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Assessment of a Particle – Turbulence Interaction Model in Conjunction with an Eulerian Two-Phase Flow Formulation

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Abstract – In the two-fluid model equations for dispersed two-phase flow, correlations arise from the phase averaging procedure relating the continuous phase fluctuations to those of the dispersed phase through response functions. Based on the work by Wang and Stock (1993) it has been possible to derive all required response functions, namely: C_k which relates the turbulence kinetic energy of the dispersed phase to that of the continuous phase, C_ν which relates the eddy viscosities (or diffusivities) of the two phases and C_i which relates the covariance of the velocity fluctuations of the two phases. These functions were implemented in a two-fluid model and applied to the prediction of dispersion in a confined air jet, laden with two particle types. Analysis of the results clarifies the differences between the new functions and those used in previous work. One obvious advantage of the proposed response model is that all functions are based on a single theory, which was not the case before.

1. Introduction

Prediction of heavy particle dispersion by fluid turbulence is relevant in a number of engineering applications. In engineering calculations it is important to keep the turbulence model as simple as possible but still able to capture the main mechanisms responsible for dispersion and enable reasonable predictions of the mean flow quantities. In previous work (Issa and Oliveira 1993, 1995) the single-phase k-ϵ model has been extended to two phase flows and shown to be capable of predicting mean velocities and particle concentration in jets of air laden with solid particles. When the phase averaged two-fluid model equations are derived a number of particle-related turbulent correlations arise, namely $\overline{\alpha_d \mathbf{u}'_d}$, $\overline{\alpha_c \mathbf{u}'_c}$, $\overline{\mathbf{u}'_d \mathbf{u}'_d}$ and $\overline{\mathbf{u}'_d \mathbf{u}'_c}$ ($\mathbf{u}'$ is the velocity fluctuation and α is the volume fraction of the continuous c, or dispersed d, phases). These must be related to fluid correlations which are known from the turbulence model. In past work we have introduced response functions: for example the particle turbulent kinetic energy was related to the fluid kinetic energy by $k_d = C_k k_c$ and similarly the particle and fluid eddy viscosities were related by $\nu_d^t = C_\nu \nu_c^t$. In Issa and Oliveira (1995) an expression for C_k was derived from a simplified momentum equation. It could capture the anisotropy of the particle velocity variance (arising from the lift force on the particle) but tended to the well known result of Tchen (see Hinze, 1975) for particulate flows in which the density ratio is high ($\sim 10^3$). Hence the particle turbulent intensity was only affected by inertia and not by possible existence of a mean drift between the two phases. Following the same approach, it has been impossible to derive an expression for C_ν. In the scope of Tchen's theory the eddy diffusivity of the particle equals that of the fluid, thus $C_\nu = 1$. However, it is well known that there is a loss of particle correlation due to a finite mean relative velocity between the phases ($\overline{u}_r$) and so the particle diffusivity should be less than that of the fluid. This was realised by Yudine (1959) who called it the crossing-trajectories effect, and was later quantified by Csanady (1963) in a way equivalent to putting $C_\nu = 1/\surd(1 + C_\beta \overline{u}_r^2/u_c'^2)$, the expression used in the previous studies referred to above. It is thus clear

that the formulation of C_ν appears completely decoupled from that of C_k, an unsatisfactory state since both are for sure related to the particle/fluid interaction by turbulence.

Recently, Wang and Stock (1993) derived an analytical formulation which enables the calculation of the particle turbulence characteristics in terms of measurable fluid turbulence quantities, in homogeneous isotropic flow. Their work differs from existing theories of particle dispersion in isotropic turbulence (Reeks 1977; Pismen and Nir 1978 and Nir and Pismen 1979) in that the results are in terms of explicit algebraic equations for the dispersion statistics of heavy particles, whereas the other authors use integral equations of difficult solution. Wang and Stock's analysis accounts for both finite particle inertia (measured by a Stokes number) and finite particle drift (measured by the crossing-trajectories parameter $\gamma = \overline{u}_r/u'_c$, where u' is the rms velocity fluctuations).

The objective of the present paper is to adapt the Wang and Stock model to the two-fluid formulation. This entails interpreting the correlations in the two-phase flow equations in terms of Wang and Stock's rms velocity ratios and particle long-time diffusion coefficients, so as to be able to define the required response functions, C_k, C_ν and C_i. The final two-fluid model is applied to the prediction of a confined jet flow laden with two types of heavy particles, and the model performance is assessed by comparison with both experimental data (Hishida and Maeda, 1991) and the previously used formulation (Issa and Oliveira 1995).

2. Equations and Turbulence Modelling

2.1. The phase averaged equations of motion

The phase averaged equations are formulated by carrying out volume averaging first followed by time averaging or by applying ensemble averaging. The resulting continuity and momentum equations for each phase (k=c for the continuous phase and $k = d$ for the dispersed phase), for steady flow are:

$$\nabla \cdot (\overline{\alpha}_k \rho_k \widetilde{\mathbf{u}}_k) = 0 \tag{1}$$

and

$$\nabla \cdot (\overline{\alpha}_k \rho_k \widetilde{\mathbf{u}}_k \widetilde{\mathbf{u}}_k) = -\overline{\alpha}_k \nabla \widetilde{p} + \overline{\alpha}_k \nabla \cdot \widetilde{\tau}_k + \nabla \cdot \overline{\alpha}_k \widetilde{\tau}_k^{\,t} + \rho_k \overline{\alpha}_k \mathbf{g} + \overline{\mathbf{F}_{D_k}} \tag{2}$$

The dependent variables are the average pressure $\widetilde{p}$, assumed to be the same acting on both phases, the time averaged volume fractions $\overline{\alpha}_k$, and the phase averaged velocities defined as:

$$\widetilde{\mathbf{u}}_k = \frac{\overline{\alpha_k \mathbf{u}_k}}{\overline{\alpha}_k}, \tag{3}$$

an expression akin to Favre-averaging in variable density flow and where the overbar denotes simple time-averaging. It is assumed that the instantaneous velocity can be decomposed in the following alternative ways:

$$\mathbf{u}_k = \widetilde{\mathbf{u}}_k + \mathbf{u}''_k = \overline{\mathbf{u}}_k + \mathbf{u}'_k \tag{4}$$

where the phase-averaged and the usual time-averaged velocity fluctuations are related by the expressions:

$$\overline{\mathbf{u}}_k = \widetilde{\mathbf{u}}_k + \overline{\mathbf{u}''_k}\, ; \quad \overline{\mathbf{u}''_k} = -\overline{\alpha_k \mathbf{u}'_k} \,/\, \overline{\alpha}_k \, ; \; (u''_k)^2 = (u'_k)^2 + \overline{\mathbf{u}''_k} \cdot \overline{\mathbf{u}''_k} \tag{5}$$

in which u represents the rms (e.g. $u' \equiv \sqrt{(\mathbf{u}' \cdot \mathbf{u}')}$). In Eq. (2), $\mathbf{F}_D$ is the drag force per unit volume, the only interphase momentum transfer relevant to the present application (since the density ratio $\rho_d/\rho_c \gg 1$, see Hinze 1975), τ is the molecular Newtonian stress and τ^t is the turbulent stress tensor given by:

$$\tilde{\tau}^{\,t}_{\,k} = -\frac{\overline{\rho_k \alpha_k \mathbf{u}''_k \mathbf{u}''_k}}{\overline{\alpha_k}}, \tag{6}$$

which is an outcome of the averaging procedure. In the next sub-sections only an overview of the turbulence modelling assumptions and equations required for closure are given, since this matter has been treated in detail in previous work. The response functions, to be derived from Wang and Stock (1993), are the main interest of the present work and are analysed with more detail in sub-section 2.7.

2.2. Turbulent stress tensor

The phase-averaged turbulent stress tensor in Eq. (6) is considered to be approximately equal to the time averaged turbulent stress (Oliveira, 1992),

$$\tilde{\tau}^{\,t}_{\,k} \simeq \overline{\tau}^{\,t}_{\,k} \quad \Leftrightarrow \quad -\frac{\overline{\rho_k \alpha_k \mathbf{u}''_k \mathbf{u}''_k}}{\overline{\alpha_k}} \simeq -\rho_k \overline{\mathbf{u}'_k \mathbf{u}'_k}$$

and the same holds for the turbulence kinetic energy,

$$\tilde{\mathrm{k}}_{\,k} \simeq \overline{\mathrm{k}}_{\,k} \quad \Leftrightarrow \quad \tfrac{1}{2}\frac{\overline{\alpha_k \mathbf{u}''_k \cdot \mathbf{u}''_k}}{\overline{\alpha_k}} \simeq \tfrac{1}{2}\overline{\mathbf{u}'_k \cdot \mathbf{u}'_k}\,.$$

Thus, by invoking the Boussinesq linear stress – strain realtionship, $\tilde{\tau}^{\,t}$ is approximated by:

$$\tilde{\tau}^{\,t}_{\,k} = \mu^t_k(\nabla \overline{\mathbf{u}}_{\,k} + \nabla \overline{\mathbf{u}}^{\,\mathrm{T}}_{\,k}) - \tfrac{2}{3}(\mu^t_k \nabla \cdot \overline{\mathbf{u}}_{\,k} + \rho_k \overline{\mathrm{k}}_{\,k})\delta, \tag{7}$$

where δ is the identity tensor. It is noted that the velocity gradients in (7) are expressed in terms of the time-averaged velocity vector, which must be related to the phase-average values by Eqs. (5) in order to obtain an explicit relation in the dependent variable $\tilde{\mathbf{u}}$. The turbulent stresses from (7) are inserted into the momentum equations (2) which become differential equations for the three velocity components of each phase $\tilde{\mathbf{u}}_{\,k}$. To solve these equations, we need to specify the eddy viscosities, μ^t_k, and the turbulence kinetic energy, $\overline{\mathrm{k}}_{\,k}$, which result from the k-ϵ equations.

2.3. The k and ϵ equations

The α-weighted equations for the transport of turbulence kinetic energy (k) and its rate of dissipation (ϵ), for the continuous phase, are written as:

$$\nabla \cdot (\overline{\alpha}_{\,c}\rho_c \tilde{\mathbf{u}}_{\,c}\tilde{\mathrm{k}}_{\,c}) = \nabla \cdot (\overline{\alpha}_{\,c}\frac{\mu^t_c}{\sigma_k}\nabla \tilde{\mathrm{k}}_{\,c}) + \overline{\alpha}_{\,c}(\mathrm{G} - \rho_c \tilde{\epsilon}_{\,c}) + \mathrm{S}^k_d \tag{8}$$

$$\nabla \cdot (\overline{\alpha}_{\,c}\rho_c \tilde{\mathbf{u}}_{\,c}\tilde{\epsilon}_{\,c}) = \nabla \cdot (\overline{\alpha}_{\,c}\frac{\mu^t_c}{\sigma_\epsilon}\nabla \tilde{\epsilon}_{\,c}) + \overline{\alpha}_{\,c}\frac{\tilde{\epsilon}_c}{\tilde{\mathrm{k}}_c}(\mathrm{C_1 G} - \mathrm{C_2}\rho_c \tilde{\epsilon}_{\,c}) + \mathrm{S}^\epsilon_d. \tag{9}$$

where the additional terms S_d account for the interaction between dispersed particles with the continuous phase turbulence. The model constants are standard ($C_2 = 1.92$, $C_\mu = 0.09$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.22$) except C_1 which takes the value of 1.60, based on Wood and Chen's (1985) calculations of round jets.

The above equations enable calculation of the eddy viscosity ($\mu_c^t = \rho_c C_\mu \widetilde{k}\,_c^2/\widetilde{\epsilon}\,_c$) and the turbulence energy of the continuous phase. Within the simplified model considered here, the turbulent quantities pertaining to the dispersed phase do not result from solution of modelled differential equations, but must be obtained via prescribed algebraic relations. Thus, the eddy viscosity and turbulence energy of the dispersed phase are related to the corresponding continuous phase values by:

$$\mu_d^t = C_\nu \, \mu_c^t \tag{10}$$

and

$$\widetilde{k}\,_d = C_k \widetilde{k}\,_c\,, \tag{11}$$

which represent the definition of the response functions for the kinetic energy (or the square of the rms velocity) and for the turbulent viscosity.

2.4. Transport of α_d by turbulence

Similar to the transport of momentum by turbulence, correlations involving volume fraction and velocity fluctuations, which represent transport of particles by turbulence, are modelled assuming the gradient eddy diffusion hypothesis:

$$\overline{\alpha_d \mathbf{u}'_c} = -\eta_c \nabla \overline{\alpha}\,_d, \tag{12}$$

$$\overline{\alpha_d \mathbf{u}'_d} = -\eta_d \nabla \overline{\alpha}\,_d. \tag{13}$$

These equations can be viewed as a definition of the turbulent diffusivities, η_k, which will be interpreted as the usual particle dispersion coefficients in section 2.7, and will then be related to the turbulent viscosities via appropriate eddy Schmidt numbers thus enabling their calculation.

2.5. Modelling the drag force

Drag is modelled assuming that the dispersed phase is a cloud of small spherical particles resulting in the linearised form:

$$\overline{\mathbf{F}}\,_{D_c} = C_f \overline{\alpha}\,_d \overline{\alpha}\,_c (\overline{\mathbf{u}}\,_d - \overline{\mathbf{u}}\,_c) \quad (C_f \equiv \frac{18\mu_c f(Re_p)}{d_p^2};\, f = 1 + 0.15 Re_p^{0.687},\; Re_p \equiv \frac{\rho_c \overline{u}_r d_p}{\mu_c}) \tag{14}$$

The relaxation time scale of an heavy particle, a measure of its inertia, is proportional to the inverse of the drag function C_f and is given by:

$$t_p = \frac{\rho_d \, d_p^2}{18\,\mu_c \, f(Re_p)} \tag{15}$$

After replacing the time average velocities in (14) by phase averaged ones, with the aid of Eq. (5), and recalling the gradient diffusion assumption (Eqs. 12 and 13), the drag term acting on the continuous phase becomes:

$$\overline{\mathbf{F}_{D_c}} = C_f \left(\overline{\alpha}_d \overline{\alpha}_c (\widetilde{\mathbf{u}}_d - \widetilde{\mathbf{u}}_c) + (\overline{\alpha}_c \eta_d + \overline{\alpha}_d \eta_c) \nabla \overline{\alpha}_d \right). \tag{16}$$

The drag on the dispersed phase is given by the same expression with opposite sign. It is then clear that drag can be decomposed into mean drag (first term on rhs of Eq. 16) plus turbulent drag (second term, proportional to the gradient of mean volume fraction). In Issa and Oliveira (1996) it is shown that the balance between these two terms can be re-cast as a diffusion equation, in the limit of a passive scalar dispersed phase, highlighting the importance of the turbulent drag term in being responsible for dispersion within the two-fluid model formulated with phase averaged velocities.

2.6. Modelling the additional terms in the k and ϵ equations

The additional source term in the k equation arises from the time average of the inner product between the instantaneous external forces and the fluctuating continuous-phase velocity. The external forces are drag (Eq. 14) and gravity, and the additional term becomes:

$$S_d^k = \overline{(\mathbf{F}_{D_c} + \rho_c \alpha_c \mathbf{g}) \cdot \mathbf{u}_c''} \simeq \\ - C_f \left(\eta_c \nabla \overline{\alpha}_d \cdot \overline{\mathbf{u}}_r + 2 \overline{\alpha}_c \overline{\alpha}_d \widetilde{k}_c (1 - C_i) - \eta_c (\eta_c - \eta_d) \nabla \overline{\alpha}_d \cdot \nabla \overline{\alpha}_d \right) \tag{17}$$

when using the usual gradient diffusion assumptions (see Issa and Oliveira, 1996). In this equation, C_i is another response function which arises from the derivation of the term. It is similar to the C_k of Eq. (11) but relates the velocity fluctuations of one phase with those of the other phase as:

$$C_i = \frac{\overline{\mathbf{u}_c' \cdot \mathbf{u}_d'}}{\overline{\mathbf{u}_c' \cdot \mathbf{u}_c'}}. \tag{18}$$

The source term in the k equation induces a dissipation equal to that term divided by the turbulence time scale ($\propto k/\epsilon$). Hence the additional source in the ϵ-equation is modelled as:

$$S_d^\epsilon = -C_3 \frac{\widetilde{\epsilon}_c}{\widetilde{k}_c} 2 C_f \overline{\alpha}_c \overline{\alpha}_d \widetilde{k}_c (1 - C_i), \tag{19}$$

where only the leading term in (17) is retained. The model constant C_3 is taken as unity in the absence of any knowledge of its actual magnitude.

2.7. Response functions: C_k, C_i and C_ν

In Issa and Oliveira (1995) the following C_k function has been derived from an approximate analysis of the particle momentum equation:

$$C_k = \frac{3 + \beta}{1 + \beta + 2\rho_d/\rho_c} \tag{20}$$

where β defined in the original reference can also be written as $\beta = 2(\rho_d/\rho_c)(t_p/t_e)$, with t_e being the time scale of the large, energy containing, eddies ($t_e = l_e/u'_c$). In the case of particulate flow, in which $\rho_d/\rho_c \gg 1$, the above formula reduces to the well known Tchen result:

$$C_k = \frac{1}{1 + t_p/T_L}, \qquad (21)$$

provided the turbulence time scale t_e is identified as the Lagrangian integral time scale of the fluid, calculated as $T_L = 0.41(k/\epsilon)$ (Calabrese and Middleman, 1979). From Tchen's theory it is also possible to conclude that $C_i = C_k$ and that, for long time dispersion, the C_ν-function simply becomes equal to unity. However, it is known that the existence of a mean relative velocity between the phases reduces the particle diffusivity (i.e., C_ν) and also affects the particle turbulent intensity (i.e., C_k), a fact which is not accounted for in the previous analysis. Wang and Stock (1993) developed a more satisfactory analysis for the case of homogeneous isotropic turbulence, where due account is taken of both effects of particle inertia and crossing trajectories, and from which it is possible to derive the response functions required here. They give the ratio of the particle to the fluid turbulent intensities from which C_k is readily derived upon squaring and summation:

$$C_k = \frac{1+St_T(\Gamma - \gamma m_T/3)}{(1+St_T\Gamma)^2} \quad (\text{with } \Gamma \equiv \surd(1 + (m_T\gamma)^2)) \qquad (22)$$

where St_T is a Stokes number defined as $St_T = t_p/T$ where T is the fluid integral time scale seen by a particle in the absence of relative velocity. In their analysis the particle turbulent intensity component aligned with the gravity vector (here considered the x-direction) turns out to be higher than those along the other two perpendicular directions. This arises because of the existence of a mean slip, which is measured by the non-dimensional crossing-trajectories parameter, $\gamma \equiv \overline{u}_r/u'_c$. It is then possible to distinguish between an axial (along x), and a radial (along y), C_k-function:

$$C_{kx} \equiv \frac{\overline{u'_d u'_d}}{u'^2_c} = \frac{1}{1+St_T\Gamma}\,; \quad C_{ky} \equiv \frac{\overline{v'_d v'_d}}{u'^2_c} = \frac{1+St_T(\Gamma - 0.5\gamma m_T)}{(1+St_T\Gamma)^2} \qquad (23)$$

where it is obvious that $C_{kx} > C_{ky}$; the total C_k in (22) comes from $C_k = \frac{1}{3}(C_{kx} + 2C_{ky})$. In other studies (e.g. in Csanady 1963, or Meek and Jones 1973), T has been identified with T_L, which is true only in the limit of negligible inertia; in fact the range of variation of T spans from T_L to T_E (the Eulerian fluid integral time scale in a frame moving with the mean fluid velocity) as inertia increases. Wang and Stock correlated T with the standard Stokes number ($St = t_p/T_E$) and obtained the empirical equation:

$$\frac{T}{T_E} = \left(1 - \frac{1-\beta}{(1+St)^{0.4(1+0.01St)}}\right) \qquad (24)$$

based on a DNS simulation in which the time scale ratio $\beta \equiv T_L/T_E$ was 0.356 . The Lagrangian time scale is difficult to measure experimentally but Sato and Yamamoto (1987) also found that β is less than one, $\beta \approx 0.3 - 0.6$. The other dimensionless parameter in Eqs. (22) and (23) is $m_T \equiv m(T/T_E)$ where m characterises the structure of the fluid turbulence, being the ratio of the following measurable quantities, $m = T_E/t_e = T_E u'_c/L_f$ (L_f: integral length scale; $t_e = L_f/u'_c$ is the eddy turnover time). The typical range of variation of m lies between 0.1 and 10, but it is usually taken as 1 (e.g. in Wang and Stock).

As for the eddy diffusivity, η_d (in Eq. 13) is identified with the particle dispersion coefficient ϵ^p_{ii} of Wang and Stock, who give an expression for $\epsilon^p_{ii}/u_c'^2\mathrm{T}$ in terms of $\mathrm{St_T}$, γ and $\mathrm{m_T}$. The diffusivity of a fluid element (a tracer) is given by $\eta_0 = u_c'^2\mathrm{T_L}$, according to classical Taylor's diffusion theory; it is then possible to write $\mathrm{C}_\eta \equiv \eta_d/\eta_0$ as:

$$\mathrm{C}_{\eta x} \equiv \frac{\eta_d}{\eta_0} = \frac{1}{\Gamma}\,\frac{\mathrm{T}}{\mathrm{T_E}}\,\frac{1}{\beta}\ ;\ \text{and } \mathrm{C}_{\eta y} = \frac{1-0.5\mathrm{m_T}\gamma/\Gamma}{\Gamma}\,\frac{\mathrm{T}}{\mathrm{T_E}}\,\frac{1}{\beta} \tag{25}$$

based on Wang and Stock's expressions for the diffusivity in the x- and y-directions. The other diffusivity, η_c in Eq. (12), is interpreted as the turbulent diffusivity of a fluid element which follows a particle trajectory, thus $\eta_c = u_c'^2\mathrm{T}^f$ where T^f is the fluid integral time scale seen by the particle. Since, in general, $u_c'^2\ \mathrm{T}^f = u_d'^2\ \mathrm{T}^p$ (Eq. 2.12 in Wang and Stock; also contained in the analysis of Reeks, 1977, and Pismen and Nir 1978), we reach the conclusion that:

$$\eta_c = \eta_d\,. \tag{26}$$

In order to relate diffusivities to turbulent viscosities, the usual Schmidt number for a passive scalar is introduced, $\eta_0 = \nu_c^t/\sigma_0$; by analogy, for turbulent diffusion of particles we write $\eta_d = \nu_d^t/\sigma_\alpha$. If now the approximation $\sigma_0 \approx \sigma_\alpha$ is invoked (note that the particle Schmidt number is often defined in different ways), then we have $\eta_d/\eta_c = \nu_d^t/\nu_c^t$ and the C_ν function defined by Eq. (10) coincides with the C_η of Eq. (25): $\mathrm{C}_\nu = \mathrm{C}_\eta$.

The remaining response function, C_i, correlates the velocity fluctuations of each phase and requires a lengthy analytical manipulation to be obtained, akin to what Wang and Stock did to obtain the particle autocorrelation $\mathrm{R}^p(\tau)$. Denoting ensemble averaging by angle brackets and using superscript p to denote that $\mathbf{u}_c'^p$ is the fluid velocity along the particle trajectory, the fluid – particle cross-correlation, from which C_i is obtained, becomes:

$$\mathrm{R}^{fp}(\tau) = <\mathbf{u}_c'^p(0)\mathbf{u}_d'(\tau)> = <\mathbf{u}_c'^p(0)\int_{-\infty}^{\tau}\frac{\mathbf{u}_c'^p(\tau_1)}{\mathrm{t}_p}\exp\left(\frac{\tau_1-\tau}{\mathrm{t}_p}\right)\mathrm{d}\tau_1> =$$

$$= \int_{-\infty}^{\tau}\frac{<\mathbf{u}_c'^p(0)\ \mathbf{u}_c'^p(\tau_1)>}{\mathrm{t}_p}\exp\left(\frac{\tau_1-\tau}{\mathrm{t}_p}\right)\mathrm{d}\tau_1 = \int_{-\infty}^{\tau}\frac{\mathrm{R}^f(\tau_1)}{\mathrm{t}_p}\exp\left(\frac{\tau_1-\tau}{\mathrm{t}_p}\right)\mathrm{d}\tau_1$$

where the particle velocity was obtained from a first integration of the particle's instantaneous Lagrangian equation of motion and $\mathrm{R}^f(\tau)$ is the fluid velocity correlation along the particle trajectory. Explicit expressions for this correlation is given in Wang and Stock; for example, for the x-direction (along the mean slip) which corresponds to the simplest expression, it is $\mathrm{R}^f(\tau) = u_c'^2\ \exp(-\Gamma\mid\tau\mid/\mathrm{T})$ (Γ as defined in Eq. 22) . Substitution of R^f from this expression in the above equation and integration yield:

$$\mathrm{R}_x^{fp}(\tau) = \frac{u_c'^2}{1+\Gamma\,\mathrm{t}_p/\mathrm{T}}\exp\left(-\Gamma\,\frac{\tau}{\mathrm{T}}\right),$$

By definition the C_i function along x-direction is simply $\mathrm{R}^{fp}(0)/u_c'^2$, so we reach the conclusion that C_i equals C_k, being given by expressions (23), similarly to what happened in the simpler theory of Tchen. It is noteworthy that this result ($\overline{\mathbf{u}_c'\cdot\mathbf{u}_d'} = \overline{\mathbf{u}_d'\cdot\mathbf{u}_d'}$) is valid even if the

corresponding velocity cross-correlation $R^{fp}(\tau)$ differs from the particle correlation $R^p(\tau)$ for a time gap different from zero (i.e. $R^{fp}(0) = R^f(0)$, but $R^{fp}(\tau) \neq R^f(\tau)$ for $\tau \neq 0$).

3. Results

Assessment of the two-fluid model embodying the new response functions described above has been done by carrying out numerical simulations and comparing predictions with the laden jet measurements of Hishida and Maeda (1991). The averaged transport equations for momentum and mass of each phase, and for the turbulence kinetic energy and turbulence dissipation of the continuous phase, are solved by a finite-volume numerical methodology. The flow configuration is shown in Fig. 1 and consists of a central pipe (D = 13 mm) carrying air and particles, issuing vertically downwards into a larger concentric pipe (D_2 = 30 mm) which carries a clean air stream at a lower velocity. Hishida and Maeda measured the mean and fluctuating velocity and the Reynolds stress of the two phases at four axial stations (x/D = 0, 5, 10 and 20), and the mass flux of the dispersed phase at the same stations except x/D = 5. Their measurements at x = 0 were used to prescribe the inlet conditions for the numerical calculations, with k evaluated as $k = \frac{1}{2}(u'^2 + 2v'^2)$ and ϵ estimated from $\epsilon = C_\mu k^{1.5}/0.03L$ ($L = D$ for $r < D/2$, and $L = (D_2 - D)/2$ for $r > D/2$).

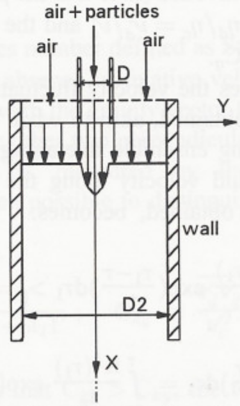
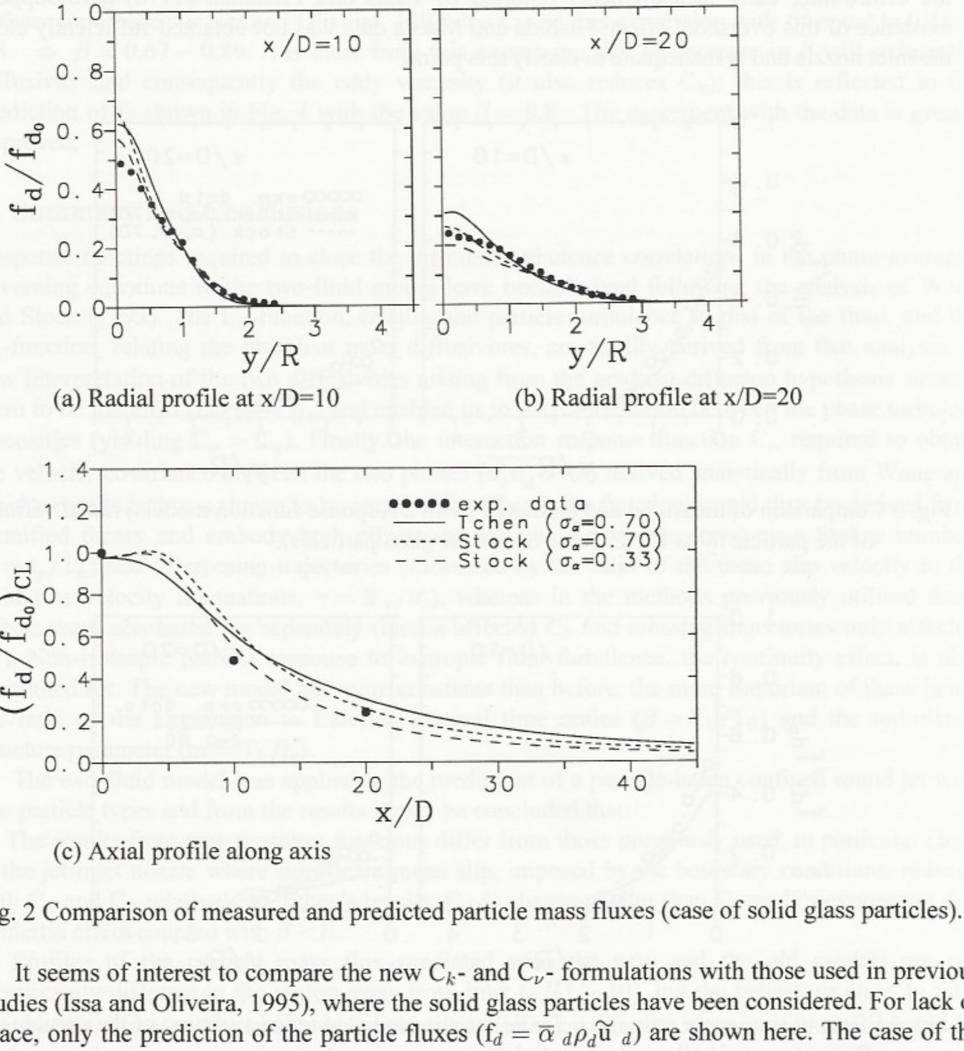


Fig.1 Sketch of the flow configuration.

Two types of particles have been considered: solid glass with average diameter 64.4 μ and density ρ_d = 2590 kg/m^3, and hollow glass with $d_p = 80.1\mu$ and ρ_d = 280 kg/m^3. The volume fractions in both cases were approximately the same ($\overline{\alpha}_{d0} \sim 2.5\ 10^{-4}$) and so the mass loading were approximately in the proportion to the particle densities: $\dot{m}_d/\dot{m}_c \simeq 0.30$ (solid glass) and 0.033 (hollow glass). The former case is thus expected to induce higher changes to the mean and turbulent fields of the single phase flow (clean air) and this was confirmed in the simulations, with a comparatively higher reduction of the turbulence intensity for the solid glass spheres. The asymptotic relaxation times based on a settling velocity from the drag law Eq. (15) are $t_{p\infty}$ = 28.4 ms for the solid glass and 5.2 ms for the hollow glass. The actual relaxation times are lower and vary across the flow, $t_p \approx$ 10 to 20 ms for the solid glass and $\approx$ 1 to 4 ms for the hollow glass. These times can be compared with the typical fluid turbulence time scale given by T_L from Eq. (21), which varied from 1.5 ms (at x/D $\simeq$ 0 to 5) to 3 ms (at x/D = 20) along the axis. If the importance of particle inertia is measured by a Stokes number defined as t_p/T_L, it is

thus clear that the solid particles have high inertia ($t_p/T_L \approx 10$) whereas the hollow glass particles have relatively small inertia ($t_p/T_L \approx 1$ to 2.5).



(a) Radial profile at x/D=10

(b) Radial profile at x/D=20

(c) Axial profile along axis

Fig. 2 Comparison of measured and predicted particle mass fluxes (case of solid glass particles).

It seems of interest to compare the new C_k- and C_ν- formulations with those used in previous studies (Issa and Oliveira, 1995), where the solid glass particles have been considered. For lack of space, only the prediction of the particle fluxes ($f_d = \overline{\alpha}_d \rho_d \widetilde{u}_d$) are shown here. The case of the solid glass particles is shown in Fig. 2, where 2 radial profiles (at x/D = 10 and 20) and the axial profile along the axis are plotted. This case has been predicted before (Issa and Oliveira, 1995) and the curve marked "Tchen" corresponds to those predictions, where C_k was given by Eq. (20) and C_ν was taken as unity. The dashed lines are the results of the newly implemented Wang and Stock model with two Schmidt numbers, of 0.7 and 0.33 (this particular value arises from: $\sigma_\alpha \equiv \nu_c^t/\eta_0 = (C_\mu k^2/\epsilon)/(u_c'^2 T_L) = (C_\mu k^2/\epsilon)/(\frac{2}{3}k \times 0.41k/\epsilon) = 0.33$) . It is seen that the particle fluxes predicted with the new response functions are closer to the data, especially for the station away from the inlet nozzle ($x/D \simeq 20$). Closer to inlet, the differences between model predictions are accentuated, with the new formulation yielding a longer "potential" core with a slight overshoot of f_d relatively to its inlet value at centre-line (f_{d0}). This behaviour may be

explained by the lower C_k in that region compared with Tchen's value, a consequence of the crossing-trajectories effect on C_k, leading to lower lateral particle fluctuations and thus less dispersion. If this effect is strong enough, it may lead to a localised agglomeration of particles in the centre-line; early measurements reported by Laats and Frishman (1970) give support to existence of this overshoot effect. Hishida and Maeda data was not obtained sufficiently close to the inlet nozzle and is inadequate to clarify this point.

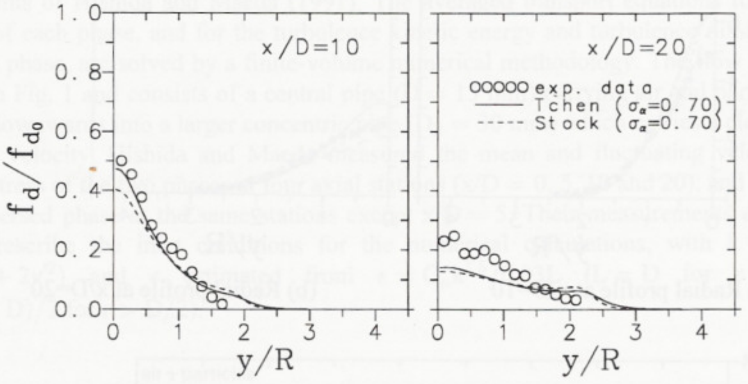


Fig. 3 Comparison of measured and predicted (with 2 response function models) radial variation of the particle mass fluxes (case of hollow glass particles).

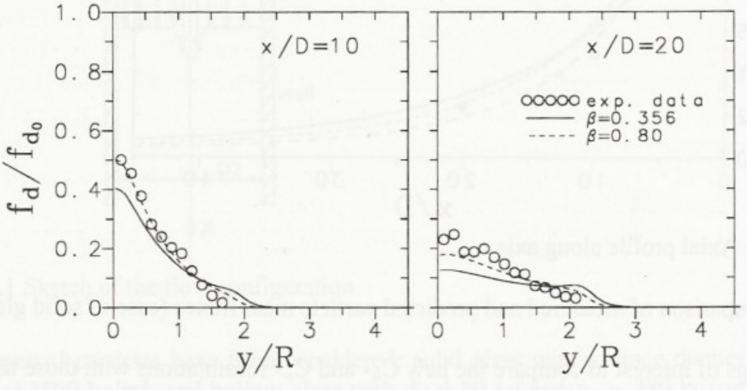


Fig. 4 Effect of time scale ratio ($\beta = T_L/T_E$) on the predicted radial variation of the particle mass fluxes (case of hollow glass particles).

Predictions of mass fluxes for the hollow glass particles are compared with the data in Figs. 3 and 4. Wang and Stock response functions with standard constants and a Schmidt number of 0.7 gives results similar to Tchen's model; both overpredict the particle dispersion at $x/D = 20$. It is remarked that if the experimental data for the two particle types is plotted together they almost coincide which is contrary to expectations. Existing theories would predict more dispersion of the lighter particles since the fluid field is approximately the same and the smaller relaxation time of these particles implies a faster equilibration with the fluid, thus leading to a reduced crossing-trajectories effects and to an increased dispersion coefficient. The new model has more empirical

coefficients (namely, β and m) and at this stage it is unknown how correct are the values assigned by Wang and Stock. The parameter β is very important in determining the magnitude of the inertia effect; the value $\beta = 0.356$ might be too low from the following consideration. For low inertia, the $C_{\nu x} = C_{\eta x}$ from Eq. (25) becomes equal to Csanady result $1/\sqrt{(1+\beta^2\gamma^2)}$ and other authors (references in Issa and Oliveira, 1993) have used this expression with β^2 equal to 0.45 or $0.8 \ \Rightarrow\ \beta = 0.67 - 0.89$. It is clear from this expression that an increase in β will reduce the diffusivity and consequently the eddy viscosity (it also reduces C_k); this is reflected in the prediction of f_d shown in Fig. 4 with the value $\beta = 0.8$. The agreement with the data is greatly improved.

4. Summary and Conclusions

Response functions required to close the different turbulence correlations in the phase-averaged governing equations in the two-fluid model have been derived following the analysis of Wang and Stock (1993). The C_k-function, relating the particle turbulence to that of the fluid, and the C_η-function, relating the turbulent mass diffusivities, are readily derived from that analysis. A new interpretation of the two diffusivities arising from the gradient diffusion hypothesis showed them to be identical (i.e. $\eta_c = \eta_d$) and enabled us to derive a relation between the phase turbulent viscosities (yielding $C_\nu = C_\eta$). Finally, the interaction response function C_i, required to obtain the velocity covariance between the two phases ($\overline{\mathbf{u}'_c\mathbf{u}'_d}$), was derived analytically from Wang and Stock's results and was shown to be equal to C_k. These new functions could thus be derived from a unified theory and embody both effects, of particle inertia (measured by a Stokes number, $St = t_p/T_E$) and of crossing-trajectories (measured by the ratio of the mean slip velocity to the fluid rms velocity fluctuations, $\gamma = \overline{u}_r/u'_c$), whereas in the methods previously utilised these effects were accounted for separately (inertia affected C_k and crossing-trajectories only affected C_ν). Non-isotropic particle response to isotropic fluid turbulence, the continuity effect, is also accounted for. The new model has more constants than before, the more important of these being the ratio of the Lagrangian to Eulerian integral time scales ($\beta = T_L/T_E$) and the turbulence structure parameter ($m = T_E/t_e$).

The two-fluid model was applied to the prediction of a particle-laden confined round jet with two particle types and from the results it may be concluded that:

- The results from new response functions differ from those previously used, in particular close to the jet inlet nozzle where significant mean slip, imposed by the boundary conditions, reduces both C_k and C_ν relatively to Tchen's results. C_ν is always greater than Csanady's expression due to inertia effect coupled with $\beta < 1$.
- Profiles of the particle mass flux predicted with the new and the old models are not significantly different in the region away from inlet ($x/D \gtrsim 10$), but the behaviour close to inlet is markedly different. The new model yields less dispersion due to a longer penetration core and a slight overshoot near inlet, for the heavier particles.
- The flux data for the lighter particle could be better matched when the prediction were made with $\beta = 0.8$ instead of $\beta = 0.356$. There is no consensus on the values for β, which may range between $0.3 - 1.0$. When β is small the diffusivity of the particles is greater than that of the fluid (Wang and Stock 1993 and Reeks 1977).

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