Modeling of hypersonic flow on a MPD thruster geometry using a PISO based method

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In the following paper we intend to present some preliminary results for a self-field MPD thruster obtained with a new algorithm developed for solving the resistive magnetohydrodynamics (MHD) equations. The numerical method is based on the well known PISO algorithm and make use of AUSM-MHD scheme for flux calculation. The model is based on a single fluid approach and, in order to obtain a more realistic and stable solution, an equation of state for real gas is also included.

I. Introduction

Nowadays numerical codes are very important tools for the research and project phase of electric propulsion systems. The application of such codes for modeling magnetoplasmadynamic thrusters is very useful because it allows us to understand relevant flow-field parameters that are very difficult to visualize and analyze in experiments. However, modeling the complex physics involved in these devices is a non-trivial task. Some of the models that were developed are based in a multi-species formulation[1] while others assume that the plasma behave as a single fluid.[2] But, assuming that a plasma is a single conducting fluid leads to some ambiguity in the definitions of plasma transport coefficients, because these are usually defined in a different way for electrons and ions. Another approach is to consider the plasma as a single fluid but with different temperatures, for heavy species and electrons.[3] With this last approach it is possible to compatibilize the simplicity of the single fluid approach and, at the same time calculate, the two distinct transport coefficients in a more realistic way.[4]

In the following paper we intend to present some preliminary results for a self-field MPD thruster. These were obtained with a new algorithm developed for the resistive magnetohydrodynamics (MHD) equations. This new algorithm is based on an all Mach version of the PISO method,[5] that was already extended and validated for ideal MHD flows.[6]

In the next section we will present the governing equations used to describe the flow and, in section 3, the numerical method is briefly sumarized. In section 4 some validation cases are presented, we start with the Hunt flow problem, were we intend to validate our method for low speed resistive MHD. Results for the Brio and Wu[7] shock tube problem are also presented and we close that section with the cloud-shock interaction test case. Finally, in section 5 we present our first results obtained for a cylindrical self-filed MPD thruster.

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II. Governing Equation

The resistive form of the MHD system of equations is given by:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \tag{1}$$

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot \left[\rho \mathbf{U}\mathbf{U} + \left(p + \frac{B^2}{2\mu_0}\right)\mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} - \tau_{visc}\right] = 0, \tag{2}$$

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$$\frac{\partial \rho e_T}{\partial t} + \nabla \cdot \left[\left(\rho e_T + p + \frac{B^2}{2\mu_0} \right) \mathbf{U} - \mathbf{U} \cdot \frac{\mathbf{BB}}{\mu_0} - \nabla \cdot (k \nabla T) \right] - \nabla \cdot \left(\frac{\mathbf{B} \times \eta \mathbf{j}}{\mu_0} \right) = 0, \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{UB} - \mathbf{BU}) + \nabla \times (\eta \mathbf{j}) = 0, \tag{4}$$

where k is the plasma thermal conductivity, η represents the plasma electric resistivity and μ_0 is the permeability of free space. The viscous stress tensor for compressible flow is given by:

$$\tau_{visc} = -\left(\frac{2}{3} \mu \nabla \cdot \mathbf{U} \right) \mathbf{I} + \mu \left[\nabla \mathbf{U} + (\nabla \mathbf{U})^T \right], \tag{5}$$

where μ is the dynamic viscosity of the plasma. Temperature is a derived quantity and can be obtained by an equation of state

$$T = \frac{1}{c_v} \left[e_T - \frac{1}{2} \left\{ (U)^2 + \frac{(B)^2}{\rho \mu_0} \right\} \right]. \tag{6}$$

Using Ampére's law, and some vector identities, the last term of equation (4) can be rewritten in terms of $\mathbf{B}$:

$$\nabla \times (\eta \mathbf{J}) = \nabla \times \left(\eta \frac{\nabla \times \mathbf{B}}{\mu_0} \right) = -\nabla \cdot \left(\eta \nabla \frac{\mathbf{B}}{\mu_0} \right). \tag{7}$$

III. Numerical Method

The numerical method that is going to be used for the MPD thruster computation was developed and validated by Xisto et al.[6] for the ideal MHD equations. This new algorithm is based on the PISO method previously developed for the Euler equations and validated for all Mach number flows.[5] In order to improve accuracy and stability in the calculation of all kinds of MHD discontinuities we make use of the AUSM-MHD scheme proposed by Han et al.[8] For multidimensional MHD cases, in order to ensure $\nabla \cdot \mathbf{B} = 0$, we make use of the hyperbolic diverge cleaning technique proposed by Dedner et al.[9] Usually MPD devices operate at high values of plasma temperature and, because of that, in order to obtain a more realistic and stable result, we use a real gas thermodynamic model.[10]

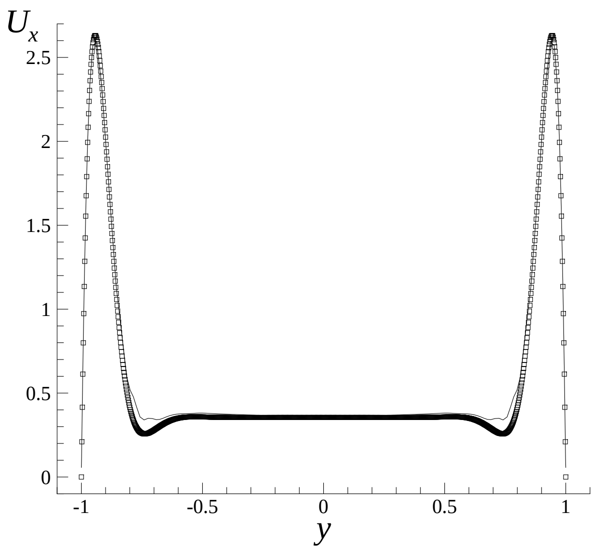
IV. Validation Cases

In Figure 1 we present the results obtained for the HUNT flow problem. This case allows us to validate the effect of Lorentz force ($\mathbf{j} \times \mathbf{B}$) in the evolution of the conducting fluid flow in a square duct with two conducting walls. An analytical solution can be obtained with the hyperbolic functions proposed by Hunt[11] or by the exponential version of those equations presented by Ni et al.[12] As we can see, the numerical result tends to agree very well with the analytical solution.

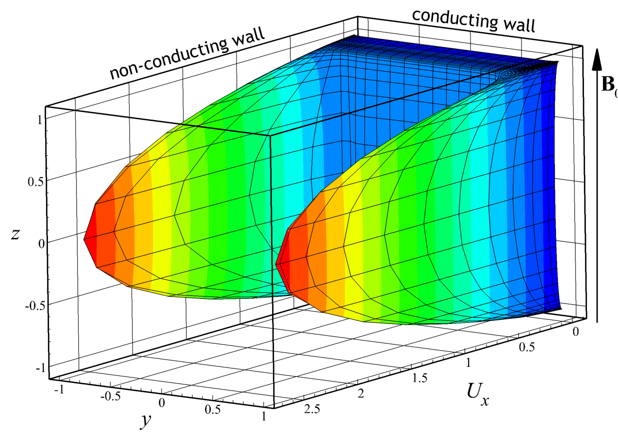
In Figure 2 we present the results obtained for a well know test case, of ideal MHD, that comprises the calculation of a disruption of a high density cloud by a strong shock wave. The figure on left shows us the density contour in a logarithm scale obtained in a 400×400 grid after $t = 0.06$ seconds. In the right picture we can see the magnetic field lines obtained at the same time. This test case does not have any analytical solution and, due to that, it can only be compared with solutions obtained by others.[13,14] Nevertheless, this is a good test case for assessing the accuracy of our method in the calculation of MHD shocks. Another parameter that can be evaluated is the robustness of the algorithm, because the strong collision between the high density cloud and the shock wave can introduce some positivity problems in the thermodynamic properties.

V. Self-field MPD Thruster simulation

In this section we present some preliminary results for a cylindrical self-field MPD thruster.[15] In these kind of systems we can assume that the induced magnetic field as only one component in the azimuthal direction. With this assumption we know that the Lorentz force ($\mathbf{j} \times \mathbf{B}$) is acting on the plasma only in

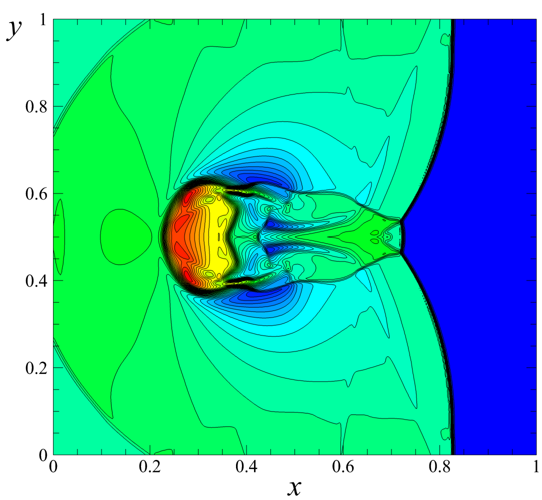


(a)

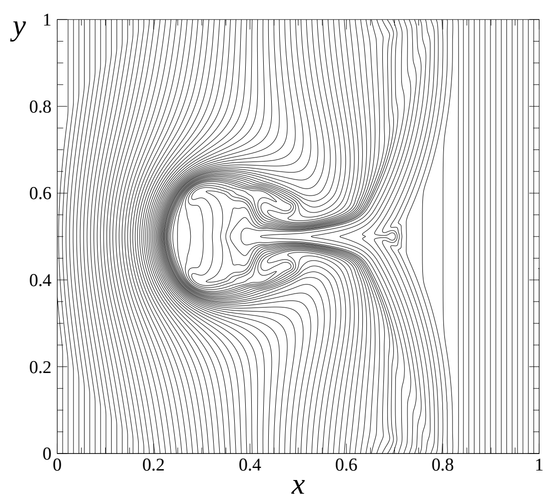


(b)

Figure 1: Results obtained for the Hunt flow problem. a) Comparison between numerical and analytical solutions. b) Distribution of the velocity profile.



(a)



(b)

Figure 2: Results obtained for the cloud-shock interaction test case. a) Contour plots for density in a logarithm scale. b) Magnetic field lines.

the axial ($j_r B_\theta$) and radial ($-j_z B_\theta$) direction and, because of that, this case can be tackled using a 2D axisymmetric grid.

In Figure 3 we present the contour plots for the axial velocity obtained for an induced magnetic field of 0.12 Tesla. As we can see the velocity increases with decreasing radius to a maximum value of 10500 m/s, this result is expected since the Lorentz force behaves in the similar way.

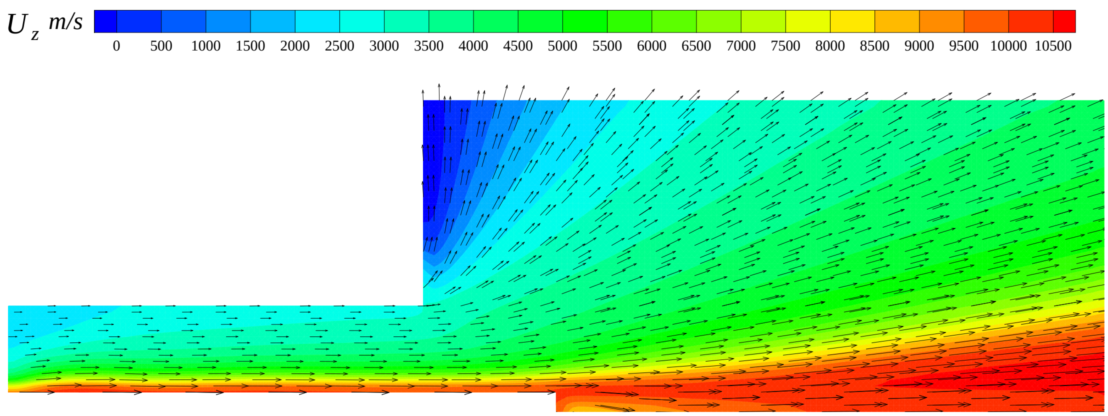


Figure 3: Contour plot for the axial velocity.

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