Scaling of purely-elastic instabilities in viscoelastic lid-driven cavity flow

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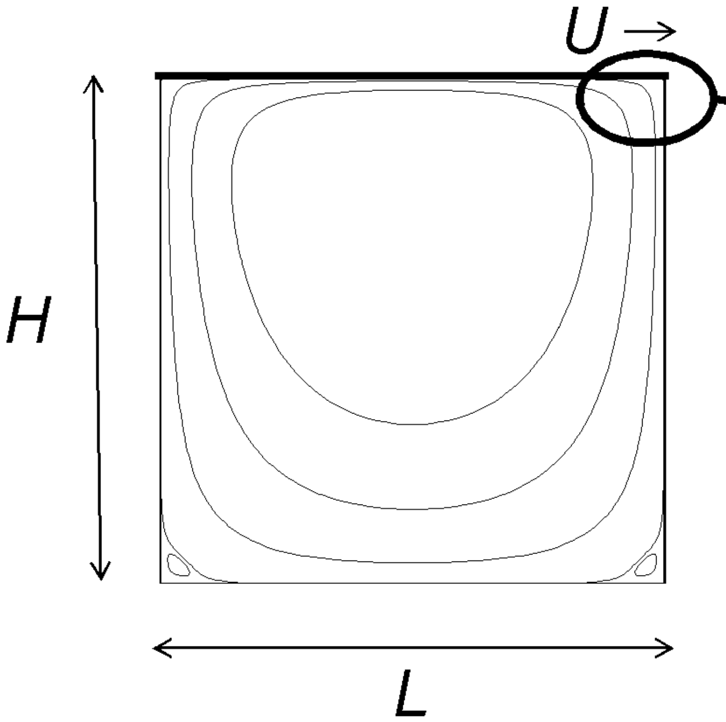
Lid-driven cavity

Well accepted that flows which display *curved streamlines* can exhibit purely elastic instabilities ($Re \to 0$) which are absent in “corresponding” Newtonian fluids. McKinley[1-2] criterion:



$$\left[\frac{\lambda U}{\Re}\frac{\tau_{11}}{\eta\dot{\gamma}}\right]^{0.5} \geq M_{crit}$$

$$\frac{1}{\Re} = \frac{a}{H} + \frac{b}{L}$$

$$\tau_{11} \approx 2\lambda\eta\dot{\gamma}^2 \qquad \dot{\gamma} \approx \frac{U}{\Re}$$

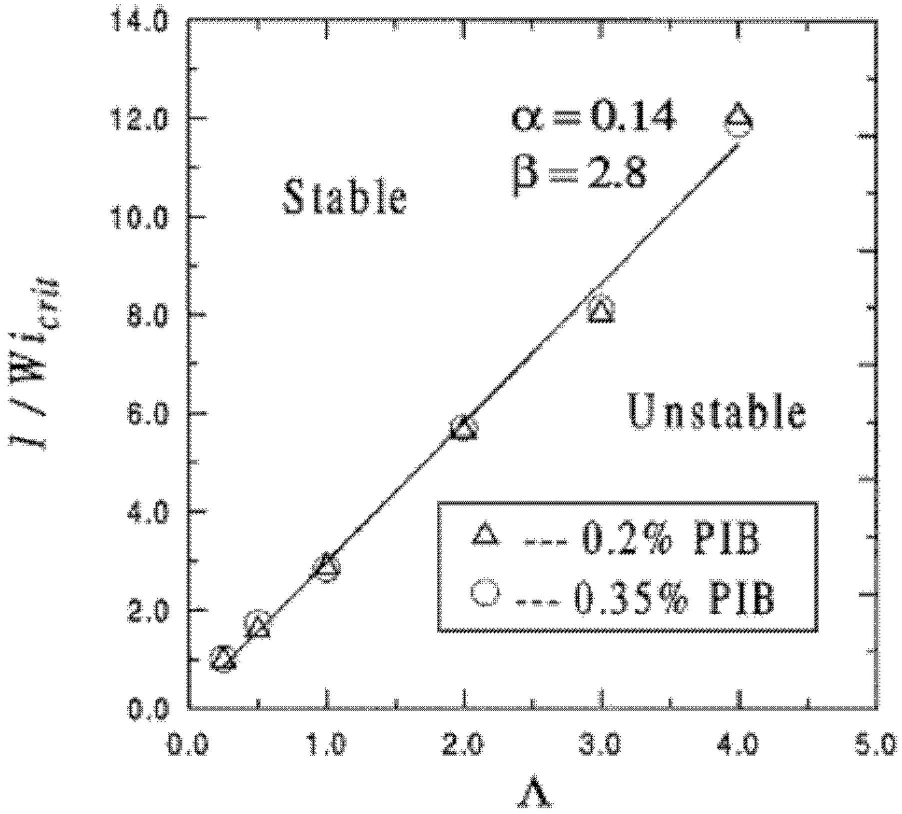
$$\boxed{\frac{1}{Wi_{CR}} \equiv \alpha + \beta\Lambda}$$

$$\Lambda = \frac{H}{L} \qquad De = \frac{\lambda U}{L} \qquad Wi = \frac{\lambda U}{H} = \frac{De}{\Lambda}$$

[1]Pakdel and McKinley *Phys. Rev. Lett.* 77(12):2459-2462 (1996)

[2]McKinley et al. *J. Non-Newt. Fluid Mech.* 67:19-47 (1996)

Lid-driven cavity



$$\boxed{\frac{1}{Wi_{CR}} \equiv \alpha + \beta\Lambda}$$

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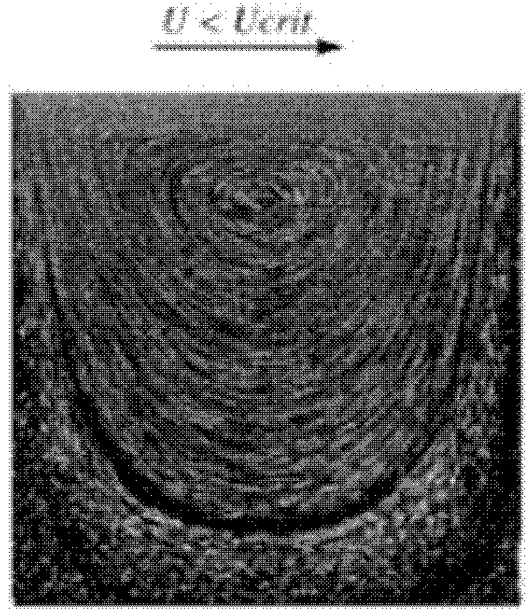
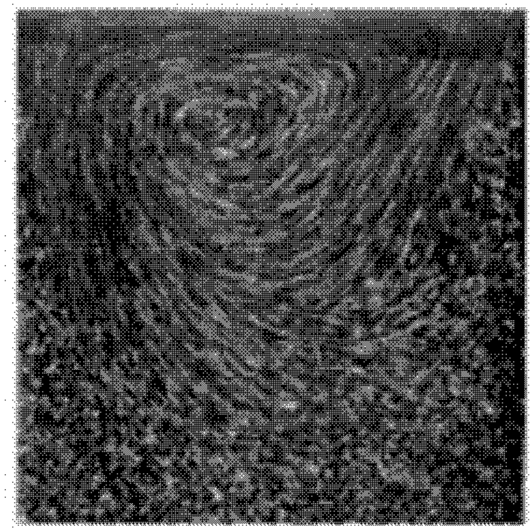


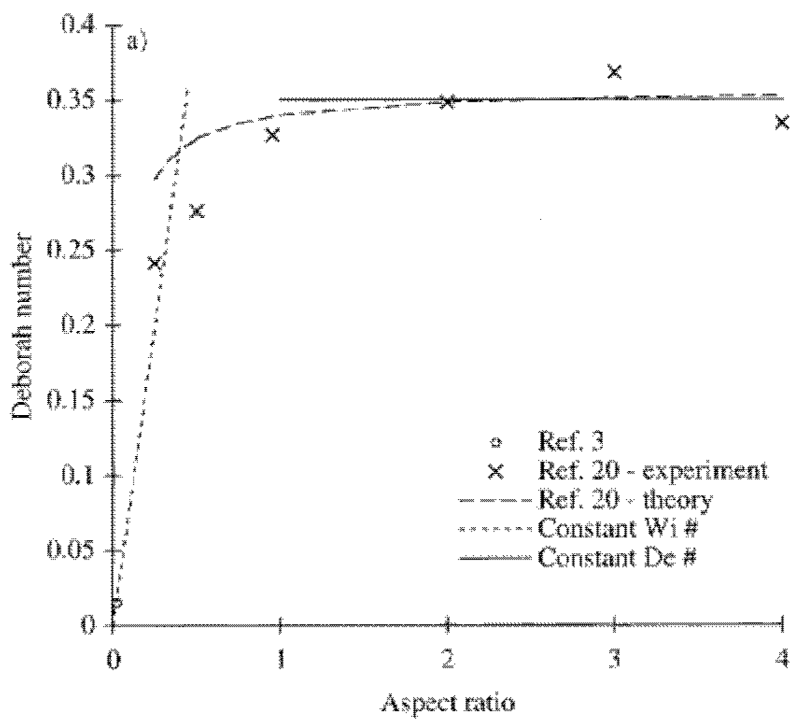
Stable ($De = Wi = 0.15$)

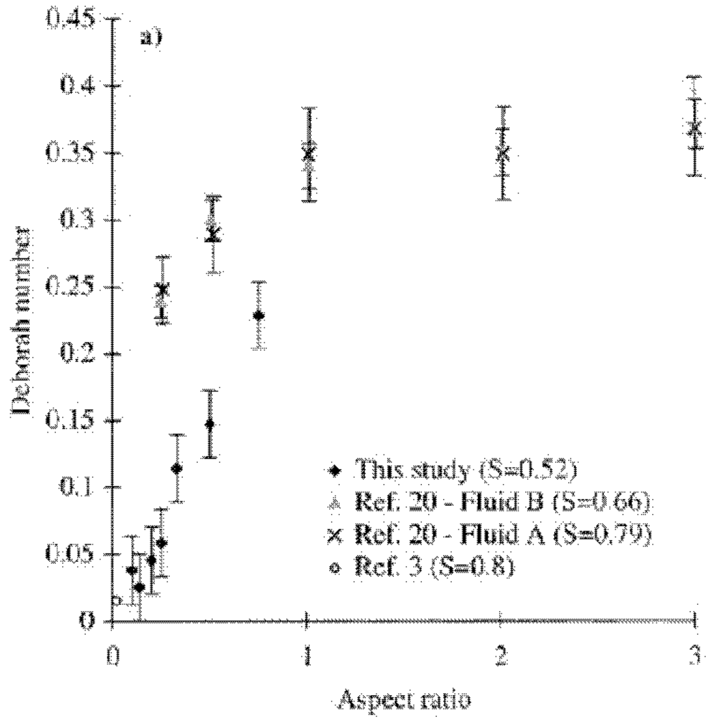


Unstable ($De = Wi = 1.5$)

Lid-driven cavity

Grillet et al.[3-4] argued that McKinley mechanism was only correct for "large" aspect ratios and that for $\Lambda < 1$ a different mechanism – an instability caused by the downstream upper corner stresses being advected to the downstream lower corner where streamline curvature is high – results in a scaling of Wi_{cr} = constant.





[3]Grillet et. al. *J. Non-Newt. Fluid Mech.* 88:99-131 (1999) [4]Grillet et al. *J. Non-Newt. Fluid Mech.* 94:15-35 (2000)

Governing equations

Assume flow is two-dimensional and incompressible

1) Mass

$$\nabla \cdot \mathbf{u} = 0$$

2) Momentum (creeping)

$$0 = -\nabla p + \nabla \cdot \boldsymbol{\tau}$$

3) Constitutive equation Upper Convected Maxwell model (UCM)

$$\boldsymbol{\tau} = \frac{\eta_p}{\lambda}(\mathbf{A} - \mathbf{I})$$

where **A** is the *conformation* tensor which evolves according to

$$\lambda\left(\frac{D\mathbf{A}}{Dt} - \nabla\boldsymbol{u} \cdot \mathbf{A} - \mathbf{A} \cdot \nabla\boldsymbol{u}\right) = (\mathbf{A} - \mathbf{I})$$

Governing equations and numerical method

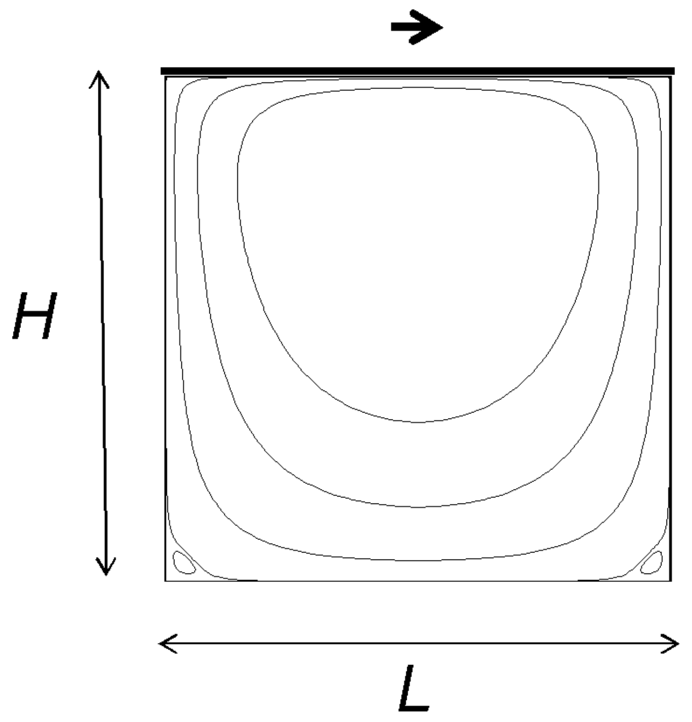
We use the log-conformation technique Θ=log **A** and then we have

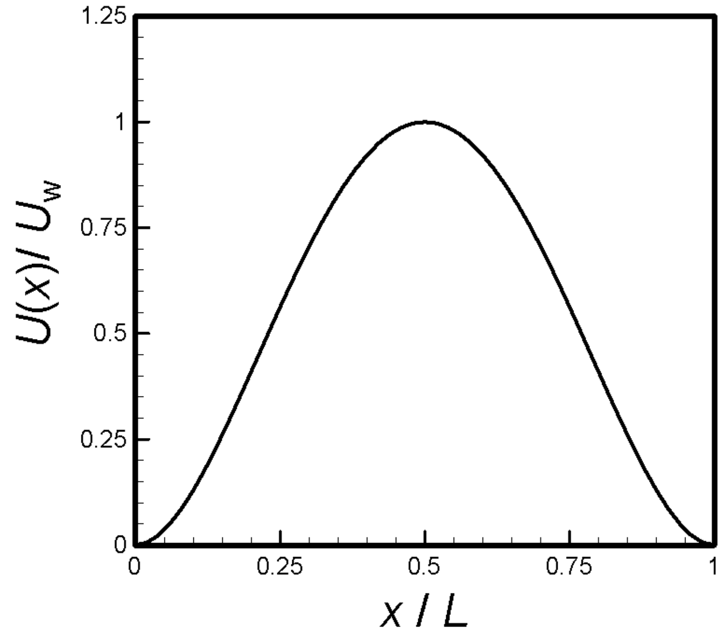
$$\frac{\partial \boldsymbol{\Theta}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\Theta} - (\mathbf{R}\boldsymbol{\Theta} - \boldsymbol{\Theta}\mathbf{R}) - 2\mathbf{E} = \frac{1}{\lambda}\left(e^{-\Theta} - \boldsymbol{I}\right)$$

where **R** is a pure rotational tensor and **E** a traceless extensional tensor

1) Finite-volume method (Oliveira et al (1998), Afonso et al. 2009))
   - fully implicit time marching pressure-correction algorithm
2) Structured, collocated and non-orthogonal meshes
3) Discretization (formally second order)
   Time: second order backward scheme
   Diffusive terms: central differences (**CDS**)
   Convective terms: **CUBISTA** (Alves et al (2003))
4) Special formulations for cell-face velocities and stresses

Boundary conditions





$$\Lambda = \frac{H}{L} \quad De = \frac{\lambda U_W}{L}$$

$$Wi = \frac{\lambda U_W}{H} = \frac{De}{\Lambda}$$

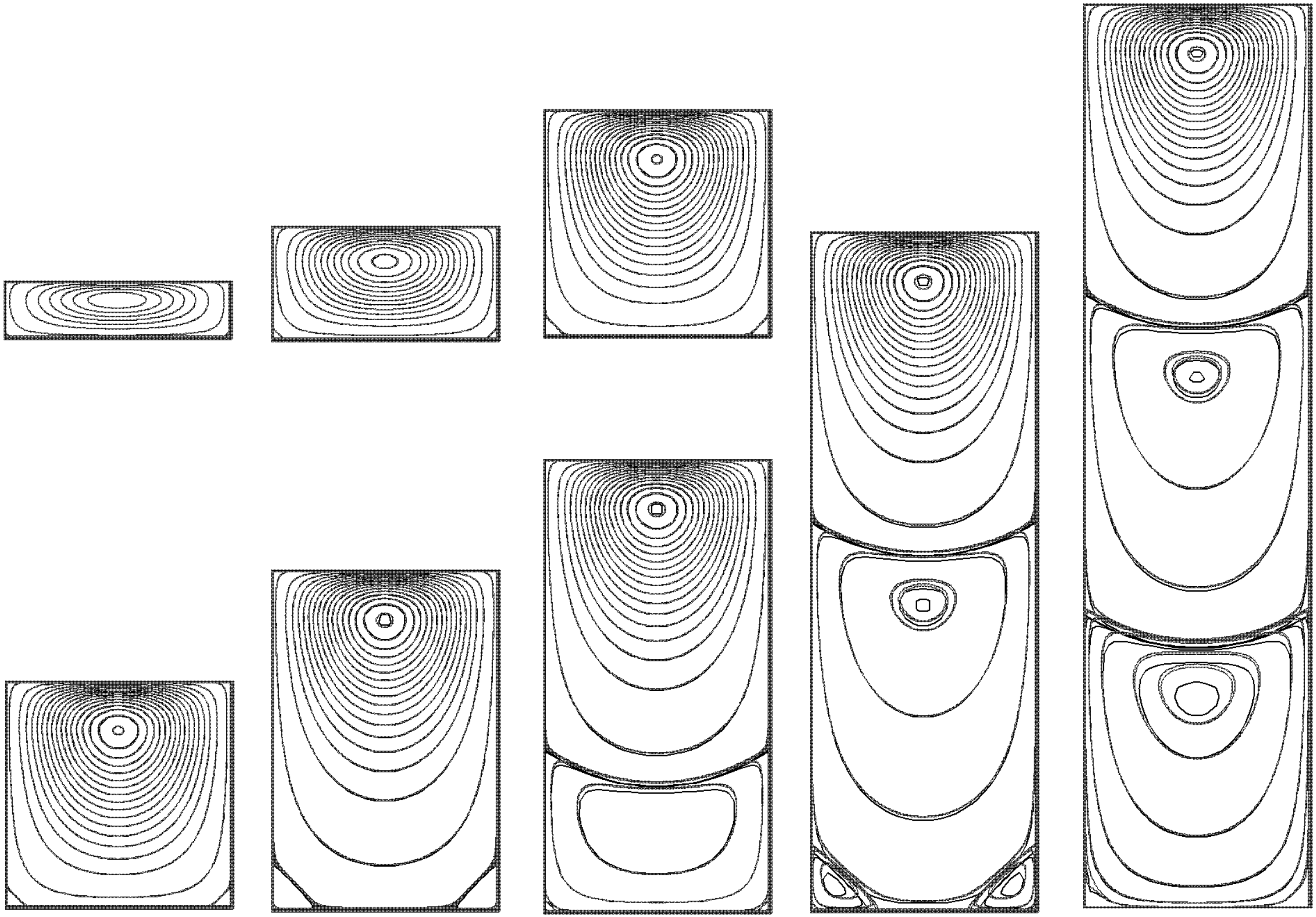
Wall regularisation[5-6]

$$U(x) = 16 U_w x^2 (1-x)^2$$

[5]Fattal and Kupferman *J. Non-Newt. Fluid Mech.* 126:23-37 (2005)

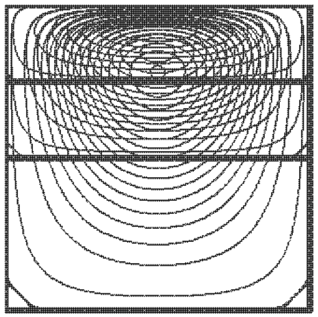
[6]Pan et al. *Int. J. Num. Meth. Fluids* 60:791-808 (2009)

Streamlines: Newtonian (*Re* = 0) M1, M2, M3

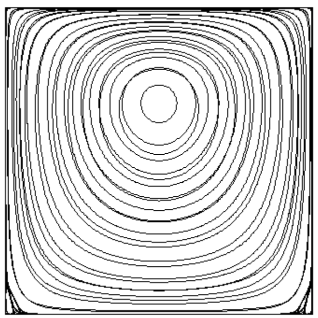


Streamlines: Newtonian (Re = 0)

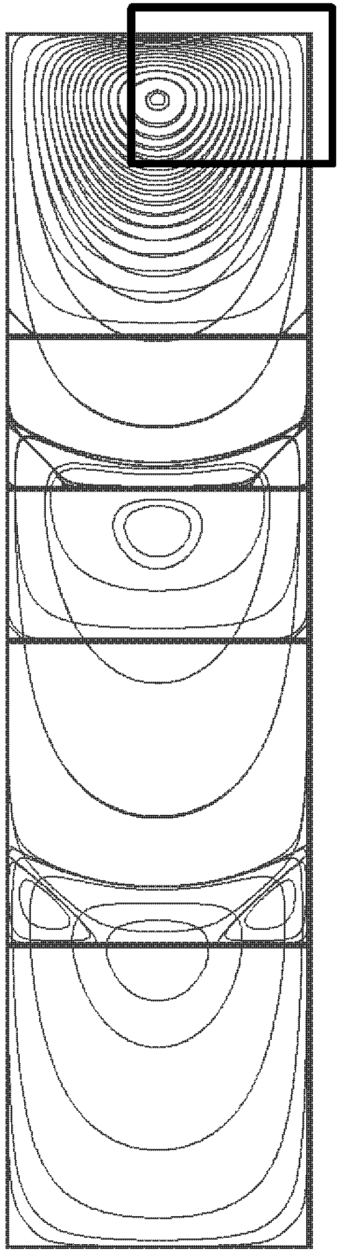
“low “ ($\Lambda \leq 1$)



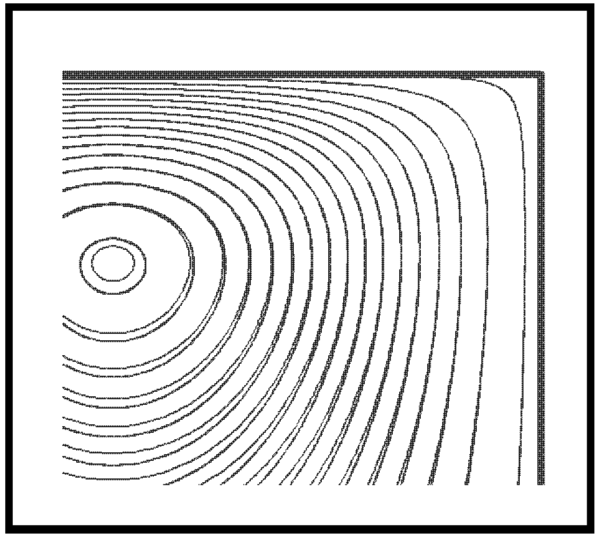
“low “ ($\Lambda \leq 0.5$)
(rescaled 0.25, 0.5)



$$\Re = f(\Lambda)$$



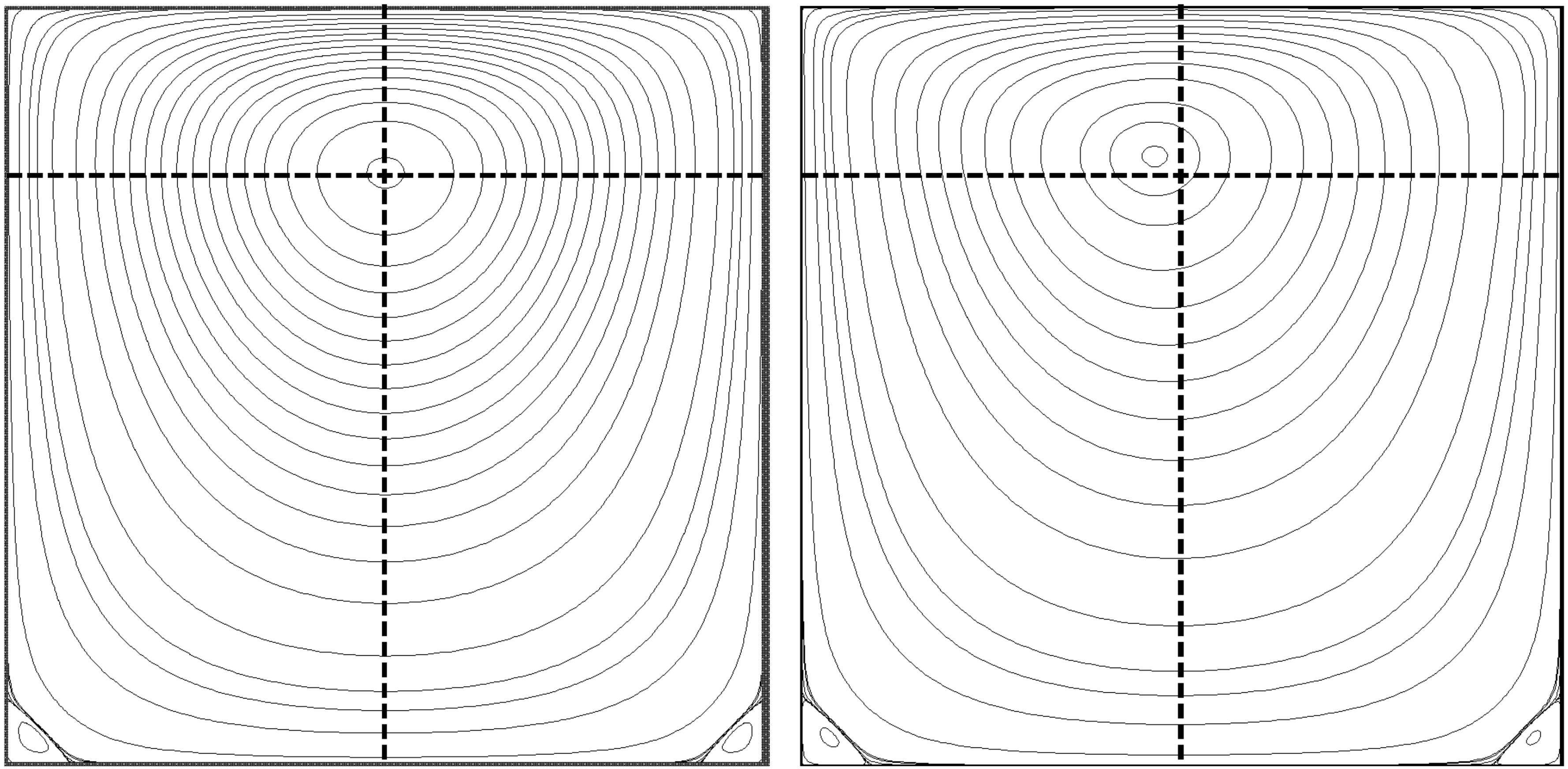
“high” ($\Lambda \geq 1$)



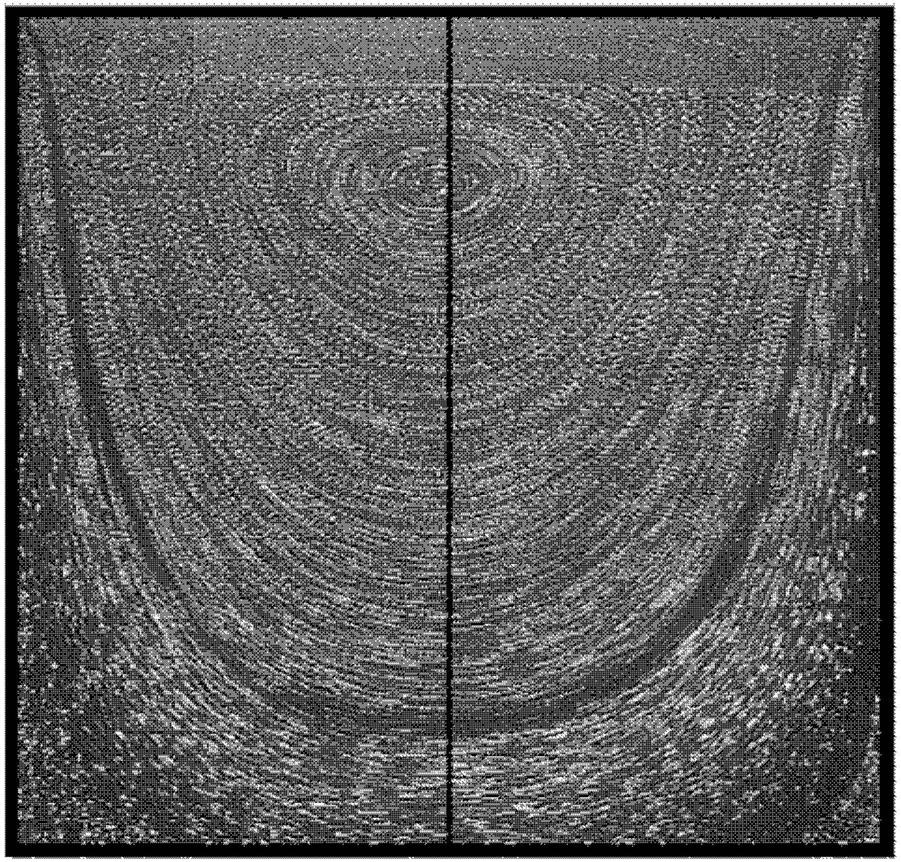
$$\Re \approx C$$

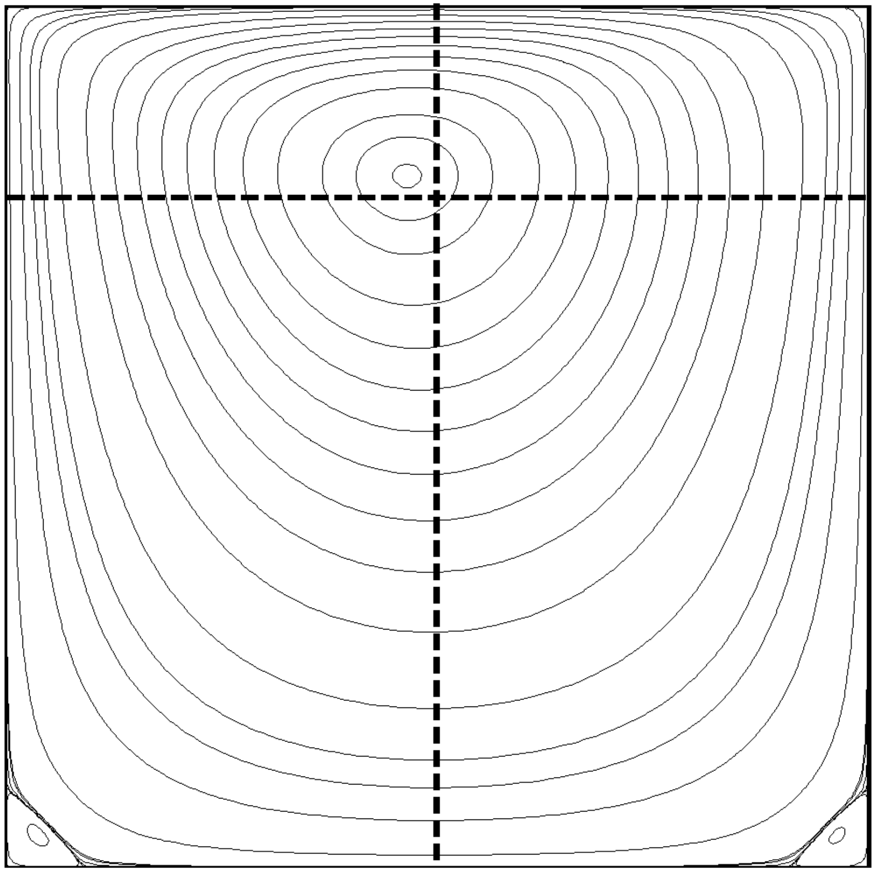
Streamlines: effect of elasticity (Re = 0) Λ=1

Newt, UCM De = 0.48



Upstream shift of recirculation consistent with experiments with Pakdel et al.[7]



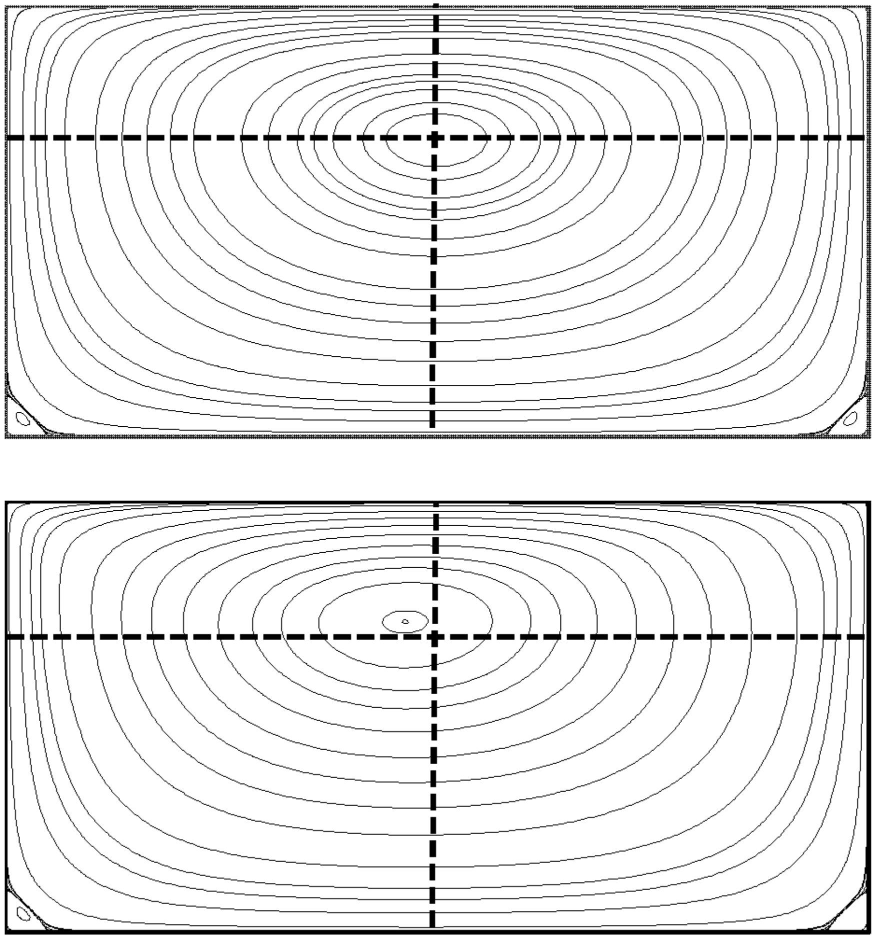


[7]Pakdel et al. *Phys. Fluids.* 9(11):3123-3140 (1997)

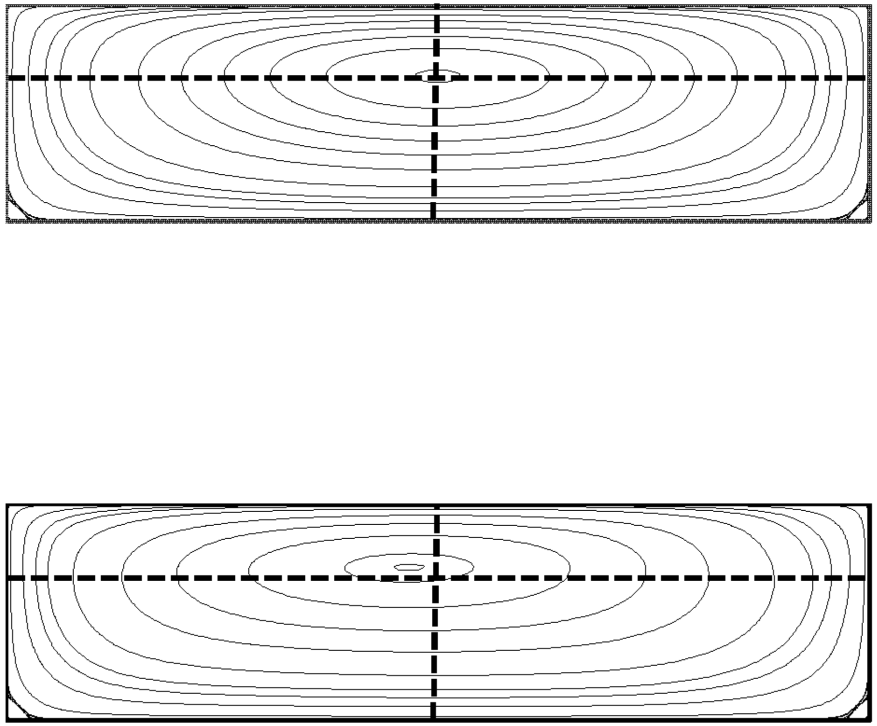
De = 0.30

Streamlines: effect of elasticity (Re = 0)

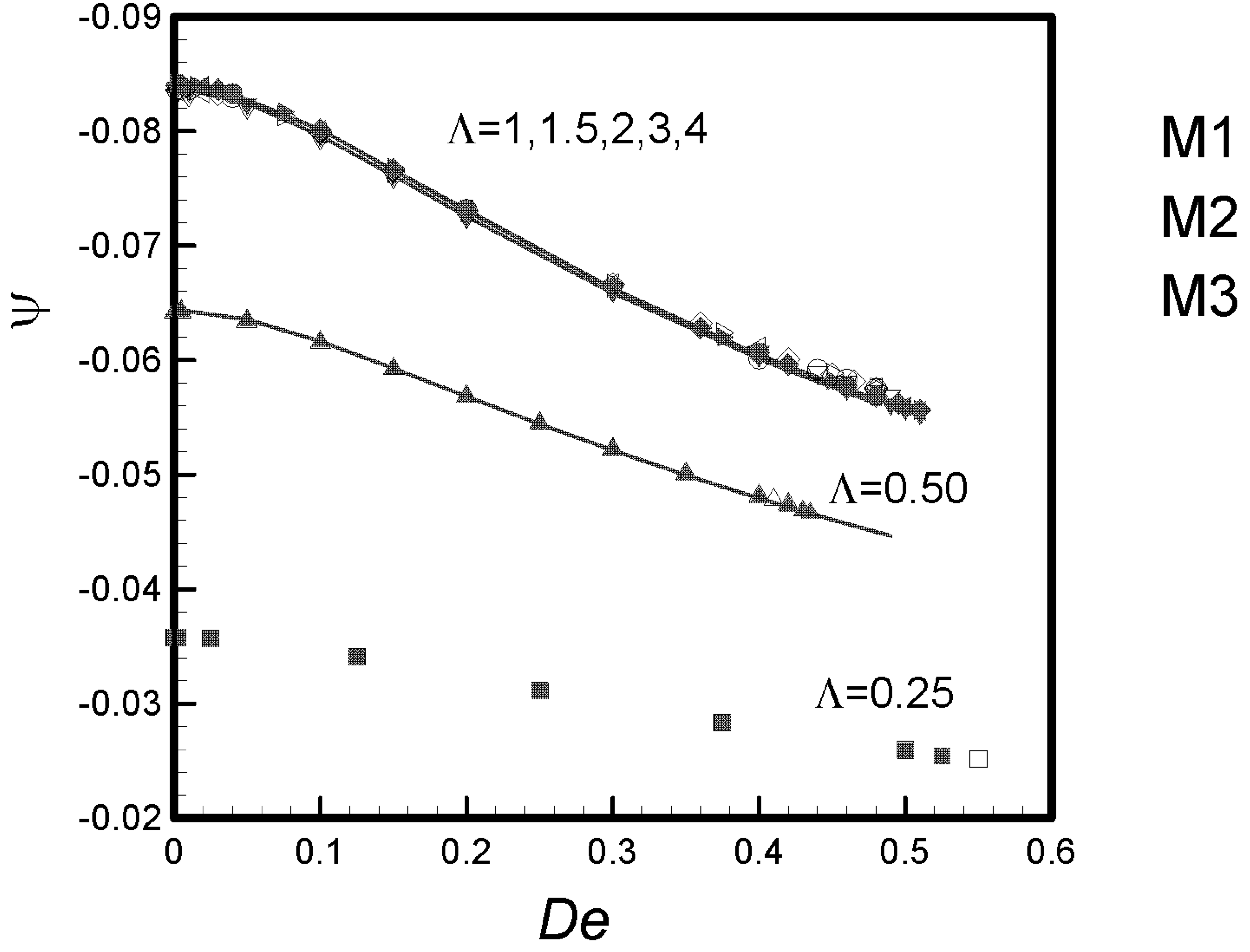
Λ=0.5 Newt, UCM De = 0.49



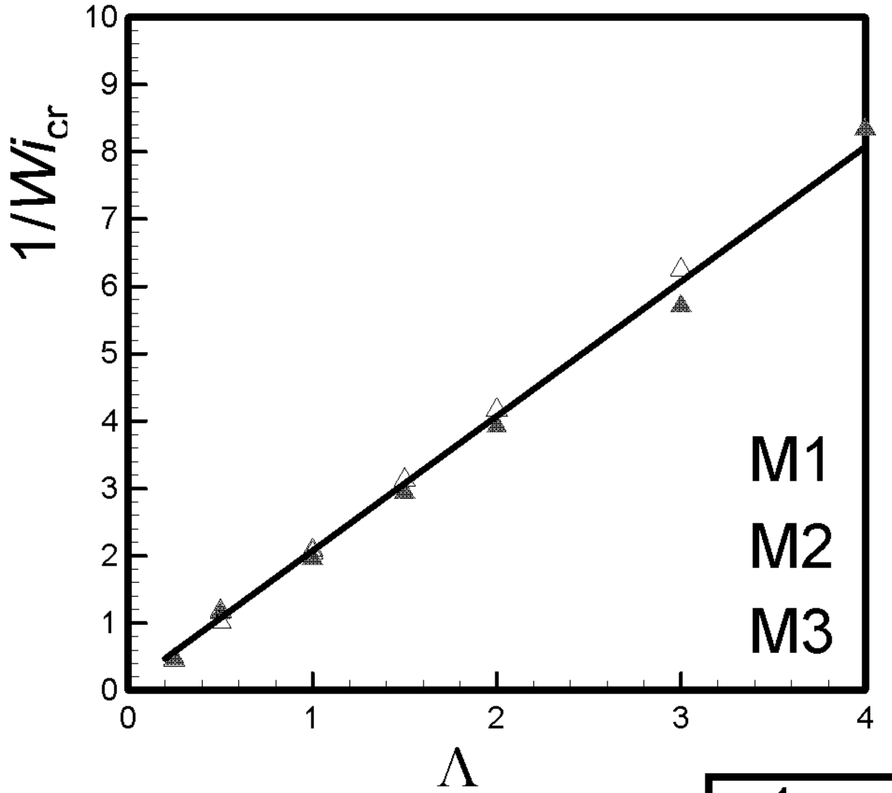
Λ=0.25 Newt, UCM De = 0.525

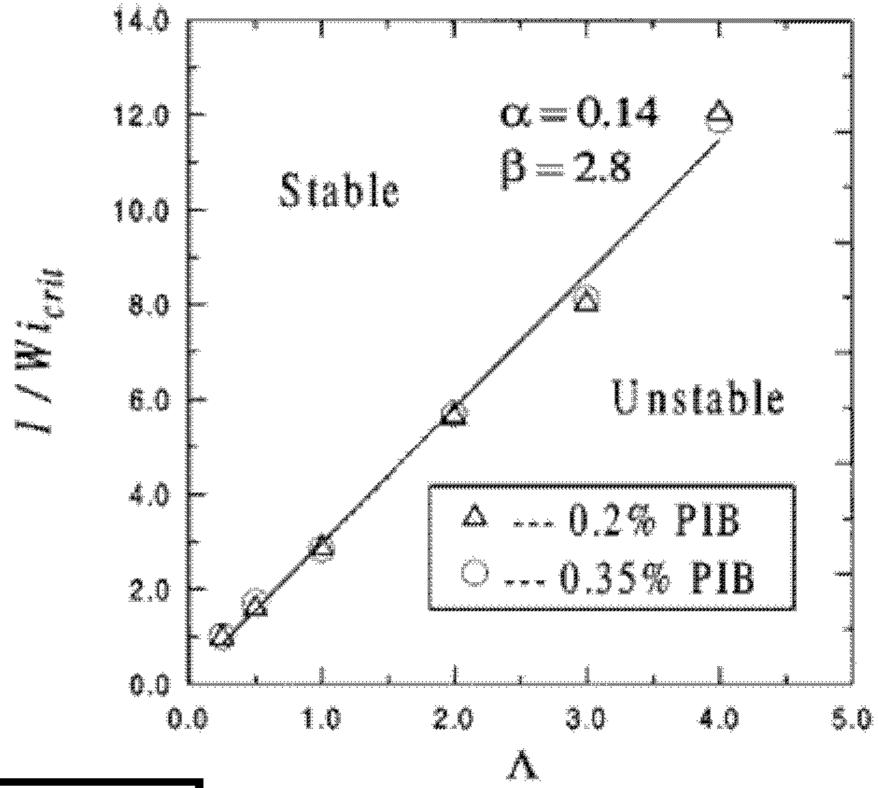


Effect of elasticity on strength of circulation



Onset of purely-elastic instability



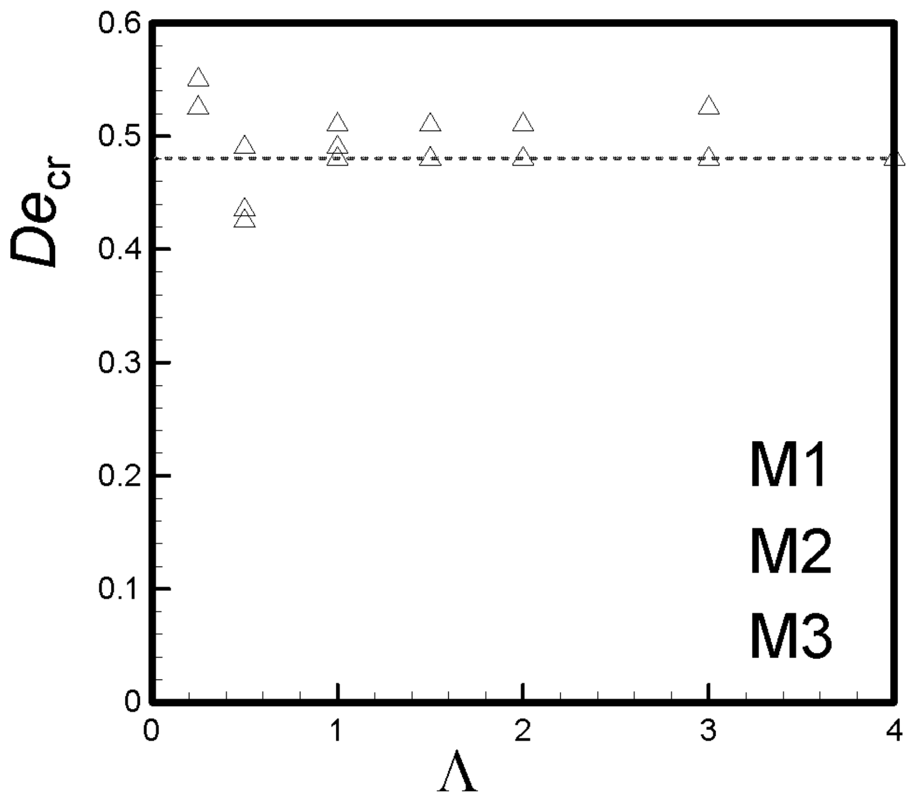


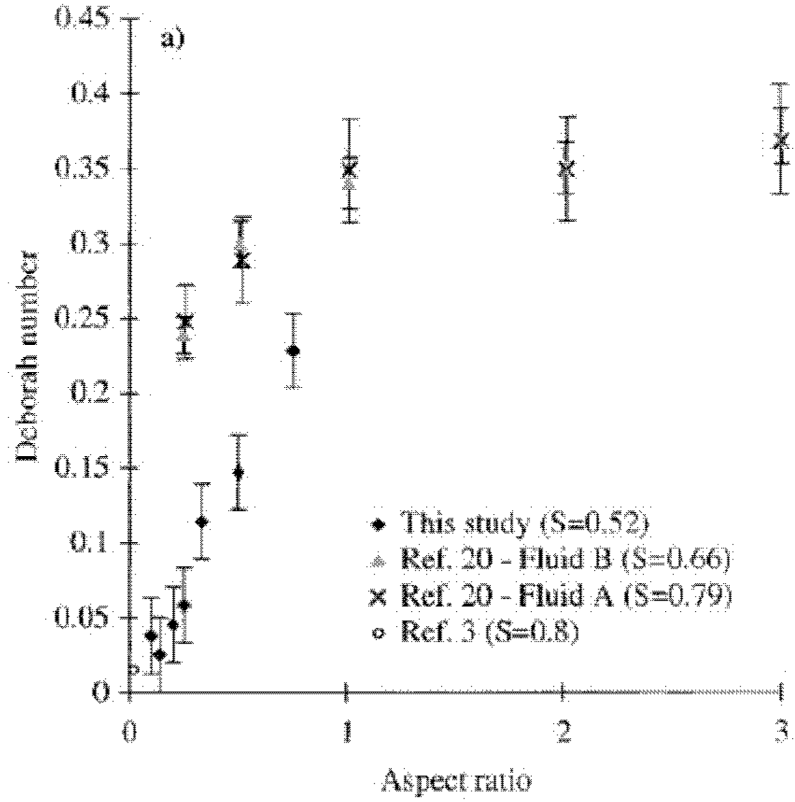
$$\boxed{\frac{1}{Wi_{CR}} \equiv \alpha + \beta\Lambda}$$

Pakdel and McKinley *Phys. Rev. Lett.* 77(12):2459-2462 (1996)

McKinley et al. *J. Non-Newt. Fluid Mech.* 67:19-47 (1996)

Onset of purely-elastic instability





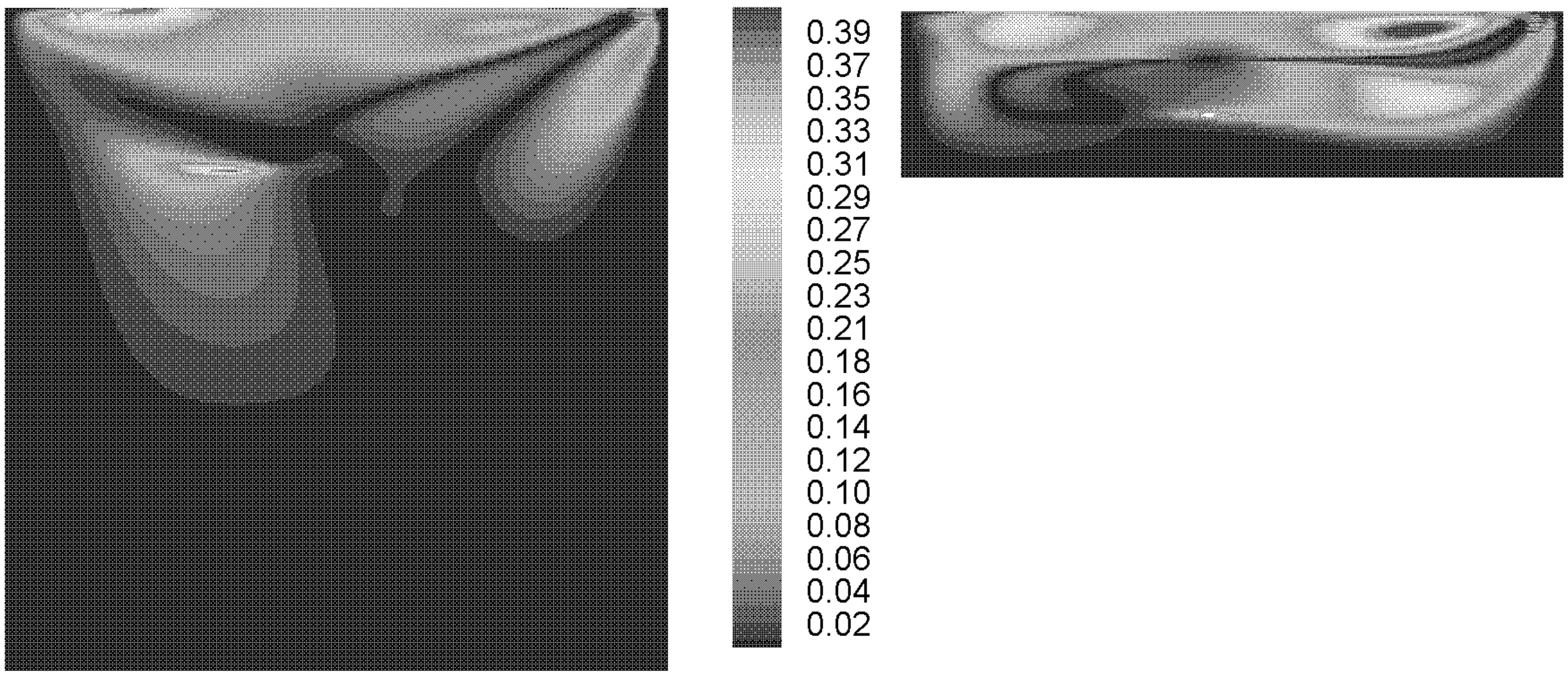
De_{cr}~0.49

[3]Grillet et. al. *J. Non-Newt. Fluid Mech.* 88:99-131 (1999) [4]Grillet et al. *J. Non-Newt. Fluid Mech.* 94:15-35 (2000)

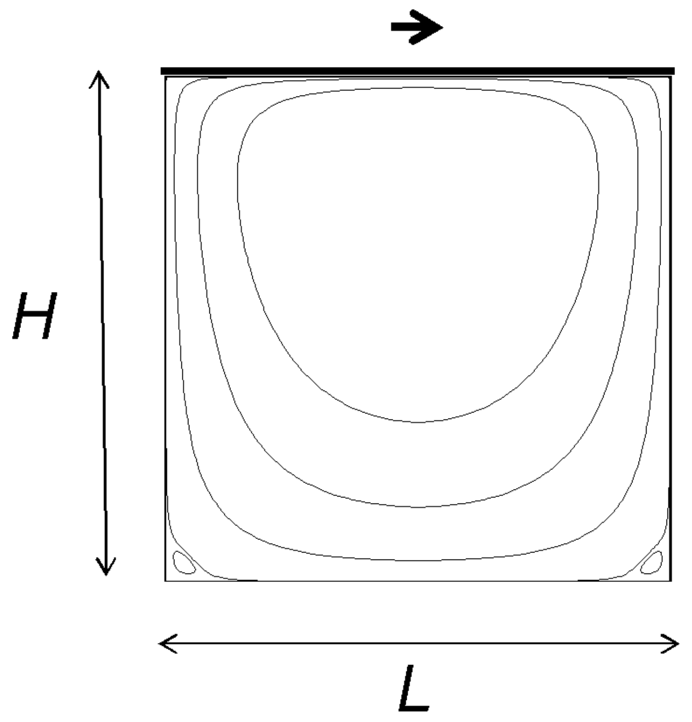
Contours of *M*

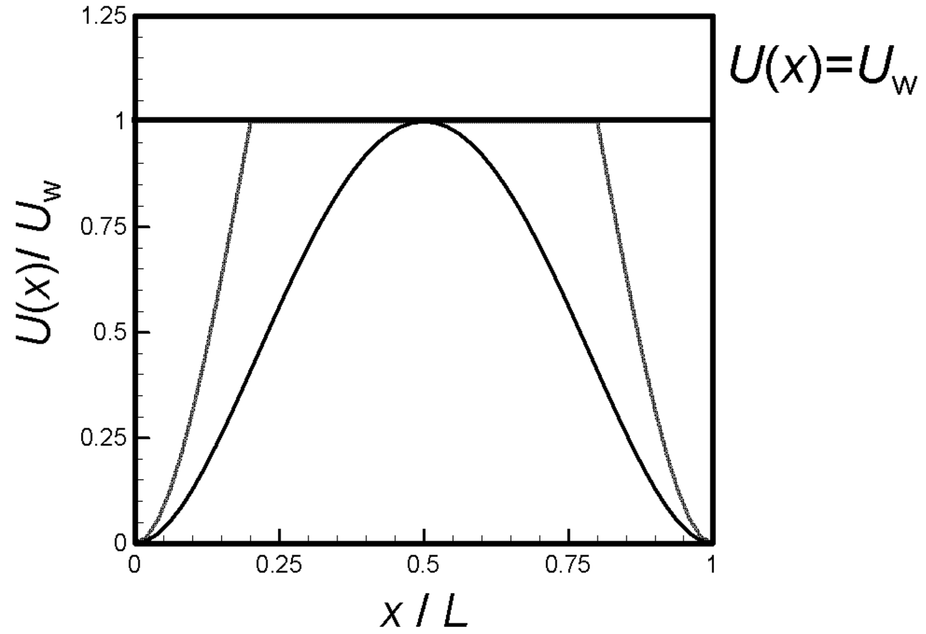
$$\left[\frac{\lambda U}{\Re}\frac{\tau_{11}}{\eta\dot{\gamma}}\right]^{0.5} \geq M_{crit}$$

Λ=1.0 UCM *De* = 0.48 **Λ=0.25 UCM *De* = 0.50**



Boundary conditions





$$\Lambda = \frac{H}{L} \quad De = \frac{\lambda U_W}{L}$$

$$Wi = \frac{\lambda U_W}{H} = \frac{De}{\Lambda}$$

Wall regularisation[5-6]

1) $0 < x/L < 1$; $U(x)=16U_w x^2(1-x)^2$

2) $0 < x/L < 0.2$, $0.8 < x/L < 1.0$; $U(x)=39.0625U_w x^2(1-x)^2$

$0.2 \leq x/L \leq 0.8$; $U(x)=U_w$

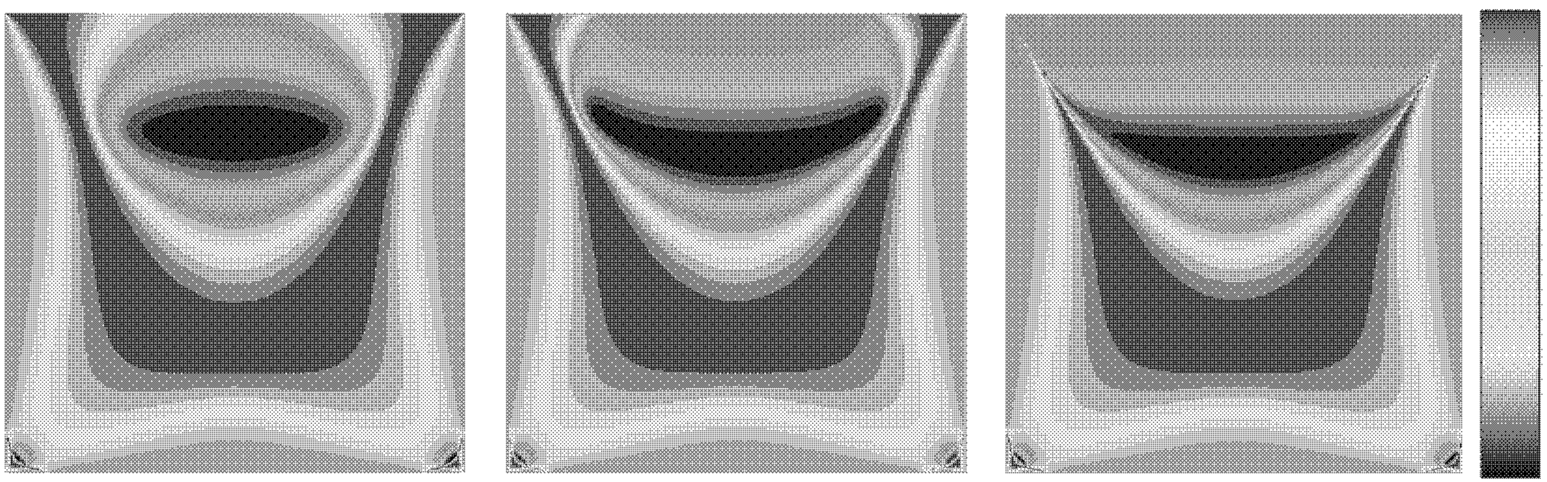
[5] Fattal and Kupferman *J. Non-Newt. Fluid Mech.* 126:23-37 (2005)

[6] Pan et al. *Int. J. Num. Meth. Fluids* 60:791-808 (2009)

Effect of wall velocity on flow type Newtonian (Re=0)

$0 < x/L < 0.2$; $U(x) = 39.0625 U_w x^2 (1-x)^2$
$0.8 < x/L < 1.0$

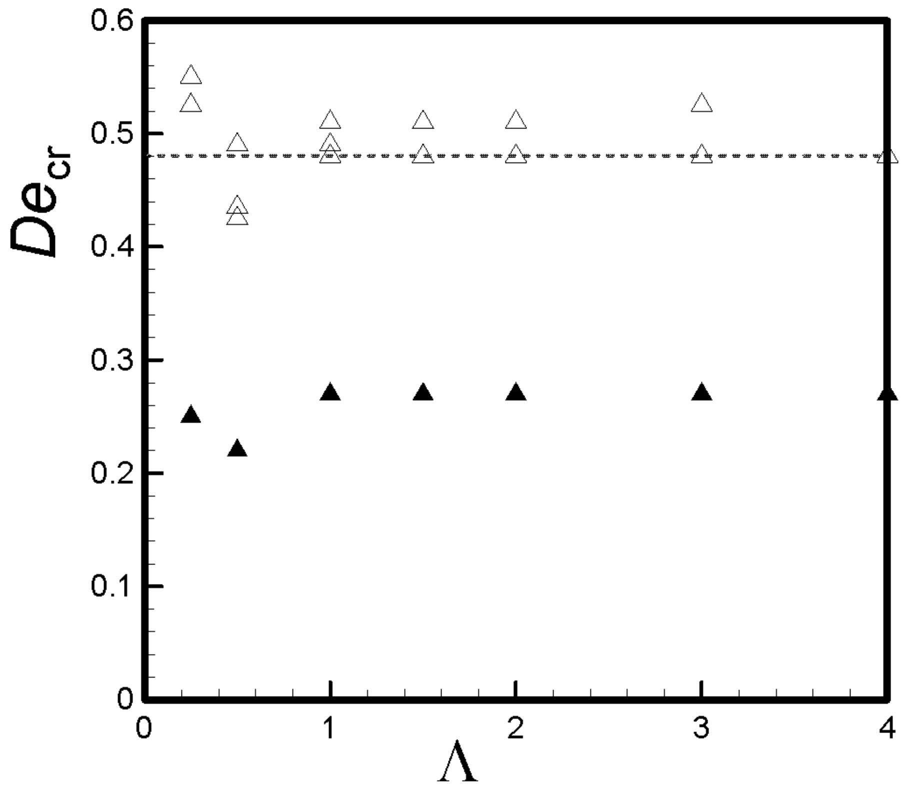
$U(x) = 16 U_w x^2 (1-x)^2$ $\quad$ $0.2 \le x/L \le 0.8$; $U(x) = U_w$ $\quad$ $U(x) = U_w$



ψ_{min} =-0.08364 $\quad$ ψ_{min} =-0.09864 $\quad$ ψ_{min} =-0.10003

$$\xi \equiv \frac{1-R}{1+R} \qquad R \equiv \frac{tr\left(\overline{\mathbf{W}}^2\right)}{tr\left(\mathbf{D}^2\right)}$$

Effect of wall regularisation on critical conditions



De_{cr}~0.49

De_{cr}~0.27

De_{cr}~0.25, 0.21

M1
M2
M3

$$De = \frac{\lambda \overline{U}}{L}$$

[3]Grillet et. al. *J. Non-Newt. Fluid Mech.* 88:99-131 (1999) [4]Grillet et al. *J. Non-Newt. Fluid Mech.* 94:15-35 (2000)

Conclusions

Viscoelastic lid-driven cavity flow has been numerically simulated and many features of the experiments have been captured:

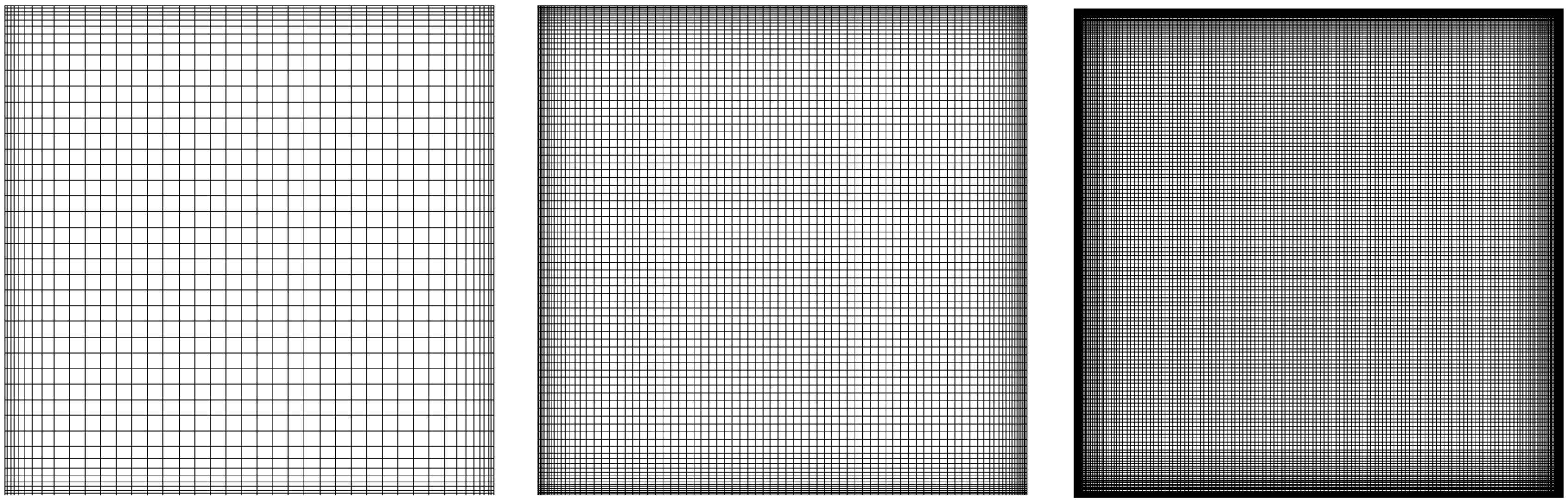
- upstream shifting of recirculation eye
- purely-elastic instability above critical wall velocity

For "tall" cavities ($\Lambda \geq 1$) the instability found to match scaling proposed by McKinley et al. (but max M_{crit} does not occur near downstream corner!)

For "short" cavities ($\Lambda < 1$) the instability found does not agree with experimental evidence, perhaps due to wall regularisation used or three–dimensional effects.

Meshes

Aspect ratio $\Lambda=H/L$	M1 $\Delta x_m/H=\Delta y_m/H$	M1 NC	M2 $\Delta x_m/H=\Delta y_m/H$	M2 NC	M3 $\Delta x_m/H=\Delta y_m/H$	M3 NC
0.25	0.0012	6765	0.0006	27307		
0.5	0.0025	3403	0.0012	13695	0.0006	54285
1.0	0.005	1681	0.0024	6889	0.0012	27225
1.5	0.005	2255	0.0024	9213	0.0012	36582
2.0	0.005	2747	0.0024	10906		
3.0	0.005	3731	0.0024	15189		
4.0	0.005	4756	0.0024	19339	0.0012	76725



The flow-type parameter is used to classify the flow locally using Astarita's criterion [25] and the normalization proposed by Thompson and co-workers [26,27], ≡ (1 − R)/(1 + R); with *R defined as R* ≡ *(tr(.**W2)/tr(D2)) where D*** is the strain-rate tensor and .**W is the relative rate of rotation tensor.**

As such, = +1 corresponds to pure extensional flow, = 0 corresponds to pure shear flowand =−1 corresponds to pure rotational flow.