been introduced, stating that rms values in terms of Lagrangian velocities equal those in terms of Eulerian velocities. This assumption is exact for homogeneous isotropic turbulence (see Hinze, 1975) but it is only a required approximation in the present non-homogeneous case. The $\mathfrak{v}_{p0}$ velocity rms appearing in Eq. (10) is evaluated from the experimental data at inlet provided by Hishida and Maeda, and the $\mathfrak{v}_{p\infty}$ rms corresponds to the u'_p predicted without accounting for non-equilibrium (is given by the solid lines in Fig. 4). The elapsed or flying time,“t”, was evaluated from the numerical results as $t = \sum \delta x_i/\overline{u}_{pi}$, where δx_i is the width of cell i along the x-direction and the sum is taken over all cells, from inlet ($x = 0$) until the x position in consideration (for the given x/D station). It is seen from Fig. 4 that these new predictions which account for non-equilibrium following Friedlander's induction period notion, are indeed very close to the axial rms u'_p data. In the stations more downstream, i.e. x/D=10 and 20, the new predictions show a rather abrupt fall of

u'_p at y/R $\approx$ 1.2 which is not seen in the data which show a gradual decay of the particle velocity fluctuations. This fact is due to the shape of the imposed velocity fluctuations at inlet u'_{p0}, which must obviously drop to zero at the edge of the inner pipe (the one carrying particles, at inlet) and, as one moves downstream, that shape is mantained by the form of expression (10) and induces the observed profiles of u'_p. It might be possible in future work to include some diffusive term in Eq. (10), or to use other equation to account for non-equilibrium, such that the decay of u'_pon the edge of the particle jet is more gradual.

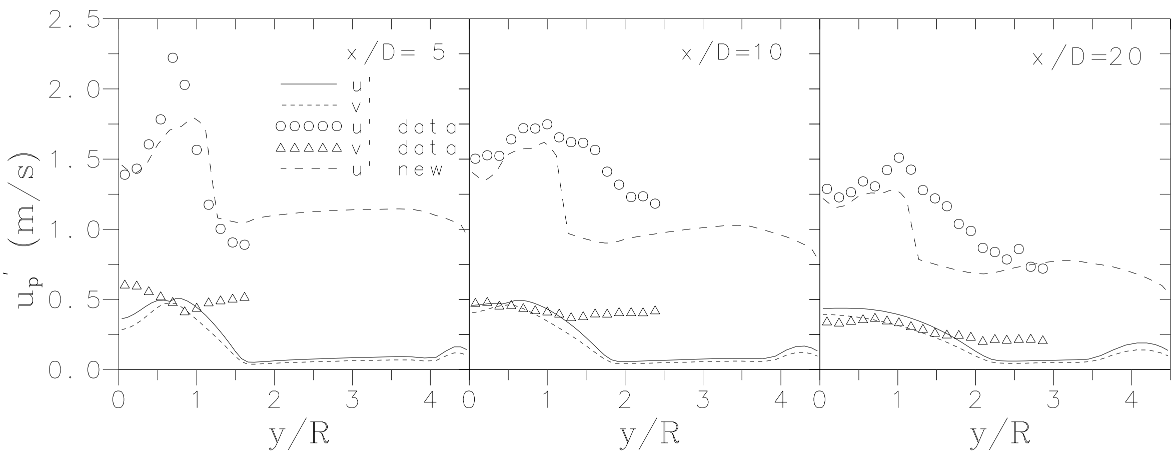

Fig. 4 Comparison of measured and predicted particle rms velocity fluctuations. "New" values based on induction period formulation.

6. Conclusions

The full two-fluid model equations are solved by a finite-volume method for the case of a confined air jet laden with solid glass particles. Turbulence effects are modelled by an extension to two-phase flows of the k-ϵ model and by response functions which explicitly and algebraically relate turbulent quantities pertaining to the dispersed phase to those of the continuous phase. Three known effects are accounted for by those response functions: inertia effect (heavier and larger particles have higher auto-correlations), crossing-trajectories effect (the presence of a mean slip reduces the particle correlation) and continuity effect (the correlation in the direction of the mean slip is higher than in the perpendicular direction).

This latter effect is however not sufficient to explain the large difference in the experimental measurements between the longitudinal (aligned with gravity, u'_p) and the transversal (v'_p) rms of the particles velocity fluctuations, and if v'_p is reasonably predicted u'_p is grossly underpredicted. It is shown that this discrepancy is due to non-equilibrium existing between the fluid turbulence and the particles response during an initial induction period which lasts for $0 < t/t_p < \simeq 3$ when the influence of the initial conditions is important. Following a proposal made by Friedlander back in 1957, it is shown that u'_p during the induction period can be seen as a weighted average between the inlet conditions u'_{p0} and the long-time (equilibrium) conditions $u'_{p\infty}$, in which the weights are a function of exp($-$ t/t$_p$) and also of the particle autocorrelation coefficient R_{pL}. For long times, these weights vanish and u'_p tends to $u'_{p\infty}$; for short times, the limit is u'_p tending to u'_{p0}. With this simple induction period model, the axial particle rms velocity fluctuations are well predicted, more so in the central part of the jet. Absence of diffusion in the model as it is, induces a sharp decay of u'_p at y/R $\approx$ 1.2, a feature not observed in the data except close to the injection plane. The induction period model is also too simplistic to account for more complex effects; for example, generation of particle turbulence by interaction of the mean particle shear with the Reynolds stresses is not accounted for and thus u'_p cannot be higher than the corresponding inlet value u'_{p0}. Still, the paper renders clear that non-equilibrium and initial-conditions effects are the most important to account for, in order to predict the right levels of axial particle velocity fluctuations.

Acknowledgements

This work has been supported by Junta Nacional de Investigação Científica e Tecnológica (JNICT-Portugal) under Project PBIC/C/CEG/2430/95.

References

Calabrese, R. V., and S. Middleman (1979) The Dispersion of Discrete Particles in a Turbulent Fluid Field, *AIChE. J.*, Vol. 25(6), pp. 1025-1035.

Csanady, G. T. (1963) Turbulent Diffusion of Heavy Particles in the Atmosphere, *J. Atm. Sci.*, vol. 20, pp. 201-208.

Friedlander, S. K. (1957) *A.I.Ch.E. Journal*, Vol. 3, p.381.

Hinze, J. O. (1975) *Turbulence*, 2^{nd} edition, MacGraw-Hill.

Hishida, K., and Maeda M. (1991), Turbulence characteristics of gas-solids two-phase confined jet (Effect of particle density). In *Proc. 5th Workshop on Two Phase Flow Predictions*, Erlangen, March 19-22, 1990, pp. 3-14.

Issa, R. I., and Oliveira P.J. (1993), Modelling of turbulent prediction in two phase flow jets. *Engineering Turbulence Modelling and Experiments 2*, Ed. W. Rodi and F. Martelli, Elsevier.

Issa, R. I., and Oliveira P.J. (1994), Numerical Prediction of Phase Separation in Two-Phase Flow Thorugh T-Junctions, *Computers and Fluids*, Vol. 23, pp. 347-372.

Issa, R.I., and Oliveira, P.J., (1997), Assessment of a Particle-Turbulence Interaction Model in Conjunction with an Eulerian Two-Phase Flow Formulation. Turbulence, Heat and Mass Transfer 2, Delft Univ. Press, pp. 759-770.

Jones, W. P., and B. E. Launder (1972) The Prediction of Laminarisation with a Two-Equation Model of Turbulence, *Int. J. Heat Mass Transf.*, Vol. 15, pp. 301-314.

Reeks, M. W., (1993), On the constitutive relations for dispersed particles in non uniform flows I. Dispersion in a simple shear flow. Physics of Fluids A, 5(4), pp. 750-761.

Simonin, O., Deutsch, E., and Boivin, M., (1993) Comparison of large eddy simulation and second-moment closure of particle fluctuacting motion in two-phase particle shear flows. 9th Symp. Turbulent Shear Flow, Kyoto, pp. 15.2.1-6.

Sommerfeld, M., and Wennerberg D. (Eds.) (1991), *Proc. 5th Workshop on Two Phase Flow Predictions*, Erlangen, KFA, March 19-22, 1990.

Tu, J.Y., and C.A.J. Fletcher (1996), Computational Modeling of Particle Dispersionm in a Backward-facing Step Flow. *ASME FED* Vol 236, pp. 283-289.

Wang, L-P., and D. E. Stock (1993) Dispersion of Heavy particles by Turbulent Motion, *J. Atmos. Sci.*, Vol. **50**, pp. 1897-1913.