Bifurcation phenomena in strong extensional flows
(in a cross-slot geometry)

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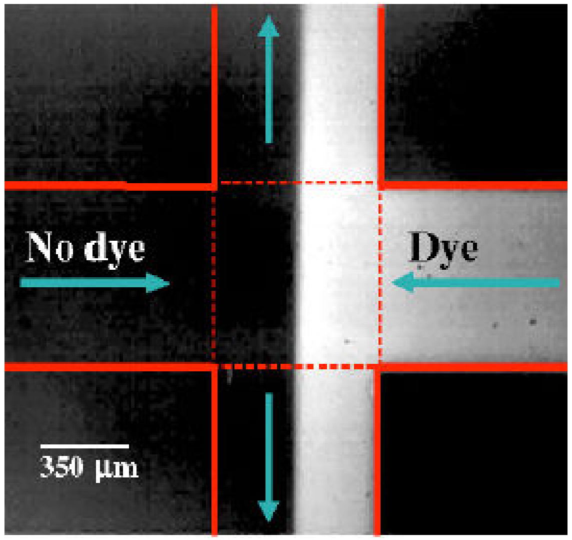
Outline

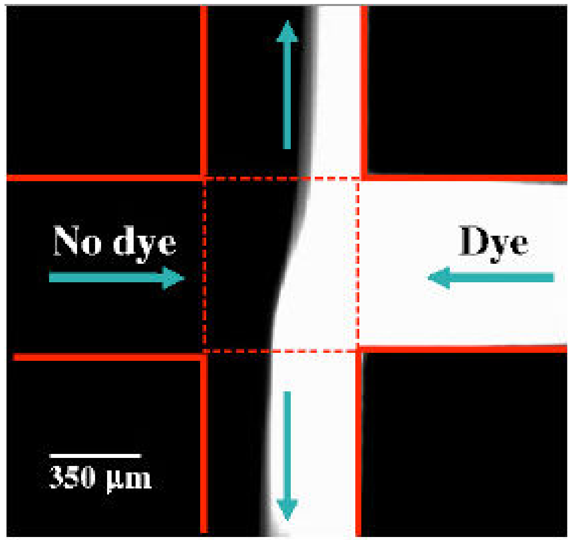
- Motivation
- Governing equations and numerical methods
- Results:
  - Weissenberg number Wi_o at the stagnation point
  - Couette correction C
  - Asymmetry parameter DQ
- Discussion:
  - Stability

Motivation – Why the cross-slot?

- Strong extensional nature and free stagnation point
- Steady-state flow asymmetries at negligible Reynolds number
- Time-dependent flow for higher Deborah number



Newtonian, Re<0.01



PAA solution, Re<0.01, De=4.5

Arratia et al., Phys Rev Lett. 96 (2006) 144502.
Same year: Pathak and Hudson, Macromolecules. 39 (2006) 8782–8792.
Earlier observations: Gardner et al., Polymer. 23 (1982) 1435–1442.

Motivation – Goals

- Accurate benchmark data:
  - Upper-Convected Maxwell (UCM) model
  - Oldroyd-B model
  - Simplified linear Phan-Thien-Than (sPTT) model

- Mechanism of Bifurcation:
  - Application of criteria developed to predict the onset of time-dependent instabilities

Governing equations

- Inertialess $(Re \to 0)$, isothermal, incompressible flow.
  - Conservation of mass: $\nabla \cdot \mathbf{u} = 0$
  - Conservation of momentum: $-\nabla p + \nabla \cdot \boldsymbol{\tau} + \beta \eta_o \nabla^2 \mathbf{u} = \mathbf{0}$

UCM model for β=0

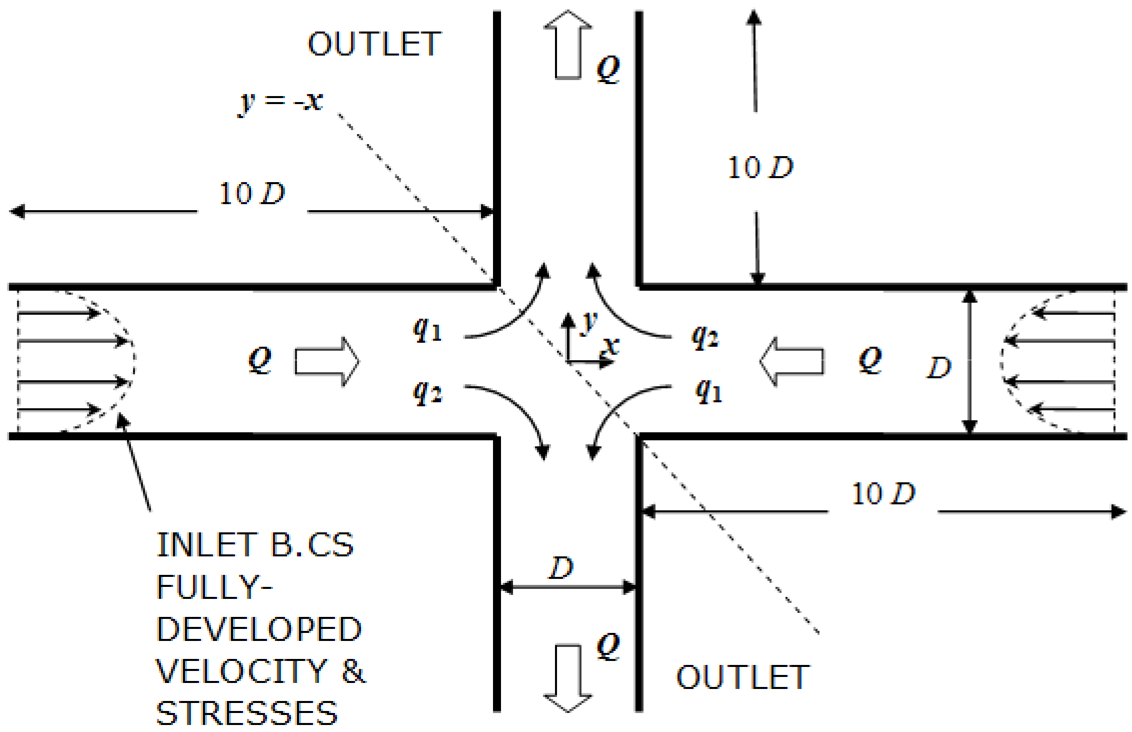
$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$

  - Constitutive equation (sPTT model):

$$\left(1 + \frac{\lambda \varepsilon}{(1-\beta)\eta_o} \mathrm{Tr}(\boldsymbol{\tau})\right)\boldsymbol{\tau} + \lambda\left[\frac{\partial \boldsymbol{\tau}}{\partial t} + \nabla \cdot \mathbf{u}\boldsymbol{\tau}\right] = (1-\beta)\eta_o\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right) + \lambda\left(\boldsymbol{\tau} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \boldsymbol{\tau}\right)$$

UCM or Oldroyd-B model for ε=0

Geometry



OUTLET
Q
$y = -x$
10 D
10 D
q_1
q_2
y
x
q_2
q_1
Q
Q
D
10 D
INLET B.CS
FULLY-
DEVELOPED
VELOCITY &
STRESSES
D
Q
OUTLET

Numerical methods

- Fully implicit finite volume method (Oliveira et al., 1998)
  - Structured, collocated and non-orthogonal meshes
  - Time-marching algorithm
  - Diffusive terms: central differences scheme (CDS)
  - Advective terms, high resolution scheme: CUBISTA (Alves et al., 2003)
  - Log-conformation technique for polymeric stress tensor (Afonso et al., 2009)

Oliveira et al., J Non-Newton Fluid. 79 (1998) 1–43.
Alves et al., Int J Numer Meth Fl. 41 (2003) 47–75.
Afonso et al., J Non-Newton Fluid. 157 (2009) 55–65.

Richardson extrapolation

- Given three meshes with cell spacing *h*, *2h* and *4h*, for variable φ,

$$p = \frac{\ln\left(\dfrac{\varphi_{2h} - \varphi_{4h}}{\varphi_h - \varphi_{2h}}\right)}{\ln 2}$$

order of convergence

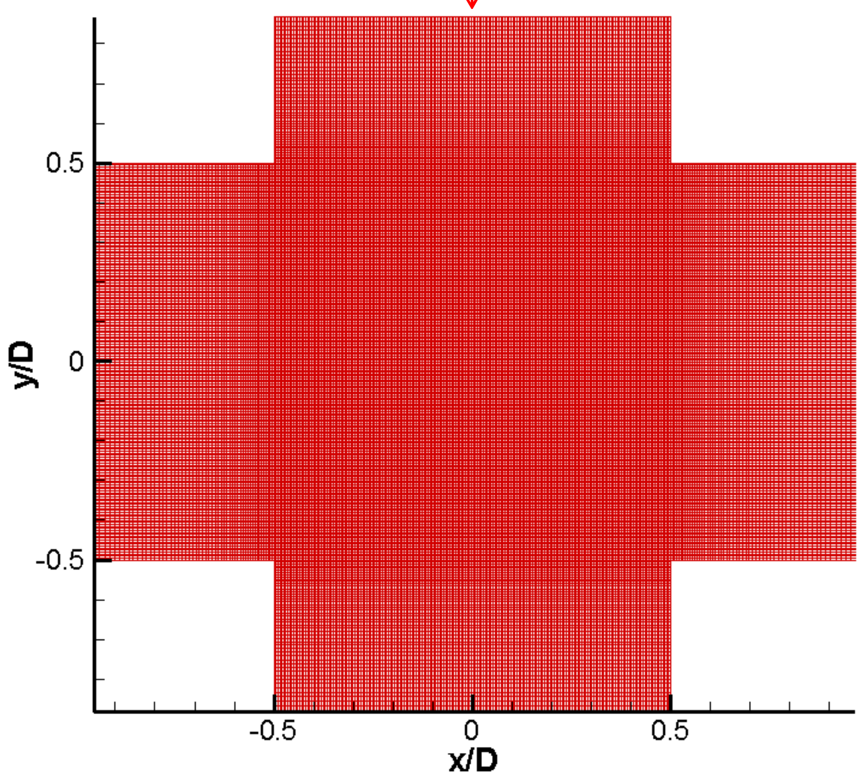
$$\varphi_0 = \frac{2^p \varphi_h - \varphi_{2h}}{2^p - 1}$$

extrapolated value

Ferziger and Peric. Int J Numer Meth Fl. 23 (1996) 1263–1274.

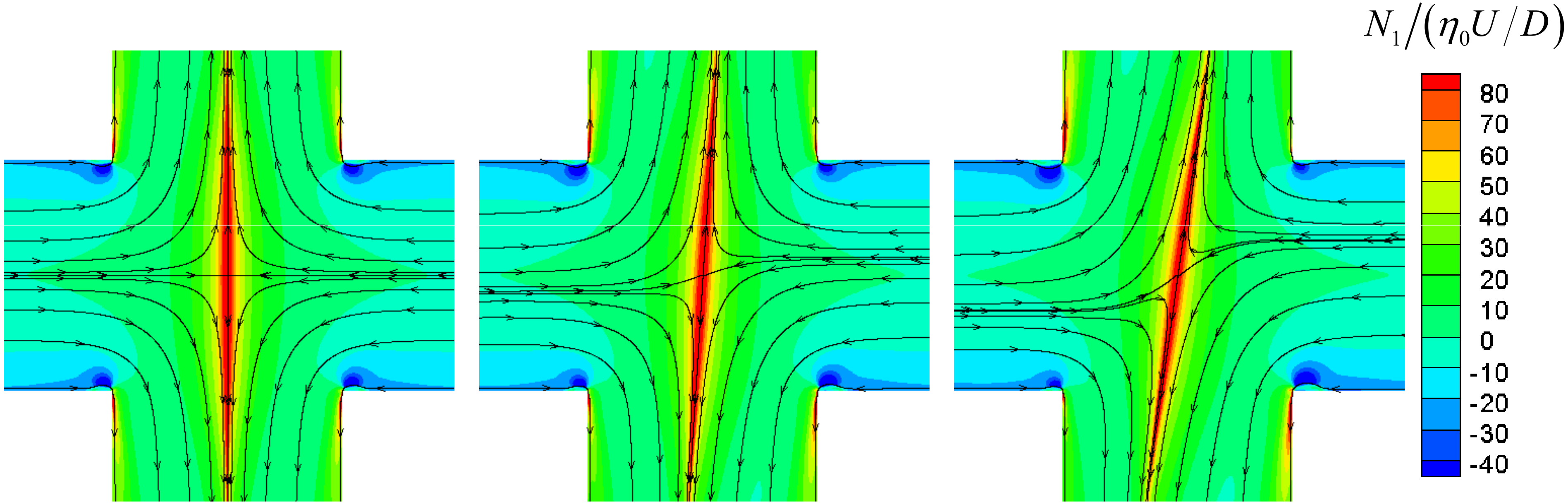
Meshes

Mesh	NC	DOF	$\frac{\Delta x_{min}}{D} = \frac{\Delta y_{min}}{D}$
M1	12801	76806	0.02
M2	50601	303606	0.01
M3	201201	1207206	0.005



Bifurcation – UCM model

Streamlines superimposed onto contour plots of N_1

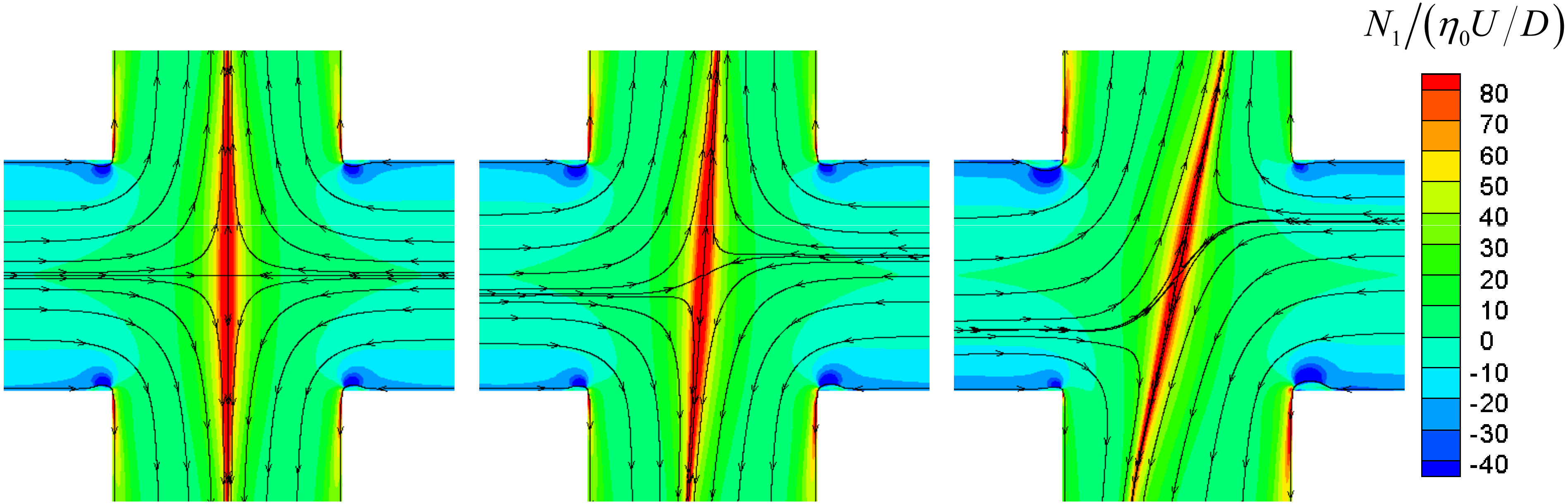


De=0.31
Steady symmetry

De=0.315
Steady asymmetry

De=0.33
Prior to unsteady flow

Bifurcation – Oldroyd-B model (β=1/9)
Streamlines superimposed onto contour plots of N_1



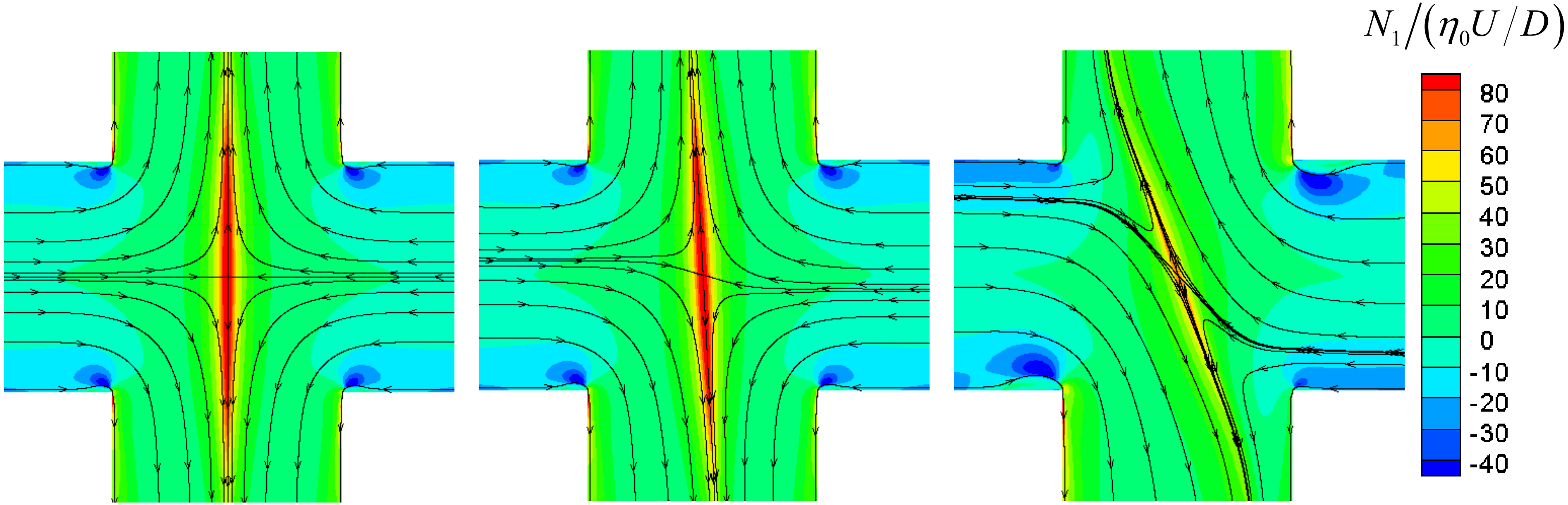
De=0.36
Steady symmetry

De=0.37
Steady asymmetry

De=0.42
Prior to unsteady flow

Bifurcation – sPTT model (β=1/9, ε=0.02)

Streamlines superimposed onto contour plots of N_1



De=0.5
Steady symmetry

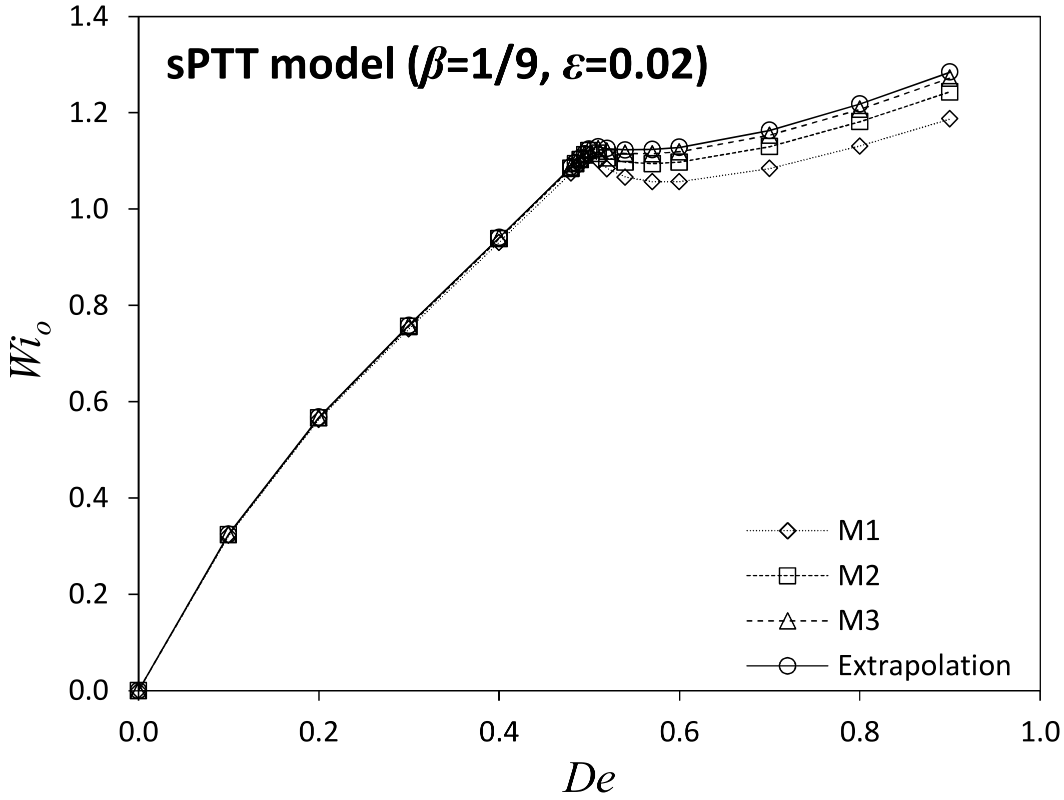
De=0.51
Steady asymmetry

De=0.9
Prior to unsteady flow

Mesh-wise convergence

Demonstration with Wi_o

$Wi_o = \lambda \dot{\varepsilon}_o$



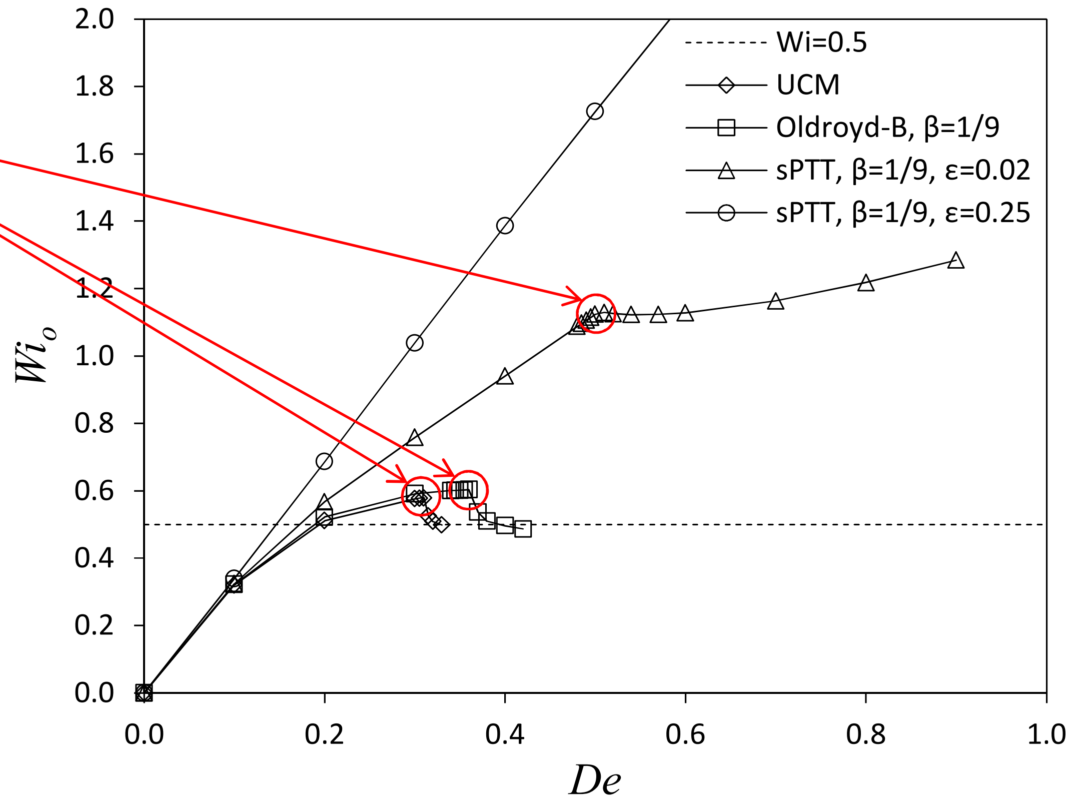
Results

Weissenberg number Wi_o at the stagnation point

Bifurcation

$Wi_o \downarrow$
=> polymer relaxes
=> stress is relieved
(Oliveira et al., 2009)

No bifurcation without sufficiently high stress
(Xi and Graham, 2009)
(Afonso et al., 2010)



Oliveira et al., J Non-Newton Fluid. 160 (2009) 31–39.
Xi and Graham, J Fluid Mech. 622 (2009) 145.
Afonso et al., J Non-Newton Fluid. 165 (2010) 743–751

Results – Couette Correction

Bifurcation

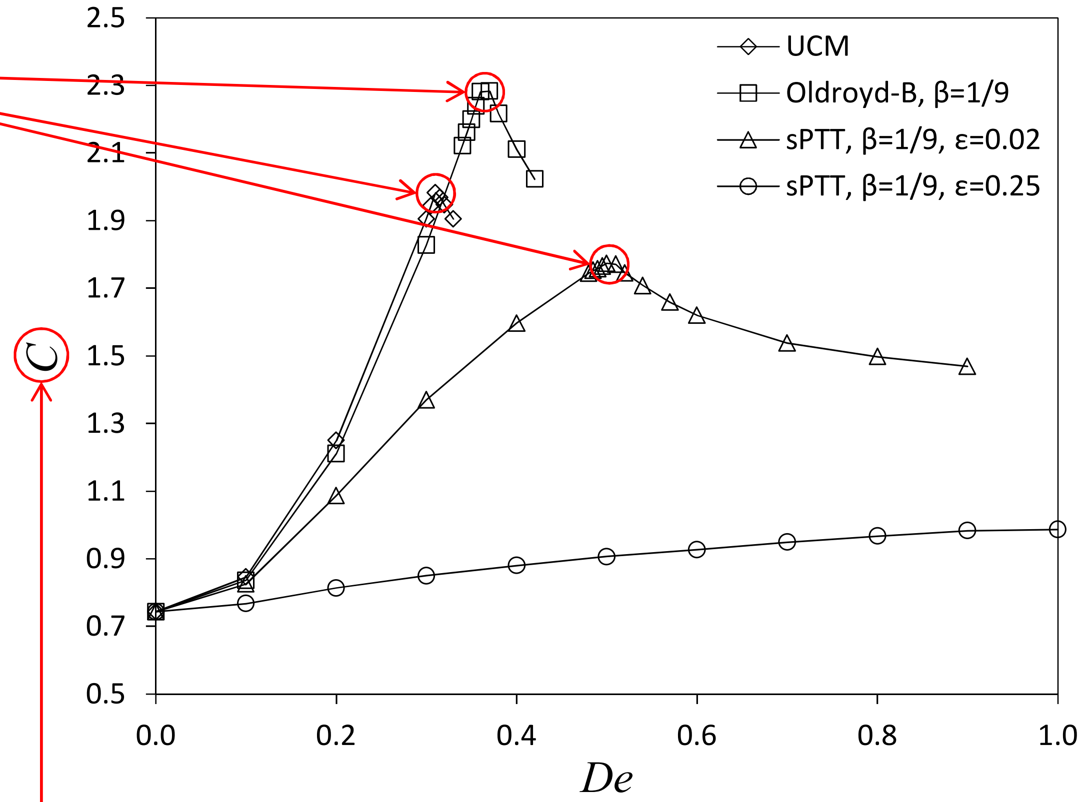
$C \downarrow$

=> Lower energy dissipation

(Poole et al., 2007)

(Oliveira et al., 2009)

(Afonso et al., 2010)



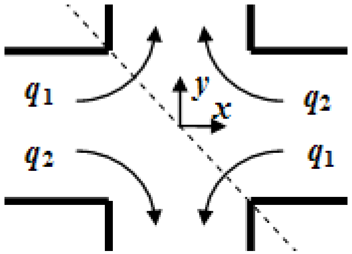
$$C \equiv \frac{\Delta p - \Delta p_{fd}}{2\tau_w}$$

Poole et al., Phys Rev Lett. 99 (2007) 164503.

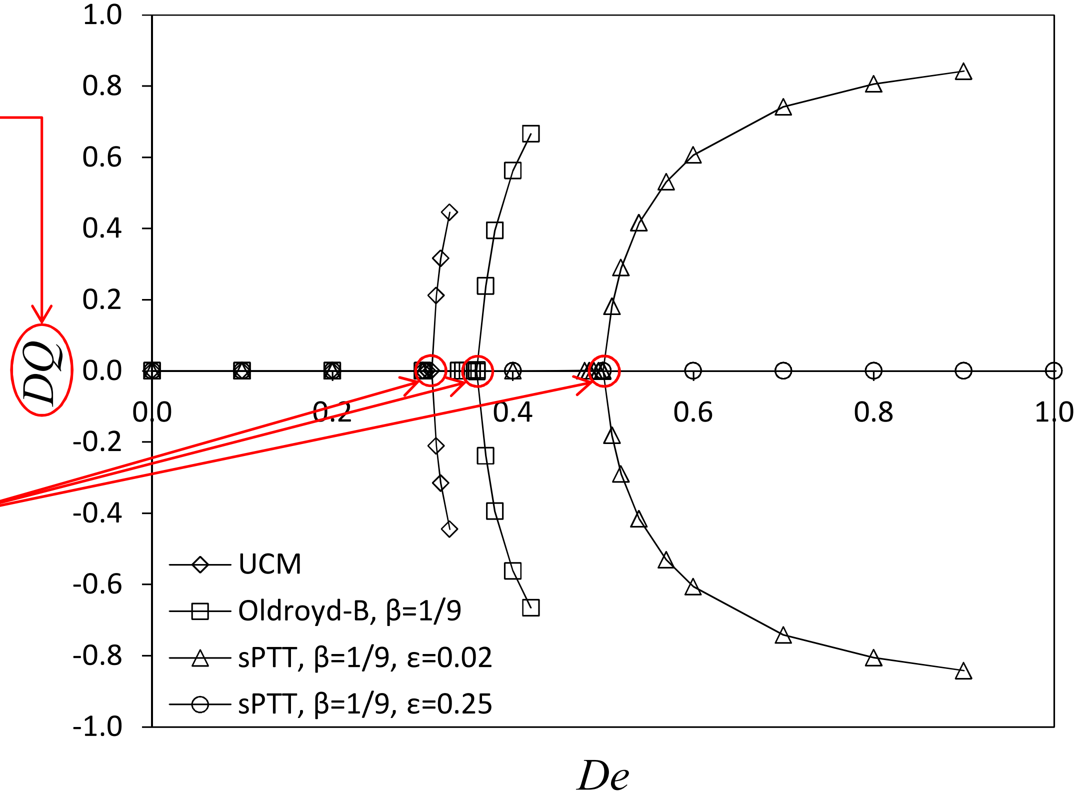
Oliveira et al., J Non-Newton Fluid. 160 (2009) 31–39.

Afonso et al., J Non-Newton Fluid. 165 (2010) 743–751.

Results – Asymmetry Parameter *DQ*



$$DQ = \frac{q_2 - q_1}{q_1 + q_2}$$



Supercritical Pitchfork Bifurcation

Locally, $DQ = A\sqrt{De - De_{CR}}$

$A = A(\varepsilon)$

$De_{CR} = De_{CR}(\beta, \varepsilon)$

(Rocha et al., 2009)

Rocha et al., J Non-Newton Fluid. 156 (2009) 58-69.

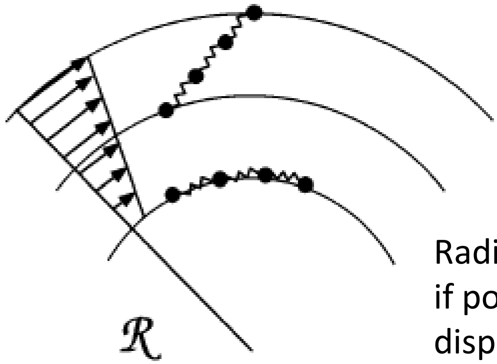
Discussion – Stability

- Effects of bifurcation
  - Stress is relieved
  - Pressure drop decreases
- => Greater stability
- Confirmation: analysis of bifurcation using instability criteria
  - M number (McKinley et al., 1996)
  - K number (Dou and Phan-Thien, 2008)

McKinley et al., J Non-Newton Fluid. 67 (1996) 19-47.
Dou and Phan-Thien, Korea-Aust Rheol J. 20 (2008) 15-26.

Discussion – M number

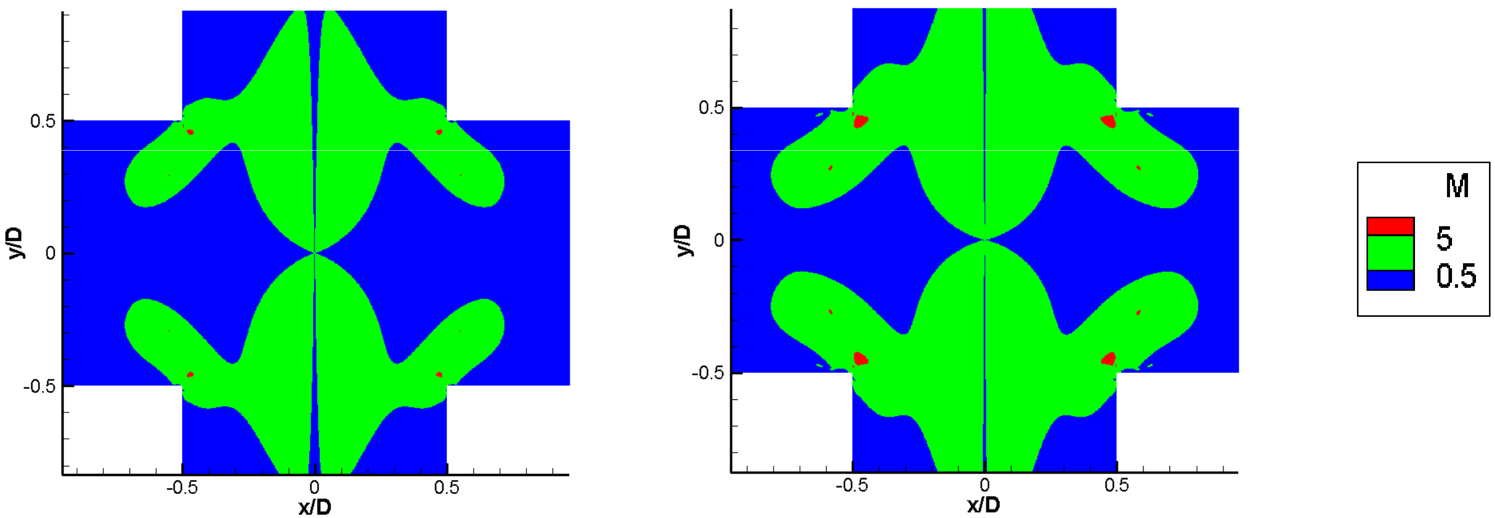
$$M = \left[\frac{\lambda U}{R} \frac{\tau_{ss}}{\eta_o \dot{\gamma}} \right]^{1/2}$$



Radial stress is generated if polymer molecules are displaced by disturbances.

(Pakdel and McKinley, 1996)

sPTT
β=1/9
ε=0.02



De=0.3 *De*=0.5

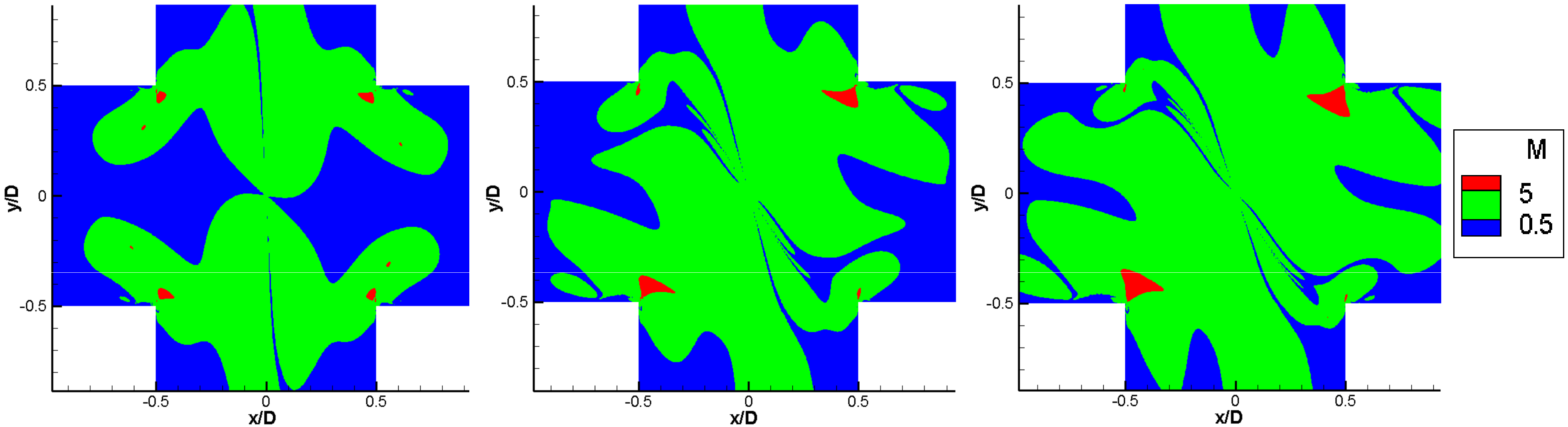
Pakdel and McKinley, Phys Rev Lett 77 (1996 2459-62.

M number definition: McKinley et al., J Non-Newton Fluid. 67 (1996) 19-47.

First description of mechanism: Larson et al., J Fluid Mech 218 (1990) 573-600.

Discussion – M number

sPTT, β=1/9, ε=0.02



De=0.51 *De*=0.7 *De*=0.9

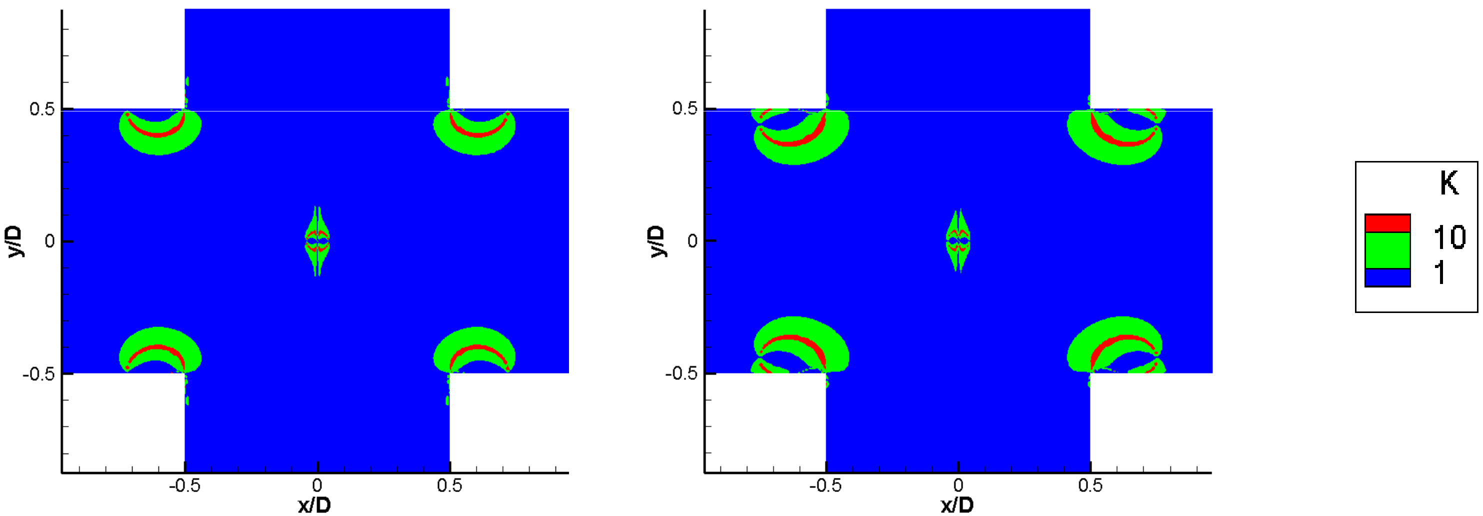
According to M, no unstable regions near the stagnation point or birefringence strand
=> M number fails to predict bifurcation

Discussion – K number

$$K = \left| \frac{\partial E / \partial n}{\partial E / \partial s} \right| \approx \left| \frac{\partial p / \partial n}{\partial p / \partial s} \right| = \left| \frac{v \dfrac{\partial p}{\partial x} - u \dfrac{\partial p}{\partial y}}{u \dfrac{\partial p}{\partial x} + v \dfrac{\partial p}{\partial y}} \right|$$

Disturbances are amplified if the transverse energy gradient is large relative to the streamwise energy gradient.

sPTT
β=1/9
ε=0.02



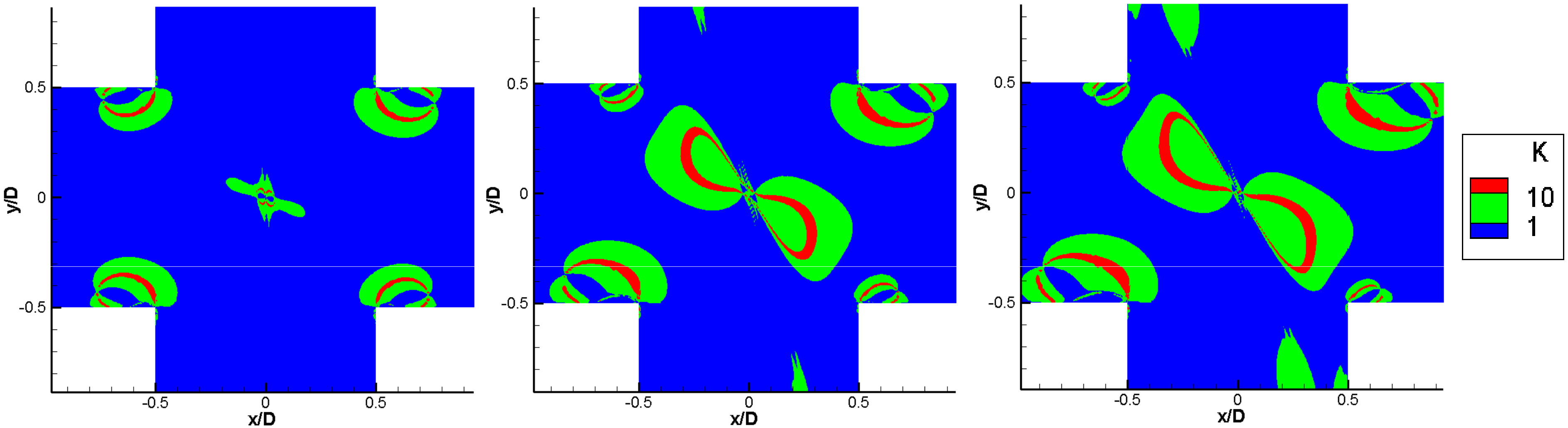
De=0.3 *De*=0.5

K number definition: Dou, 2005, arXiv:nlin/0501049v2.

Adaptation to viscoelastic flows: Dou and Phan-Thien, Korea-Aust Rheol J. 20 (2008) 15-26.

Discussion – K number

sPTT, β=1/9, ε=0.02



De=0.51 De=0.7 De=0.9

According to K, the birefringence strand tends to instability only after bifurcation
=> K number fails to predict bifurcation

Conclusion

- Bifurcation improves stability.
  - Stress is relieved, pressure drop is minimized.

- Instability criteria fail to predict bifurcation.
  - Both the M and K numbers provide no indication that bifurcation is about to occur.

- => The two known types of elastic instability, steady bifurcation and time-dependent flow, apparently do not share a common cause.

Acknowledgements

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Thank you for your attention!