Viscoelastic lid-driven cavity flow scaling of purely-elastic instabilities

\[ \frac{V}{D} = \frac{H}{J} = \frac{M}{2J} \quad \frac{D}{J} = \frac{L}{J} \quad \frac{L}{J} = V \]

\[ \frac{V}{D} \approx L \quad \frac{2J}{2J} \approx \frac{2J}{H} \quad \frac{1}{v} = \frac{1}{v} \]

\[ \frac{L}{q + H/n} = \frac{v}{1} \]

\[ W \leq \left[ \frac{2J \cdot \frac{L}{J}}{H \cdot \frac{2J}{H}} \right] \]

Lid-driven cavity

Well accepted that flows which display curved streamlines can exhibit purely elastic instabilities (Re ≈ 0), which are absent in corresponding Newtonian fluids.
Unstable

Stable

Lid-driven cavity

\[ \frac{Wf}{\alpha^2} + \frac{V}{1} \approx \frac{Wc}{\alpha} \]


Constant

Lower corner where streamwise curvature is high – results in a scalloping of Wf = Wf′.

By the downstream upper corner stress being advected to the downstream aspect ratio and that for a different mechanism – an instability caused.

Grillet et al. argued that Mc Kinley mechanism was only correct for "large"

Lid-driven cavity
\[(I - \mathbf{v}) = \left( n_\Delta \cdot \mathbf{v} - \mathbf{v} \cdot n_\Delta - \frac{\partial d}{\partial t} \right) \gamma \]

where \( A \) is the conformation tensor which evolves according to

\[(I - \mathbf{v}) \frac{\gamma}{dt} = \mathbf{1} \]

Upper Convected Maxwell model (UCM)

3) Constitutive Equation

\[\mathbf{1} \cdot \Delta + d/dt \Delta = 0\]

\[0 = \mathbf{n} \cdot \Delta\]

2) Momentum (creeping)

1) Mass

Assume flow is two-dimensional and incompressible

Governing equations
(4) Special formulations for cell-face velocities and stresses

**Convection terms:** CUBISTA

**Diffusive terms:** Centred differences (CDS)

Time: second order backward scheme

(3) Discretization (formally second order)

Structured, collocated and non-orthogonal meshes

Fully implicit time marching pressure-correction algorithm

(2) Finite-volume method (Oliviera et al. (1998), Alonso et al. 2009)

(1) Finite-volume method (Oliviera et al. (1998), Alonso et al. 2009)

\[
(I - \frac{\gamma}{\rho}) \frac{\partial}{\partial t} = \Delta \mathbf{E} - \Theta (\mathbf{R} - \Theta \mathbf{Q}) - \Theta (\Delta \cdot \mathbf{n}) + \frac{\mu}{\Theta} \mathbf{E}
\]

where \( \mathbf{R} \) is a pure rotational tensor and \( \mathbf{E} \) a traceless extensional tensor.

We use the log-conformation technique \( \Theta = \log \mathbf{A} \) and then we have

Governing equations and numerical method
\[ \frac{\frac{\gamma}{\nu}}{\frac{L}{H}} = \frac{\nu}{H} \]

Boundary conditions
Streamlines: Newtonian (Re = 0) M1, M2, M3
\[ C \approx \mathcal{C} \]

\[(v)f = \mathfrak{w}\]

(rescaled: \(0.25, 0.5\))

(low: \(v \geq 0.5\))

(high: \(v \geq 1\))

Streamlines: Newtonian (\(Re = 0\))
Neu't, UCM D0 = 0.48
Streamlines: effect of elasticity (Re = 0) v=1
Upstream shift of recirculation consistent with experiments with Pakdel et al.
Streamlines: effect of elasticity ($Re = 0$)
Effect of elasticity on strength of circulation
\[
V + \alpha \equiv \frac{W_{cr}}{1}
\]

\text{Closure of purely-elastic instability}
Dec~0.49

V

M3
M2
M1

Onset of purely-elastic instability
$\text{Contours of } M$

$V = 0.25 \text{ UCM } D_e = 0.50$

$V = 1.0 \text{ UCM } D_e = 0.48$
\[ \frac{V}{\partial t} + \nabla \cdot \mathbf{u} = 0 \]

Wall Regularization:

1. \( \hat{u}(x) = \) \( \frac{1}{2} \) \( x^2 \) \( 1 - x^2 \) \( x > 0, x^2 < 1 \)
2. \( \hat{u}(x) = \) \( 39.0625 \) \( x^2 \) \( 1 - x^2 \) \( x > 1 \)
3. \( \hat{u}(x) = \) \( 0.2 \) \( x \leq 0.8 \)
4. \( \hat{u}(x) = \) \( 0 \) \( x > 1 \)

Boundary Conditions:

L = \frac{L}{H} = \frac{V}{\gamma} = \frac{H}{\gamma} = \frac{W}{\gamma}
\[
\left( \frac{\partial}{\partial z} \right)^2 \psi = \frac{\Gamma}{\rho} \\
\left( \frac{\partial}{\partial z} \right)^2 \psi = \frac{\Gamma}{\rho - 1} \equiv \chi
\]

\( \psi_{\text{min}} = -0.1003 \) \( \psi_{\text{min}} = -0.08364 \) \( \psi_{\text{min}} = -0.08364 \)

Effect of wall velocity on flow type Newtonian (\( \text{Re} = 0 \))
$D_e \approx 0.25, 0.21$

\[ \frac{L}{\nu} D_e = \gamma \]

$D_e \approx 0.27$

$M_3$

$M_2$

$D_e \approx 0.49$

Effect of wall regularization on critical conditions
three-dimensional effects.
Experimental evidence, perhaps due to wall regularization used or
For "short" cavities ($\gamma > 1$) the instability found does not agree with
downstream corner(s) proposed by McKinley et al. (but max $M_{\infty}$ does not occur near
For "tall" cavities ($\gamma < 1$) the instability found to match scaling

- purely-elastic instability above critical wall velocity
- upstream shifting of recirculation eye
- Viscoclastic lid-driven cavity flow has been numerically simulated and

Conclusions
Flow corresponds to pure rotational flow corresponds to pure shear flow and $-1$ extensional flow, $0$ corresponds to pure extensional flow. As such, $+1$ corresponds to pure relative rate of rotation tensor. $W$ is the strain-rate tensor and $\dot{\varepsilon}$ is the rate of strain tensor defined as $R' \equiv (\text{tr} \cdot W)(W \cdot D)$, where $D \equiv (1 - R')/(1 + R')$; with $R'$ defined as $R'$ Thompson and co-workers [26, 27].

As trlicts criterion [25] and used to classify the flow locally using

The flow-type parameter is