NUMERICAL STUDY OF 2D LID-DRIVEN CAVITY FLOW AND ELASTIC RECOIL WITH THE FENE-CR MODEL

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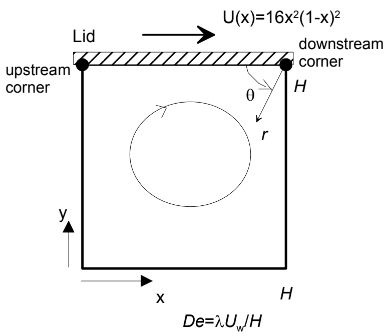
PURPOSE

- viscoelastic flow in square cavity
- FENE-CR model, Boger fluid
- Re=0: **steady** first, followed by **elastic recoil**
- fundamental flow (benchmark results)
- understand near-corner behaviour

$$De = \lambda U_w / \mathcal{L}$$

$$H / \mathcal{L} = 1$$



MOTIVATION -

previous work with NN cavity flow

Pakdel, Spiegelberg, McKinley "Cavity Flows of Elastic Liquids", Phys. Fluids 9 (1997)

Grillet, Yang, Khomami, Shaqfeh, "Modeling of viscoelastic lid driven cavity flow using fe simulations", JNNFM 88, (1999)

Renardy, "Stress integration for the UCM in a driven cavity", JNNFM 112 (2003)

Sahin, Owens, "Novel FVM applied to lid-driven cavity…", IJNMF 42 (2003) *Newtonian, good review (regularization)*

(after the present work): Fattal, Kupferman, "Log conformation tensor", JNNFM 123, 281 (2004)

EQUATIONS

Mass

$$\nabla \cdot \boldsymbol{u} = 0$$

Motion

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \nabla \cdot \left[\eta_s (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T) \right]$$

Constitutive: FENE-MCR model

$$\lambda_{eq} \frac{D\boldsymbol{\tau}}{Dt} + \boldsymbol{\tau} = \eta_p (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}}) + \lambda_{eq} (\boldsymbol{\tau} \cdot \nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}} \cdot \boldsymbol{\tau})$$

$$\beta = \eta_s / \eta_0 = 0.79$$

$$L^2 = 100$$

$$\lambda_{eq} = \lambda / \left\{ (L^2 + \frac{\lambda}{\eta_p} tr(\boldsymbol{\tau})) / (L^2 - 1) \right\}$$

FINITE VOLUME METHOD

Mass conservation: fluxes

$$\sum_f F_f = 0$$

Momentum conservation:

$$a_P \boldsymbol{u}_P^{**} = \sum_F a_F \boldsymbol{u}_F^{**} - \nabla p^* + S_u[\nabla \cdot \boldsymbol{\tau}^{(n+1)}] + S_u^{HOS} + \frac{\rho V}{\delta t}\left(2.0\boldsymbol{u}_P^{(n)} - 0.5\boldsymbol{u}_P^{(n-1)}\right)$$

Stress evolution

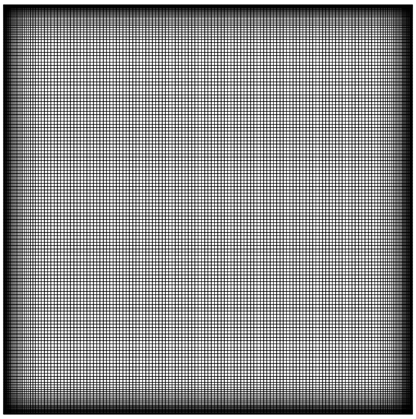
$$a_P^\tau \boldsymbol{\tau}_P^{(n+1)} = \sum_F a_F^\tau \boldsymbol{\tau}_F^{(n+1)} + S_\tau[\nabla \boldsymbol{u}^*] + S_\tau^{HOS} + \frac{\lambda_{ef} V}{\delta t}\left(2.0\boldsymbol{\tau}_P^{(n)} - 0.5\boldsymbol{\tau}_P^{(n-1)}\right)$$

Time advancement: 2nd order backward scheme

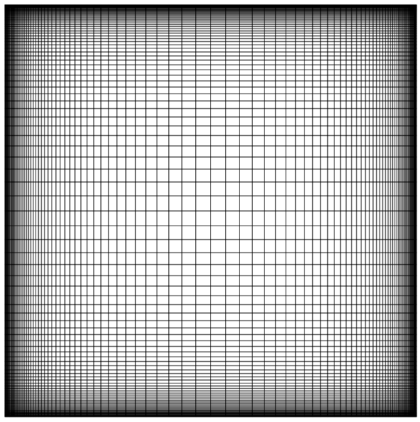
Details of meshes

Grid	Type	Numb. Cells	Min/Max cell space	Cell Expan. Factor	Delta t
1	40x40 U	1,600	0.025	1.0	0.0025
2	100x100 U	10,000	0.01	1.0	0.001
3	100x100 NU	10,000	0.001/ 0.036	1.0761	0.001/ 0.0005
4	160x160 NU	25,600	0.001/ 0.008	1.0732	0.001/ 0.0005

Mesh-4: 160x160 NU

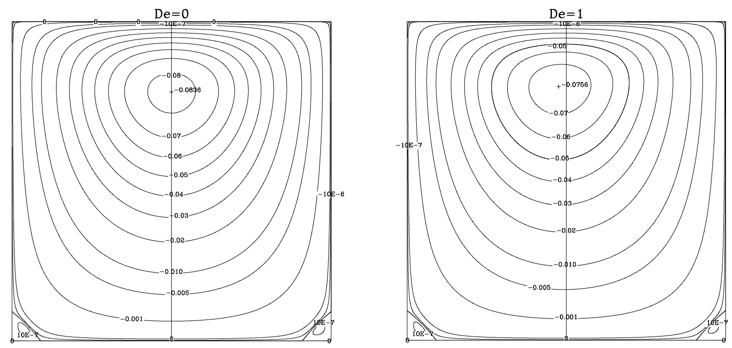


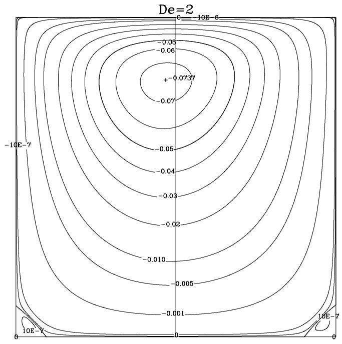
Mesh-3: 100x100 NU

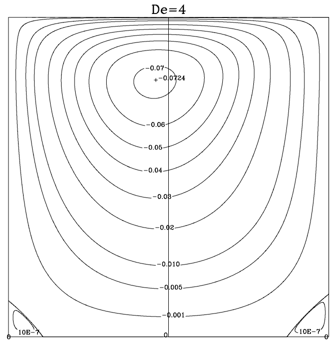


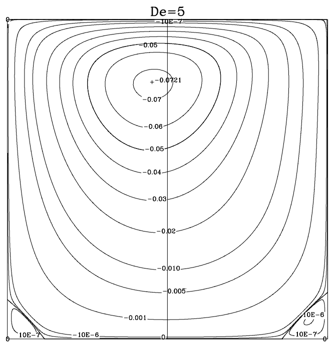
Steady flow: Streamlines

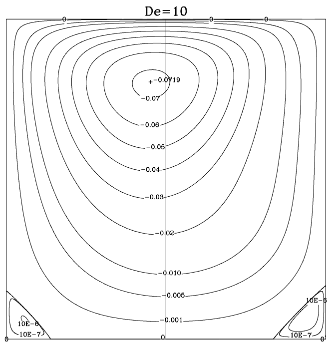
(mesh 160x160 NU)




De=2

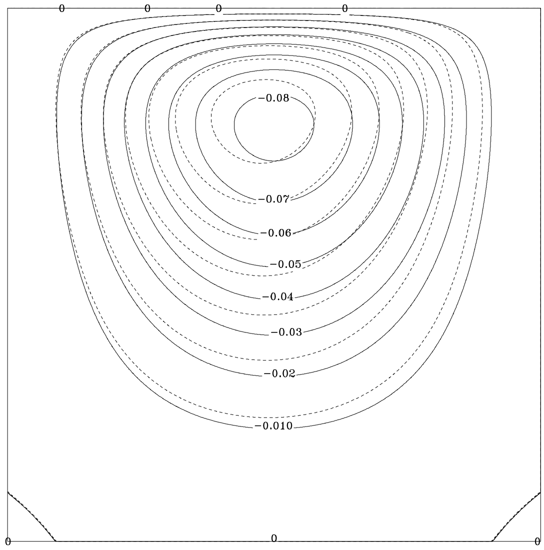

De=4


De=5

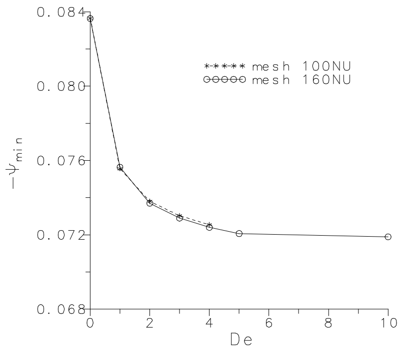

De=10

Streamline Comparison:

Newtonian (solid), Viscoleastic De=1 (dashed)



Variation of main eddy
recirculation with elasticity



Re=0

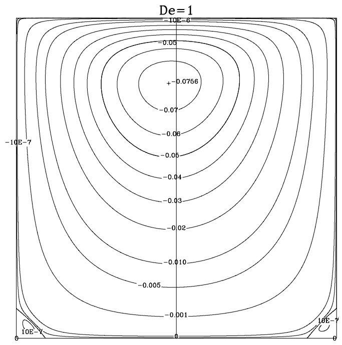
Note:

(unregularized)

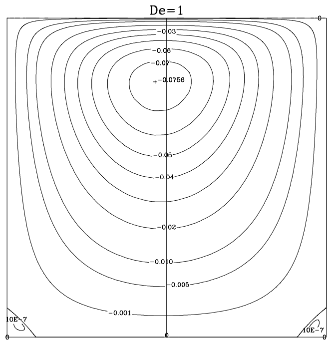
Barragy & Carey C&F (1997): -0.1000758
Botella & Peyret C&F (1998): -0.10007627
Present: -0.10006 (mesh 200 U)
Sahin & Owens:-0.100054

Mesh refinement – Streamlines: De=1

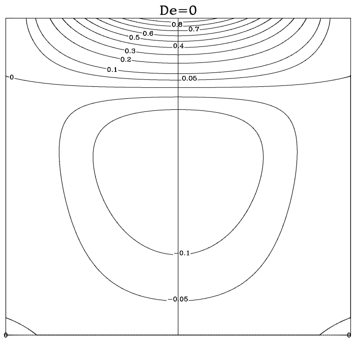
Mesh-4 160x160 NU

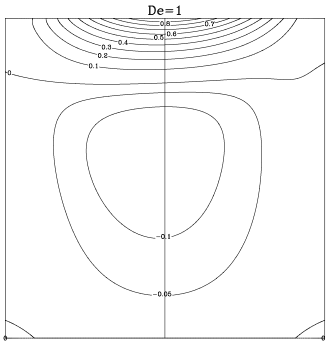


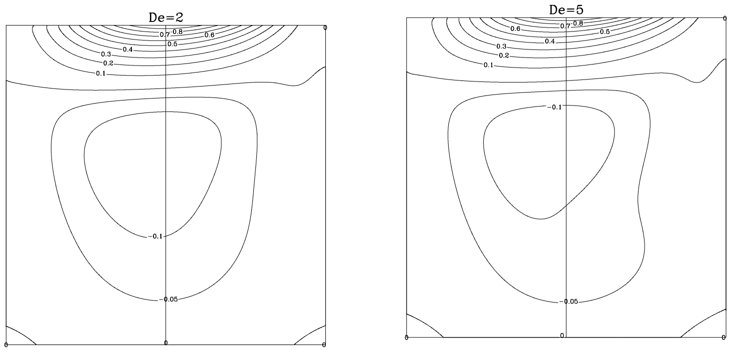
Mesh-3 100x100 NU



U-VELOCITY CONTOURS



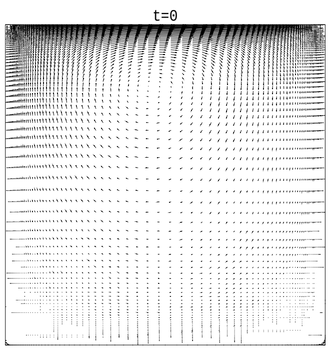




N1: **Tkk**: **Lambda**:

Time dependent flow: recoil from De=4

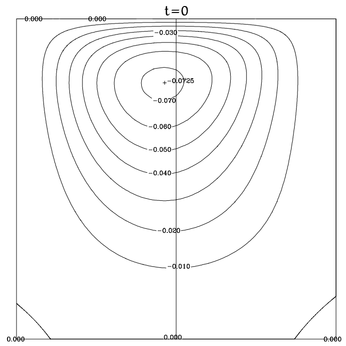
Velocity Vectors



$$time = t \times 10^{-2} \qquad \Delta t = 0.5 \times 10^{-4}$$

Time dependent flow: recoil from De=4

Instantaneous Streamlines (mesh 100x100 NU)



$$time = t \times 10^{-2}$$

Time dependent flow: **Other examples**

recoil from **De=1** and **2**

(mesh-4 160x160 NU)

$\Delta t = 0.5 \times 10^{-4}$

up to:

$\Delta t = 5 \times 10^{-4}$

De=1; M4

Streamlines

1-normal stress

Velocity Vectors

De=2; M4

De=2; M4

De=2; M4

Newtonian case:

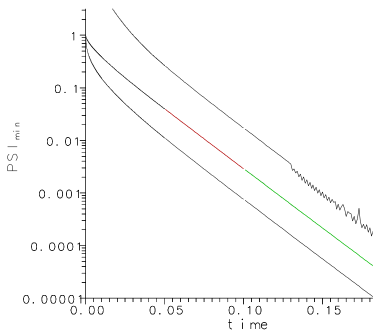
Decay of max. velocity and max. and min. streamfunction

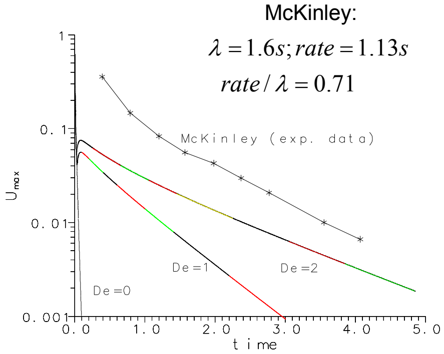
Viscoelastic cases: recoil

McKinley:

$\lambda = 1.6s; rate = 1.13s$

$rate/\lambda = 0.71$





T=1/52.0

T(Stokes)=1/52.99

Re=0; Mesh 160x160 NU

De=0, T=0.019

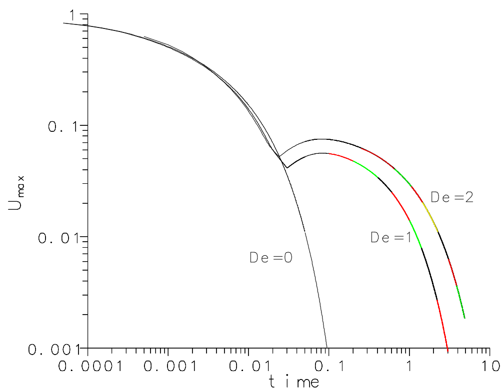
De=1, T=0.75

De=2, T=1.48

T/De=0.75; 0.74

Velocity decay: compare Newtonian and viscoelastic

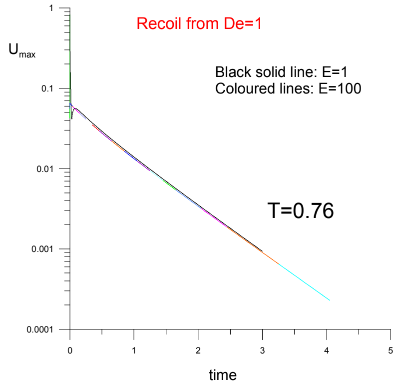
initial: diffusion time; later: relaxation time

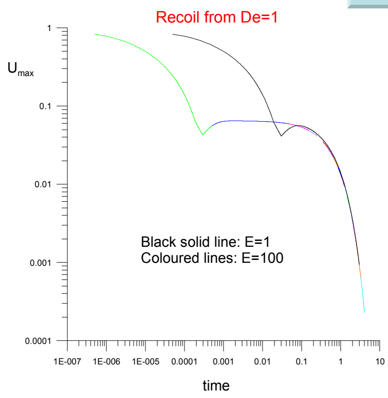


Effect of Elasticity Number

(different time scales)

$$E = \frac{\eta_0 \lambda}{\rho H^2} = \frac{De}{\mathrm{Re}}$$

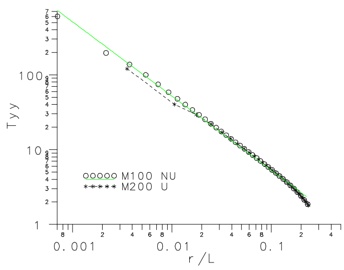




3 time scales: $t_C = \frac{H}{U}; t_D = \frac{H^2}{\eta_0/\rho}; \lambda$

$\Delta t = 0.5\times10^{-6} \rightarrow 8\times10^{-5}$

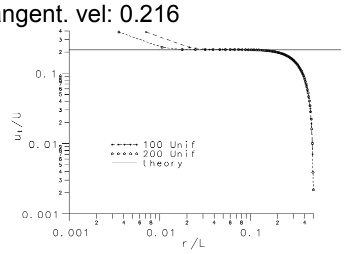
Corner behaviour and mesh refinement: Newtonian



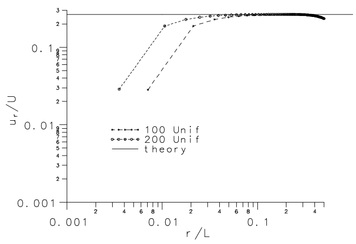
Original problem

Best fit slope: -0.99

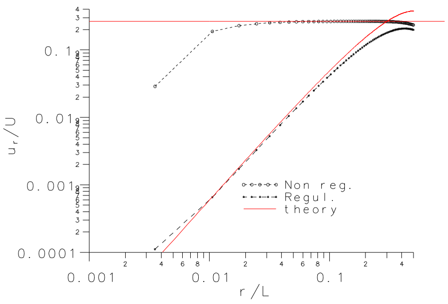
Tangent. vel: 0.216



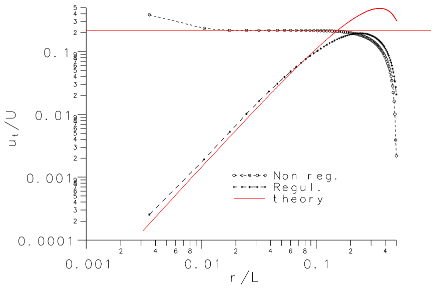
Radial vel.: 0.266



Newtonian: Regularized and Unregularized Problems



Radial vel. at 45 degrees

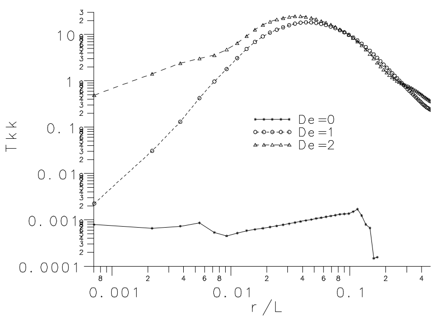


Tangential vel.

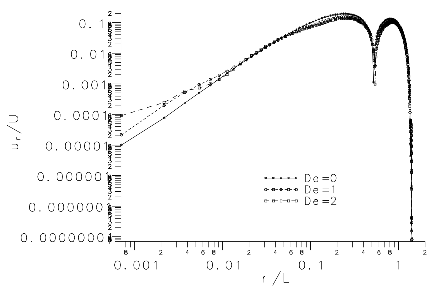
(Mesh 200x200 U)

Viscoelastic : corner bahaviour

(regularized problem)



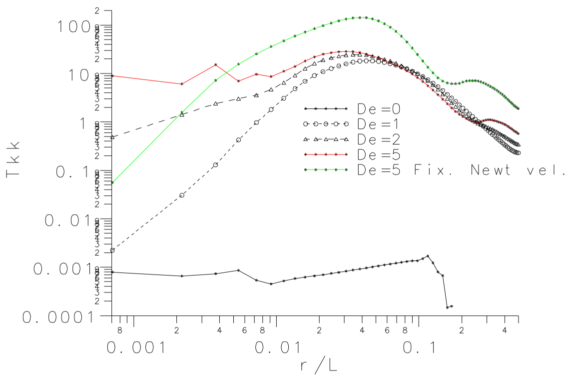
Trace of stress (45. deg.)



Radial velocity

Mesh 160x160 NU

Viscoelastic: effect of using a fixed velocity (newtonian)



CONCLUSIONS

- Regularized cavity amenable to higher De simulations
- FENE-CR, L2=100, gives solutions for De from 0 to 10
- Recoil and main vortex structures are captured
- Waiting for more detailed comparison
- All this for 2D simulations

ACKNOWLEDGMENTS

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