Purely elastic instabilities in a cross-slot flow

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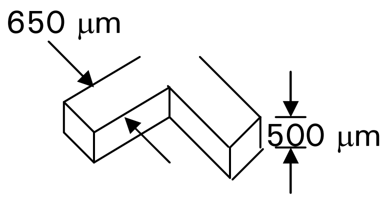
Outline

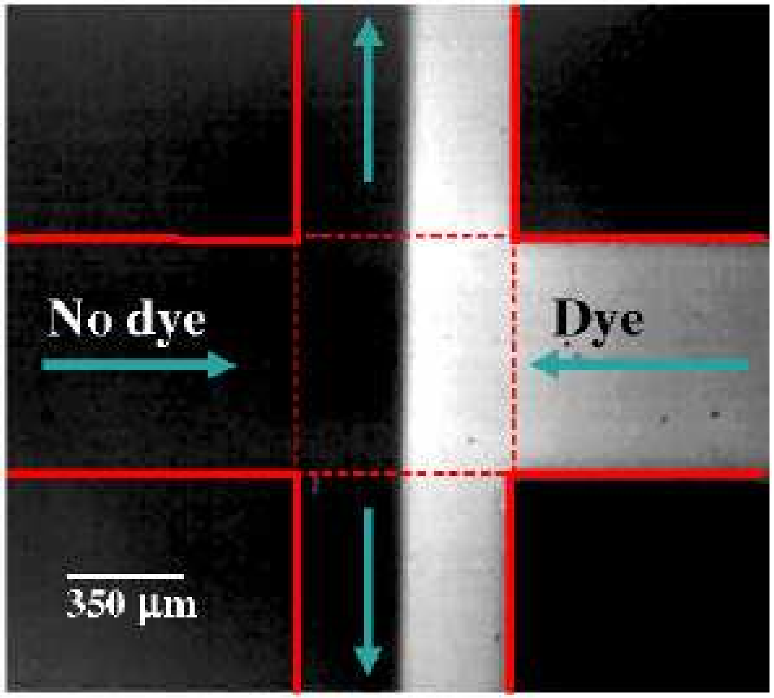
- *Motivation and previous work (UCM)*
- *Governing equations/numerical method*
- *Effect of solvent viscosity (Oldroyd-B) and inertia*
- *Effect of extensional viscosity (PTT)*
- *Conclusions*

Motivation

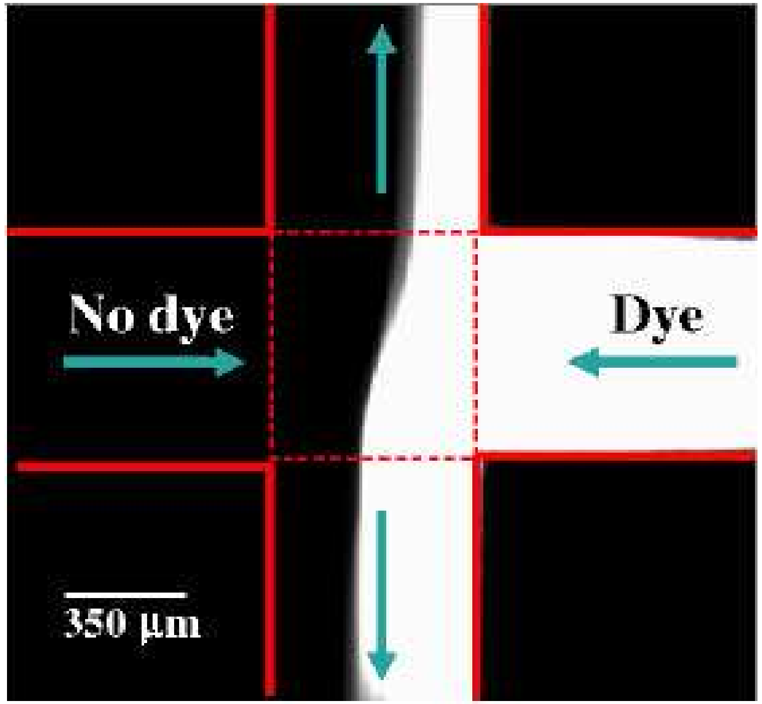
Arratia et al, *Physical Review Letters* **96,** 144502 (2006)

Microfluidic flow in a "cross channel" geometry





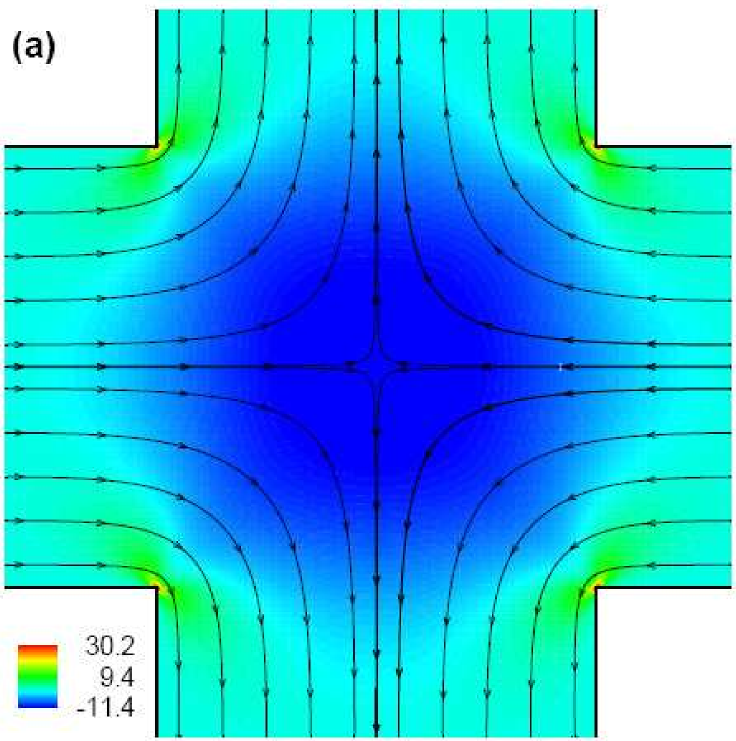
Newtonian:
Re < 10^{-2}



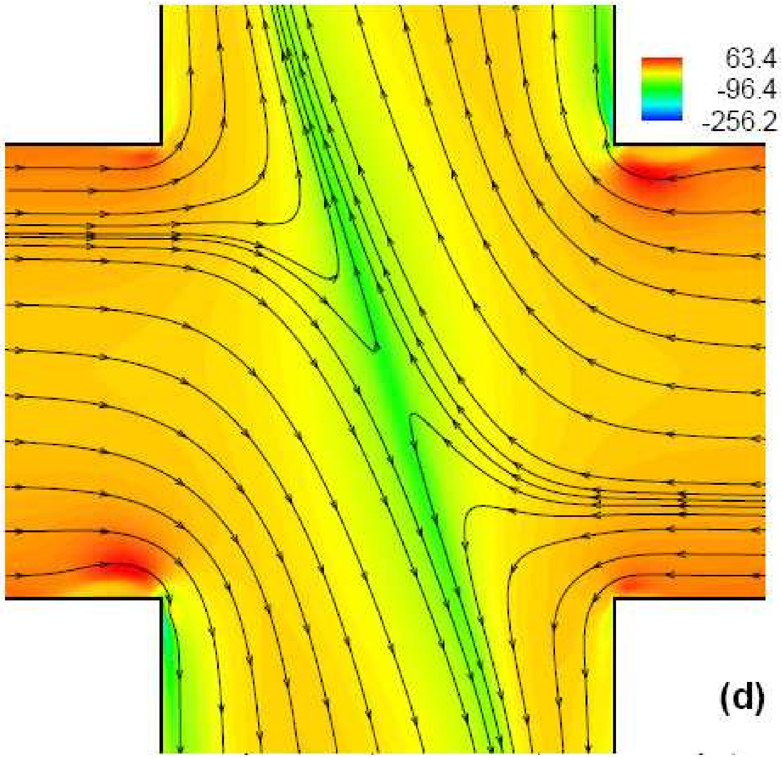
PAA Boger fluid:
Re < 10^{-2} (De=4.5)

Poole, Alves and Oliveira, accepted in *Physical Review Letters* (2007)

Successfully used a numerical technique with a *simple* viscoelastic constitutive equation (UCM) to model this steady asymmetry under creeping-flow conditions.



Newtonian



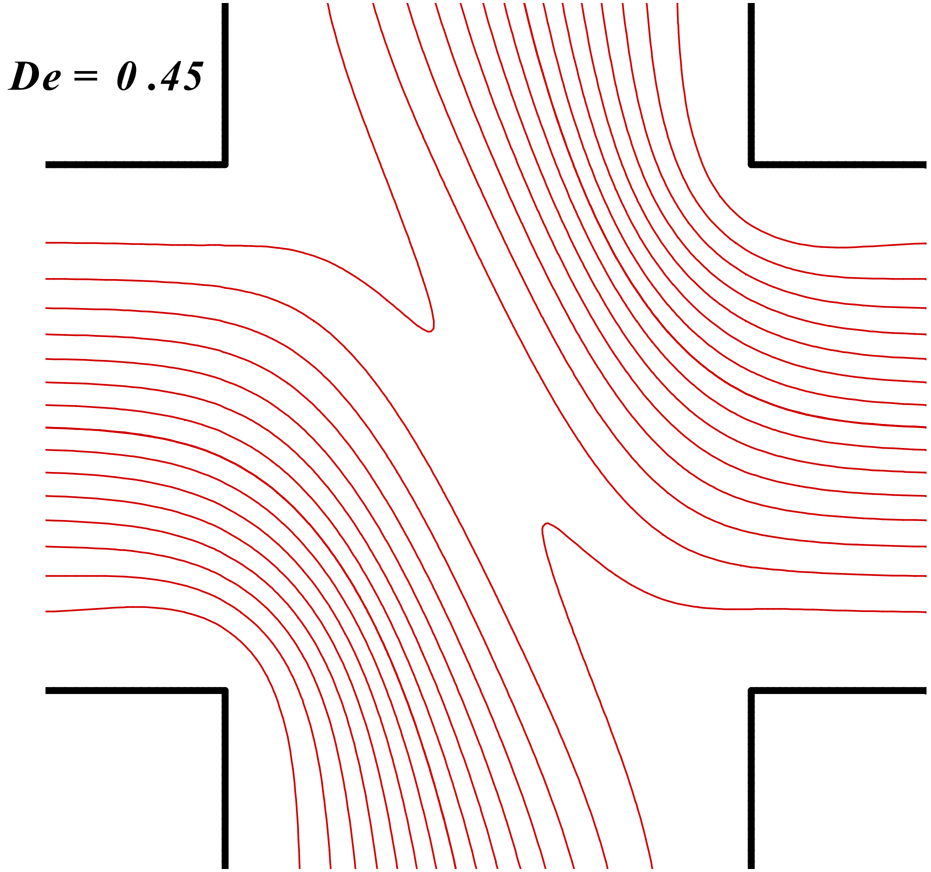
De=0.45

Motivation and Previous work

Poole, Alves and Oliveira, accepted in *Physical Review Letters* (2007)

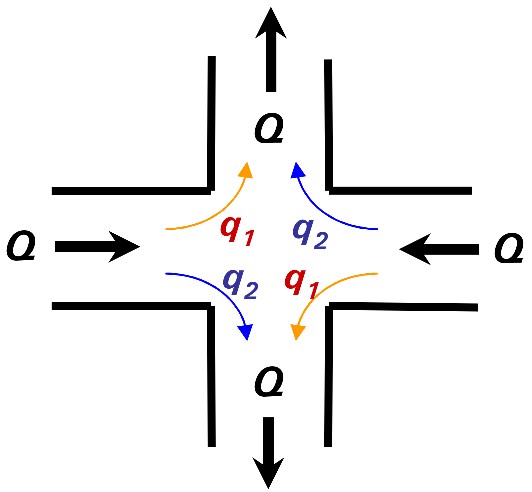
> Successfully used a numerical technique with a *simple* viscoelastic constitutive equation (UCM) to model this steady asymmetry under creeping-flow conditions.

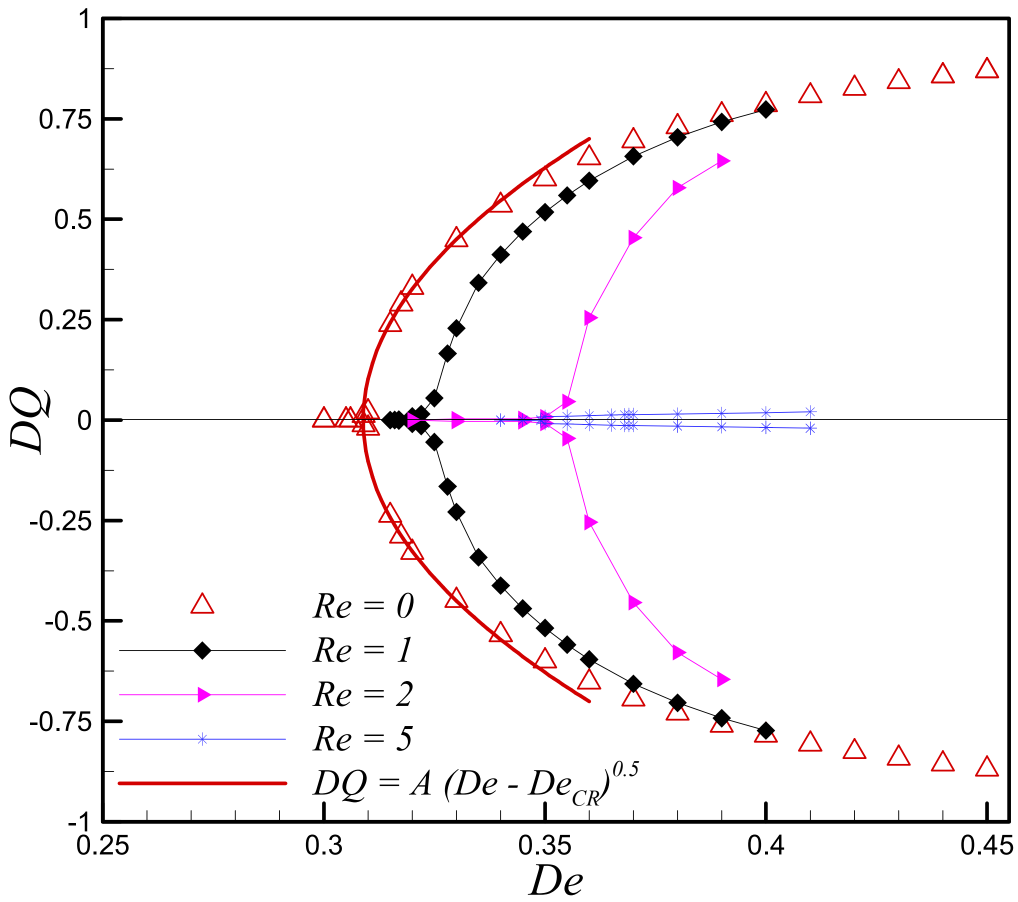
Streamlines →



Poole, Alves and Oliveira, accepted in *Physical Review Letters* (2007)

Purely-elastic: inertia decreases the degree of asymmetry and stabilizes the flow





Flow asymmetry:

$$DQ = \frac{q_2 - q_1}{q_2 + q_1} = \frac{q_2 - q_1}{Q}$$

DQ=0 → symmetric

DQ=±1 → completely asymmetric

Motivation and Previous work

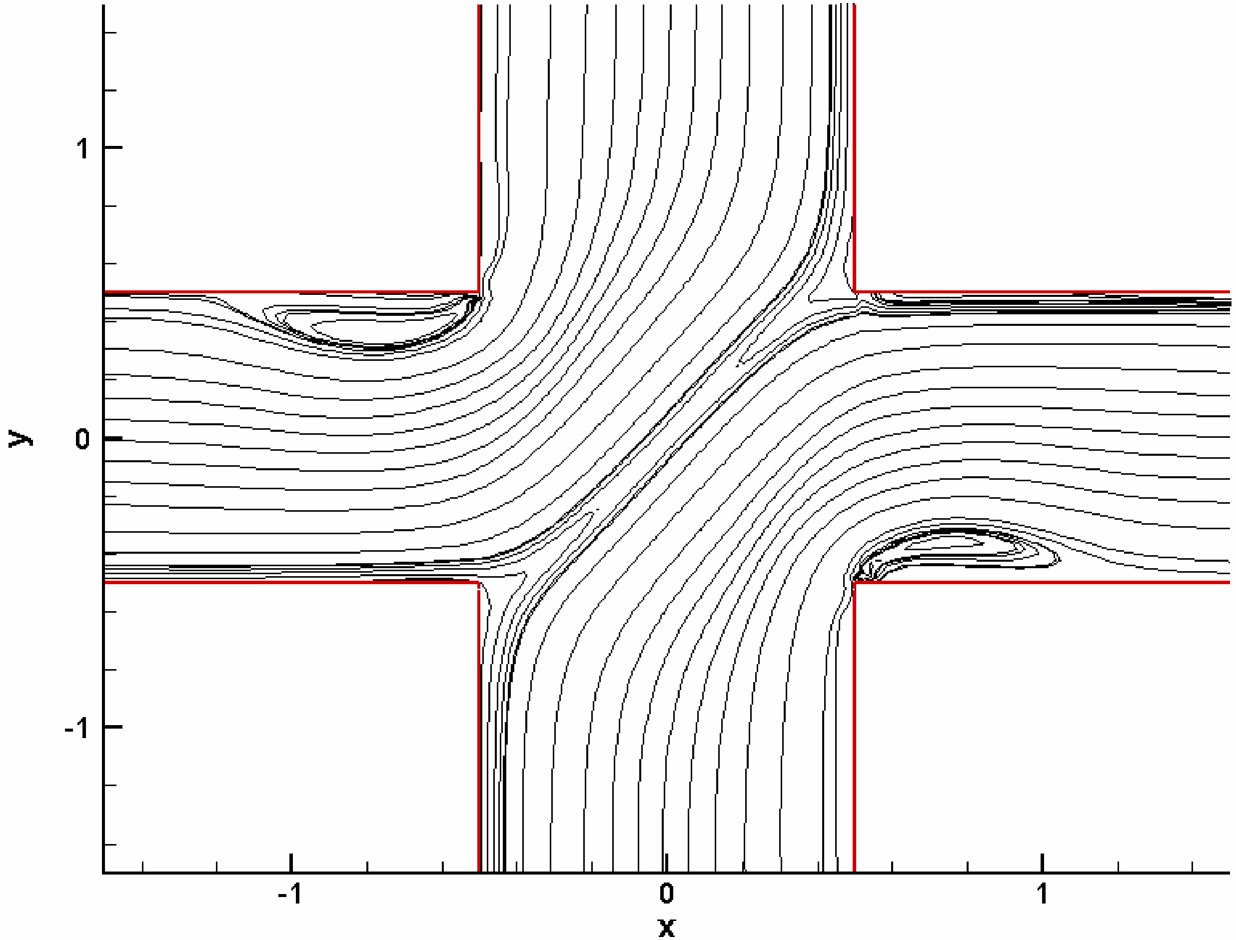
Afonso, Alves, Pinho and Oliveira, XV IWNMNNF, Rhodes (2007)

Secondary instability in which the flow becomes **unsteady** and fluctuates non-periodically in time.

UCM model

De=1.0

Re=1.0 →



Governing equations

- Incompressible **Viscoelastic fluid**

$$\nabla \cdot \mathbf{u} = 0$$ **(Mass conservation)**

$$\beta \equiv \frac{\eta_s}{\eta_o} = \frac{\eta_s}{\eta_s + \eta_P}$$

$$\lambda \left(\frac{D\mathbf{A}}{Dt} - (\nabla \mathbf{u})\mathbf{A} - \mathbf{A}(\nabla \mathbf{u})^T \right) = g(\mathbf{A})$$ **(Momentum conservation)**

$$\lambda \left(\frac{D\mathbf{A}}{Dt} - (\nabla \mathbf{u})\mathbf{A} - \mathbf{A}(\nabla \mathbf{u})^T \right) = f(\mathbf{A})$$ **(Constitutive equation, based on the conformation tensor, A)**

- **Examples:**

$$f(\mathbf{A}) = \begin{cases} (\mathbf{A} - \mathbf{I}), \\ (\mathbf{A} - \mathbf{I}), \\ -Y(\mathrm{tr}\mathbf{A})(\mathbf{A} - \mathbf{I}), \end{cases}$$

Upper Convected Maxwell - UCM (β=0)

Oldroyd-B (0<β<1)

Phan-Thien and Tanner (0<β<1),

with

$$Y(\mathrm{tr}\mathbf{A}) = \begin{cases} 1 + \varepsilon(\mathrm{tr}\mathbf{A} - 3) & \text{(linear)} \\ \exp\left[\varepsilon(\mathrm{tr}\mathbf{A} - 3)\right] & \text{(exponential)} \end{cases}$$

- **Finite-Volume Method**

  Oliveira, Pinho and Pinto (1998)

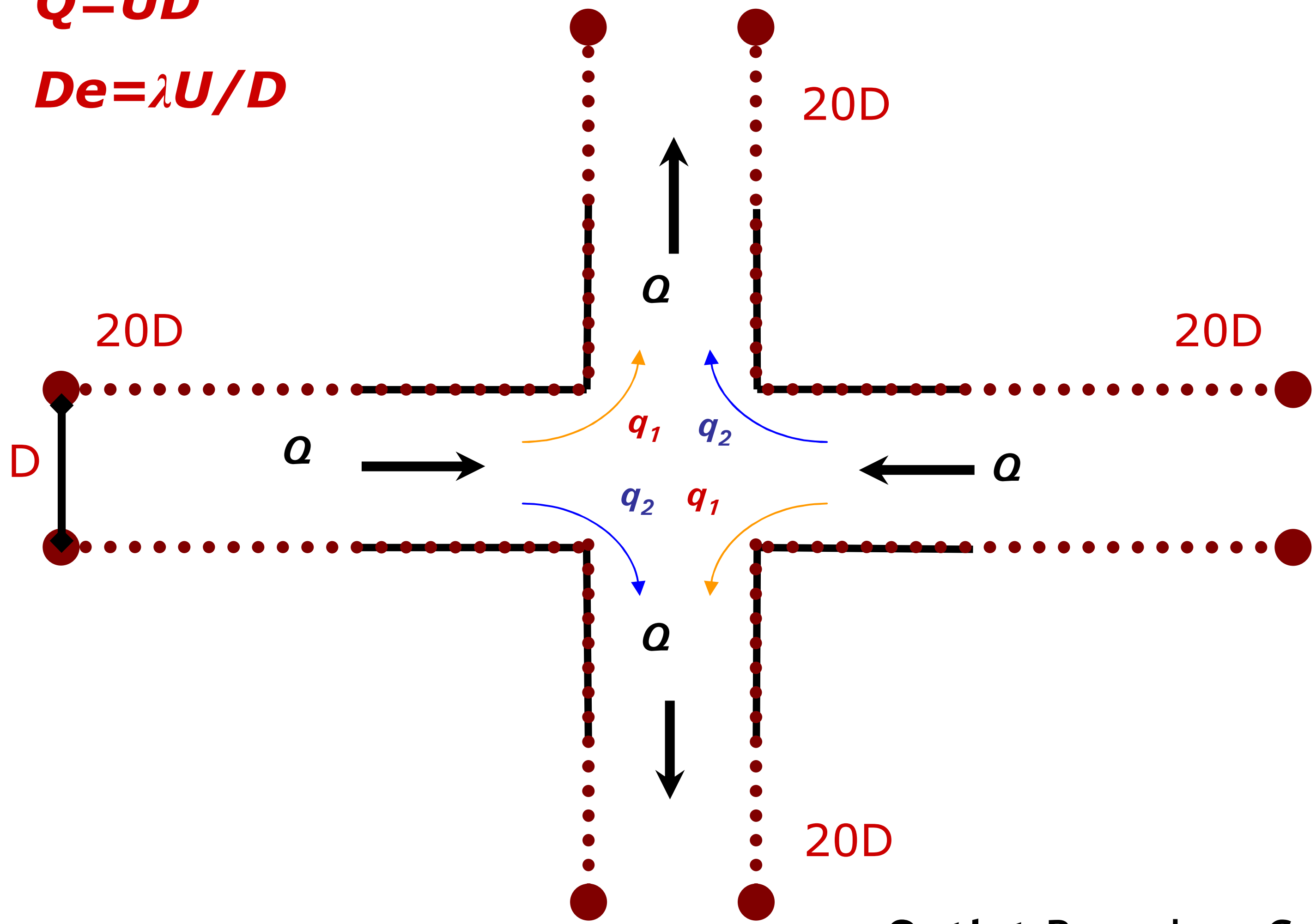
  Oliveira and Pinho (1999)

  - Structured, collocated and non-orthogonal meshes.
  - Discretization (formally 2nd order)
    - Diffusive terms: central differences (CDS)
    - Advective terms, high resolution scheme: **CUBISTA**

      Alves, Pinho and Oliveira (2003)
  - Dependent variables evaluated at cell centers;
  - Special formulations for cell-face velocities and stresses;

$Q=UD$

$De=\lambda U/D$



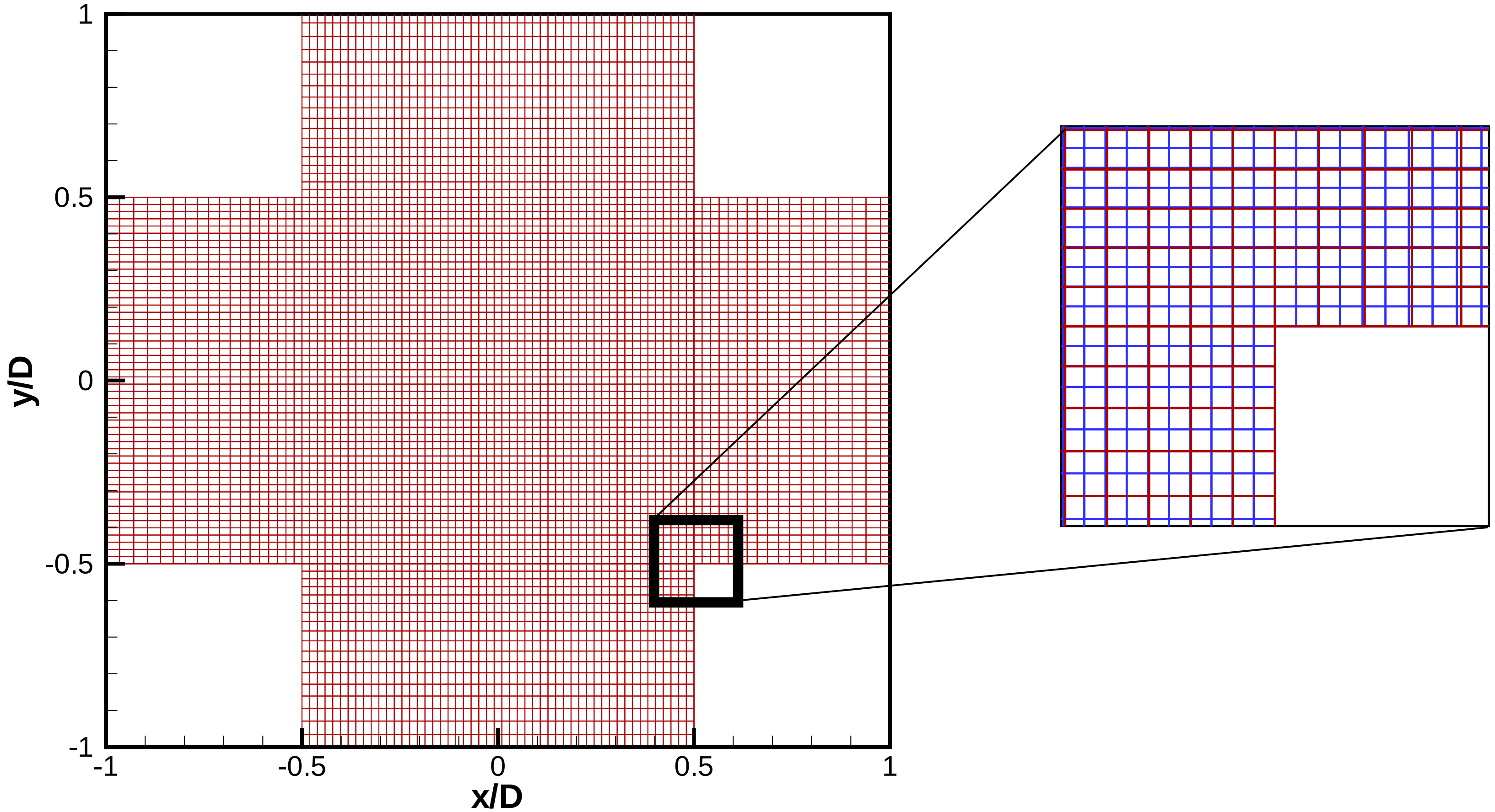
Inlet Boundary Conditions:

Fully-developed $u(y)$ and $\tau(y)$

Outlet Boundary Conditions:

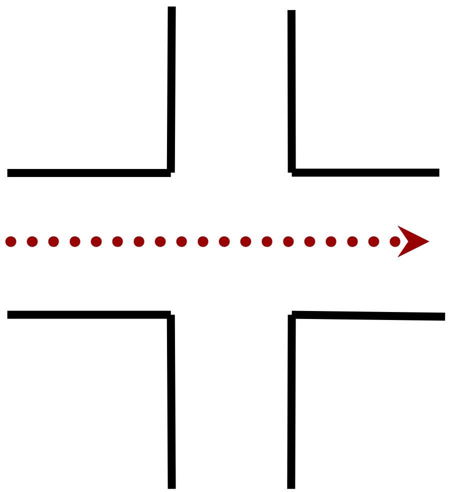
$\partial\phi/\partial y = 0$

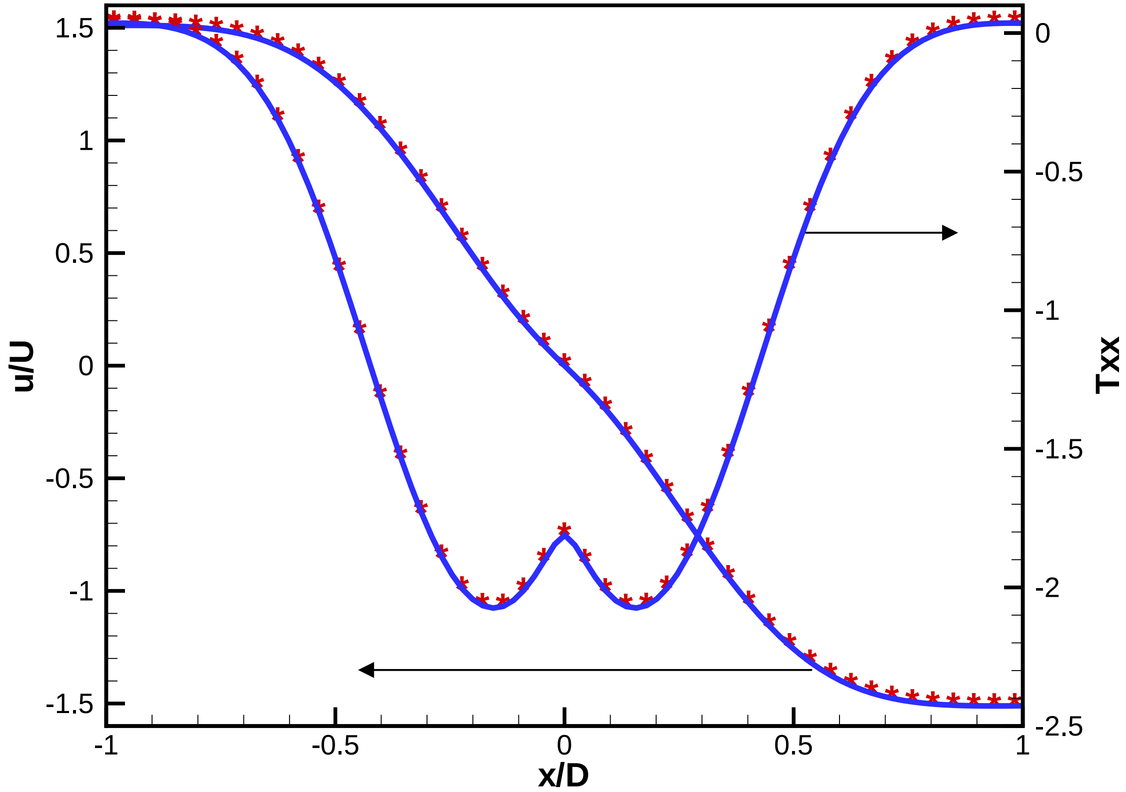
Effect of mesh refinement



	NC	DOF	$(\Delta x_{MIN})/D$
M1	12 801	76806	0.02
M2	50 601	303606	0.01

Effect of mesh refinement (Oldroyd-B, β=1/9, De=0.35 and Re=0)





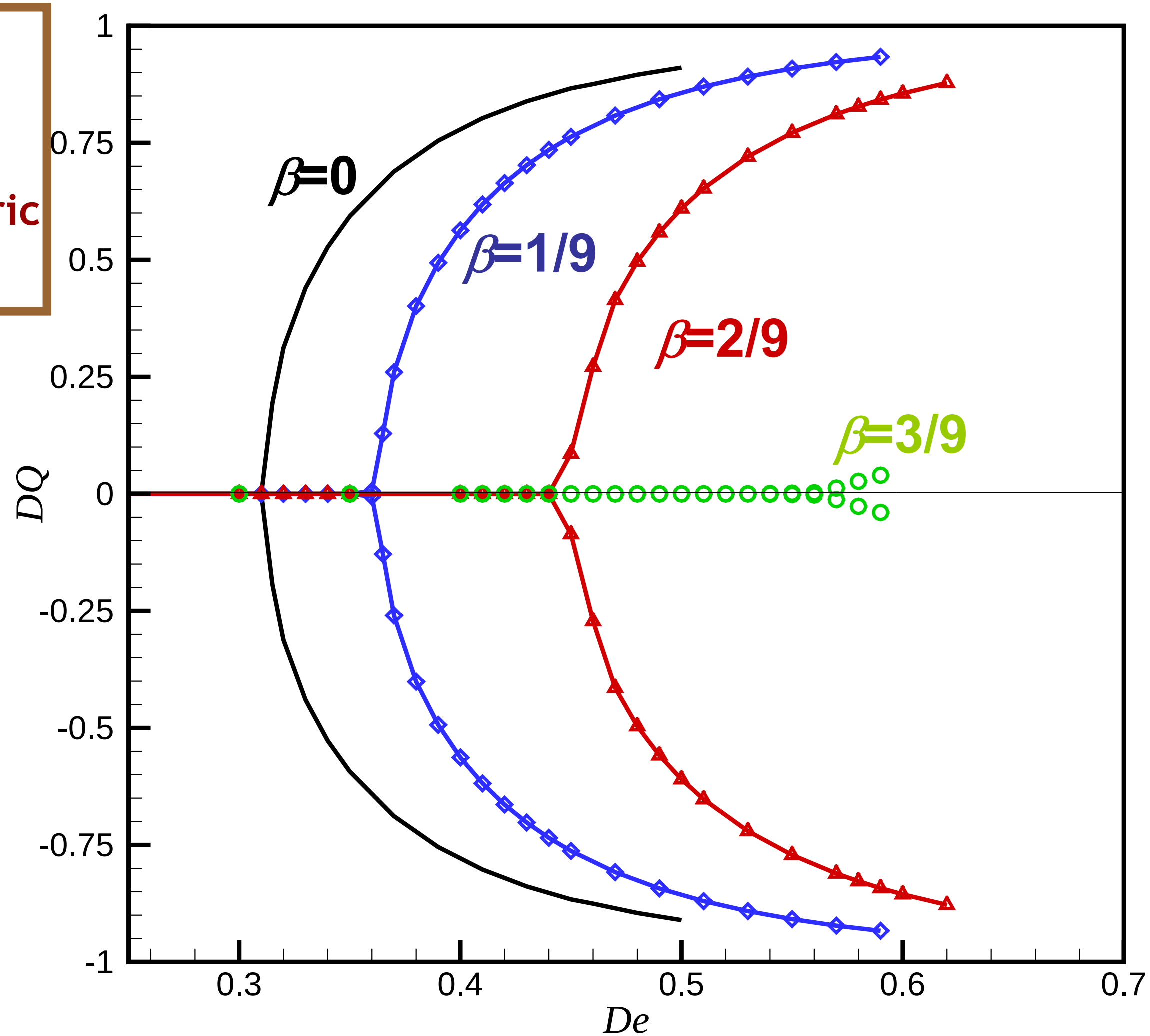
	NC	DOF	$(\Delta x_{MIN})/D$
M1	12 801	76806	0.02
M2	50 601	303606	0.01

Oldroyd-B Results – β effect

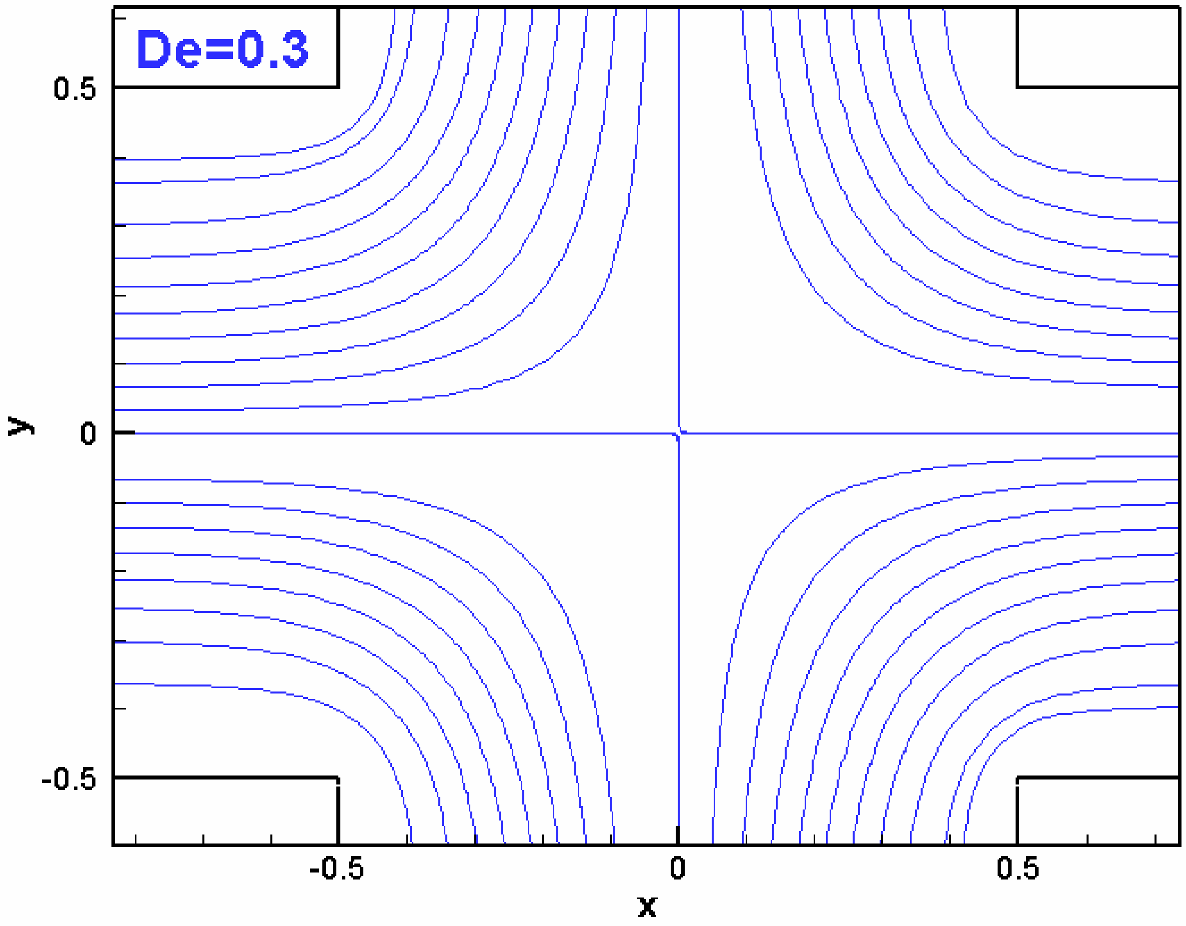
- Effect of increasing solvent viscosity (creeping flow)

Increase of critical De_{CR}

For β>3/9 flow became asymmetric unsteady;



- Streamlines (creeping flow and β=1/9)

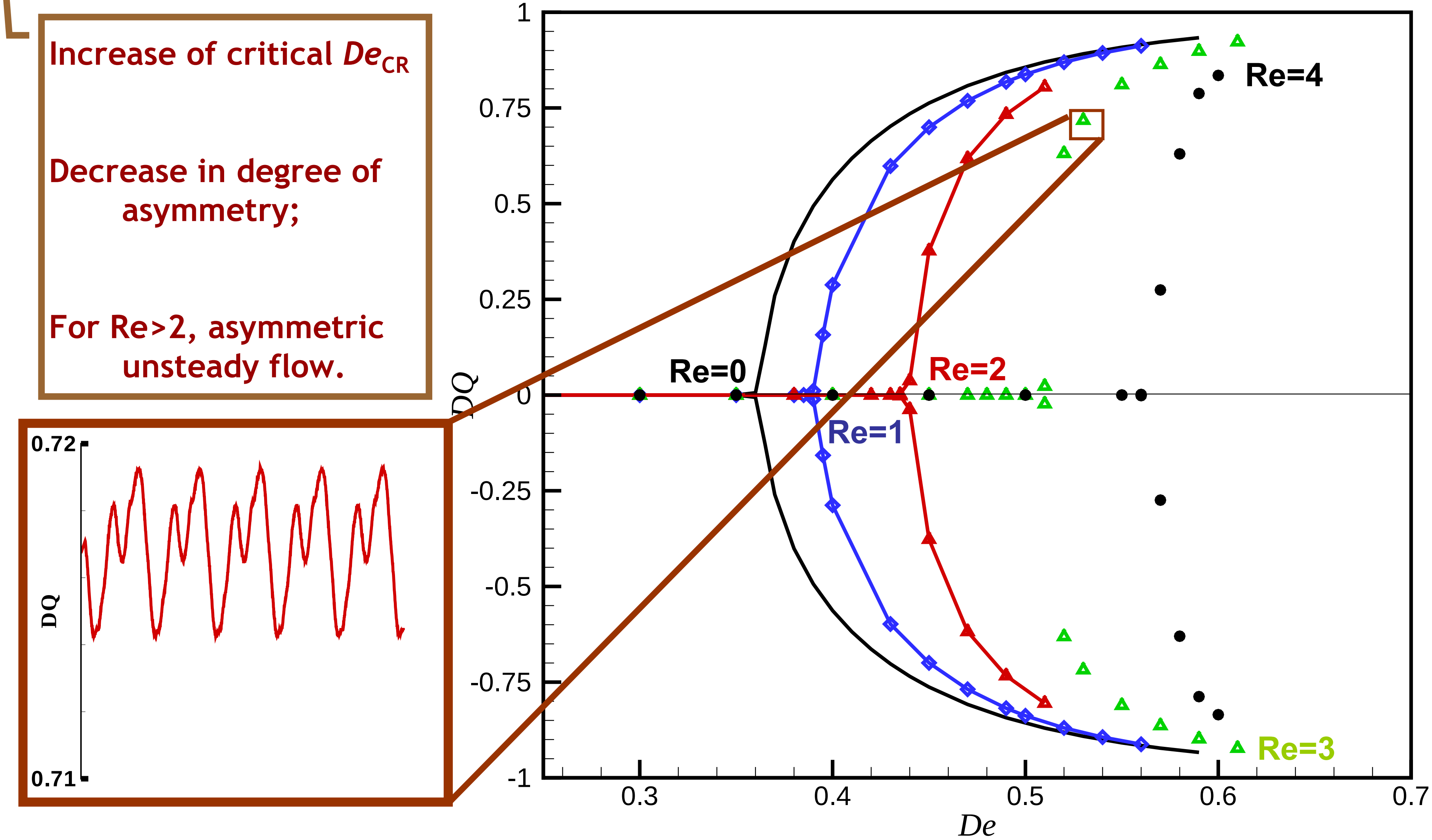


- Effect of increasing Reynolds number (β=1/9)

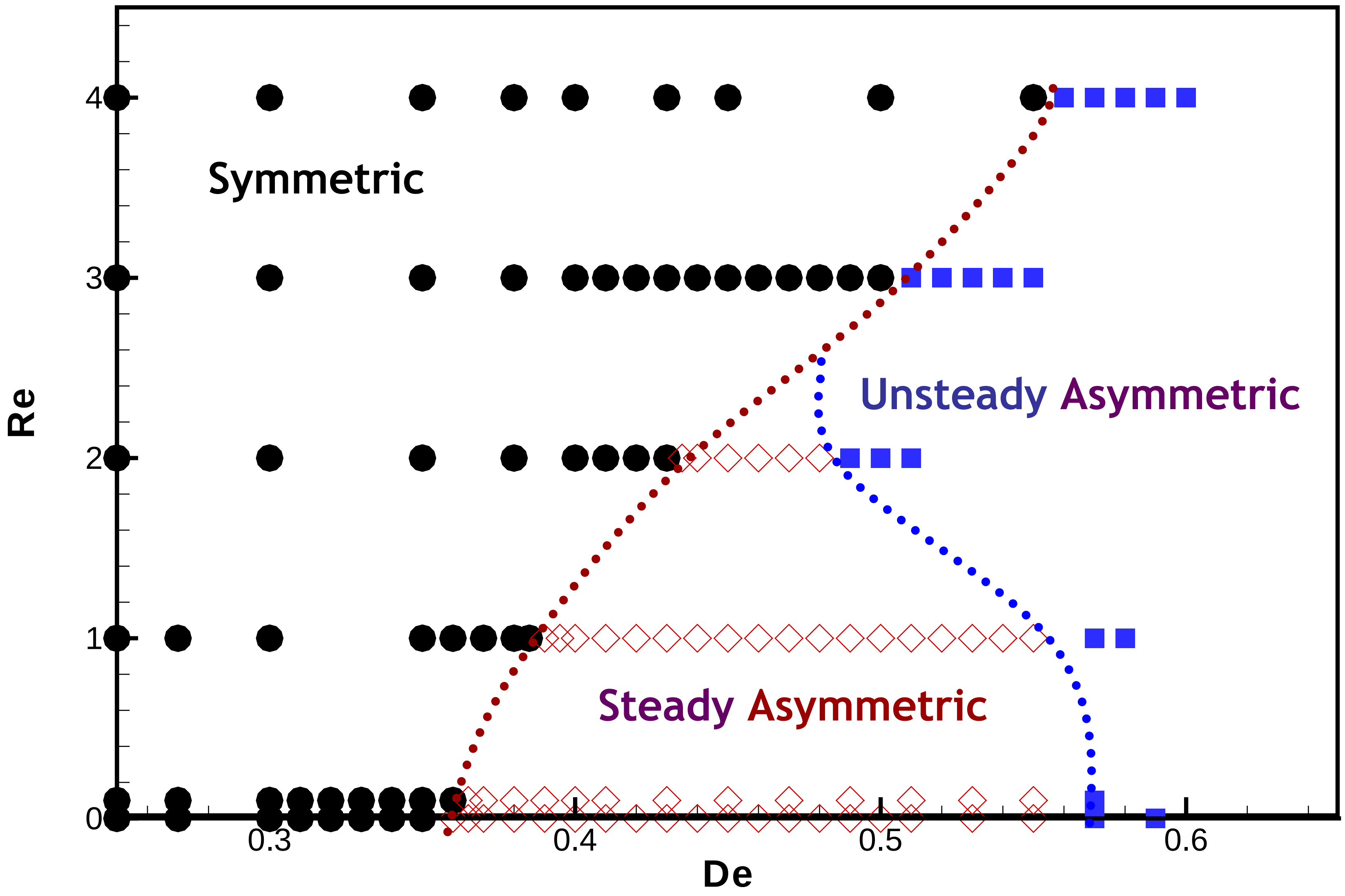
Increase of critical De_{CR}

Decrease in degree of asymmetry;

For Re>2, asymmetric unsteady flow.



• *Re*.vs.*De* map (β=1/9)



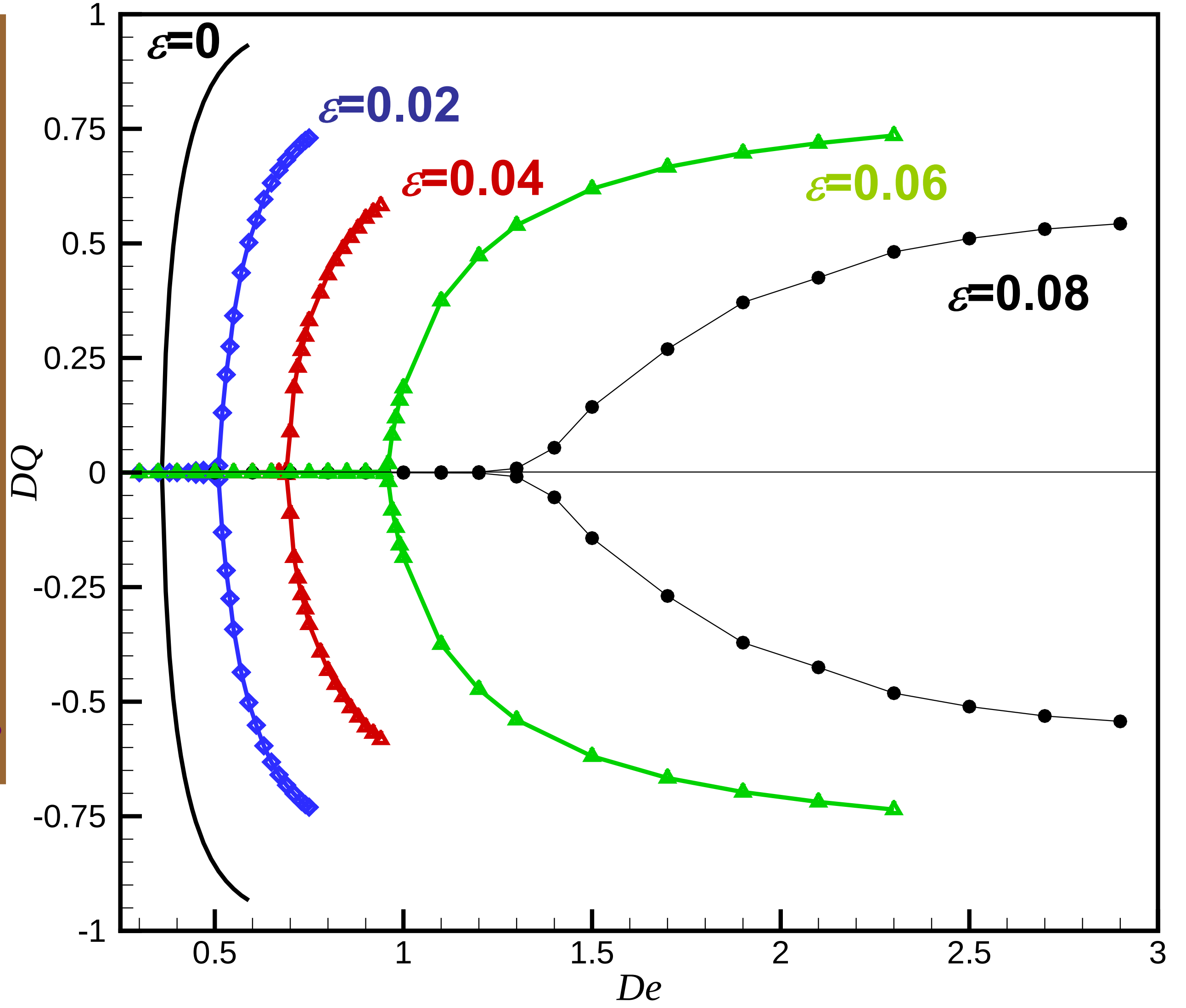
- Effect of varying ε parameter in PTT model (creeping flow and β=1/9)

Increase of critical De_{CR}

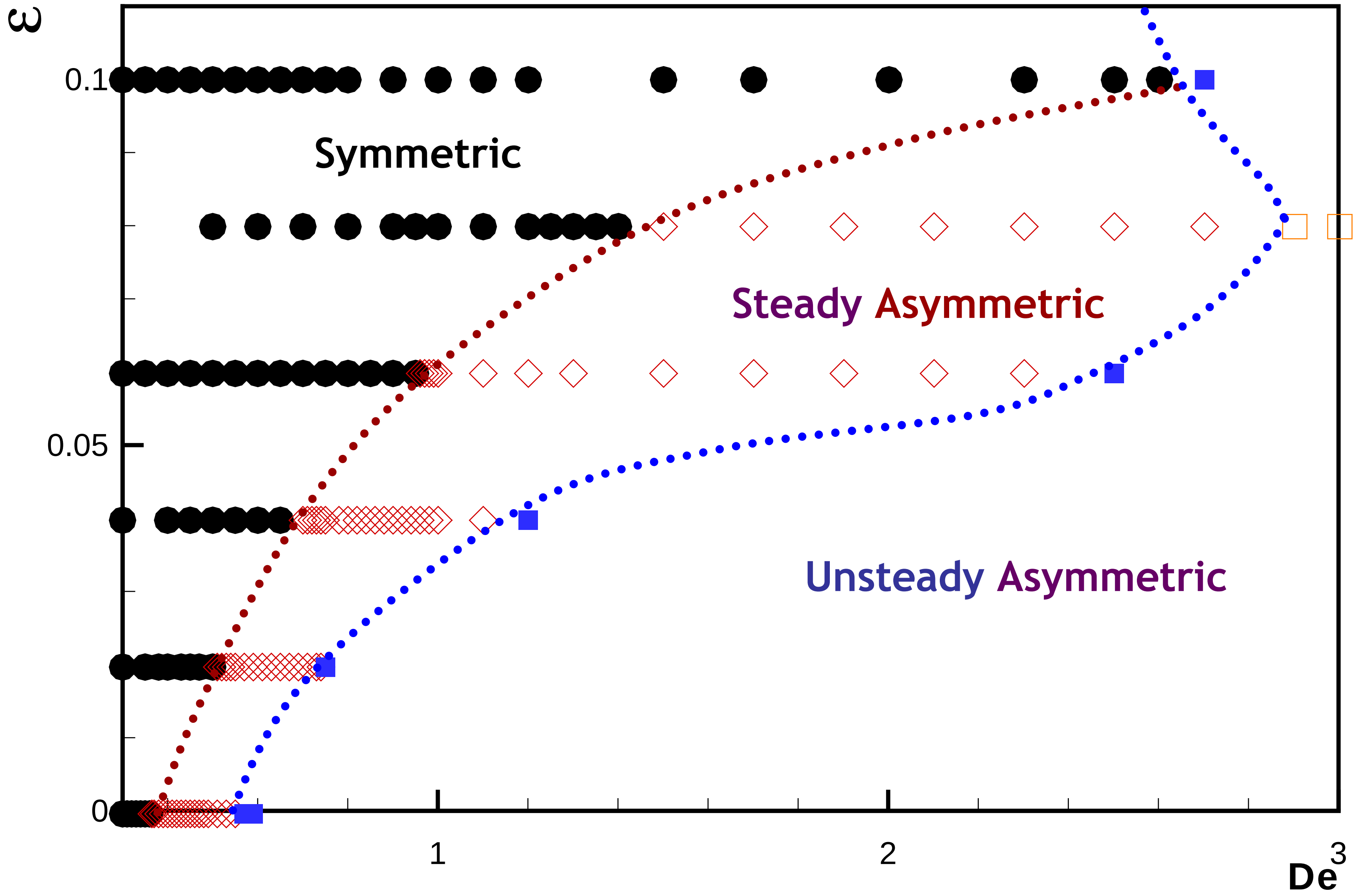
Decrease in degree of asymmetry (ε<0.04);

Increase in degree of asymmetry and in maximum *De* (ε>0.04);

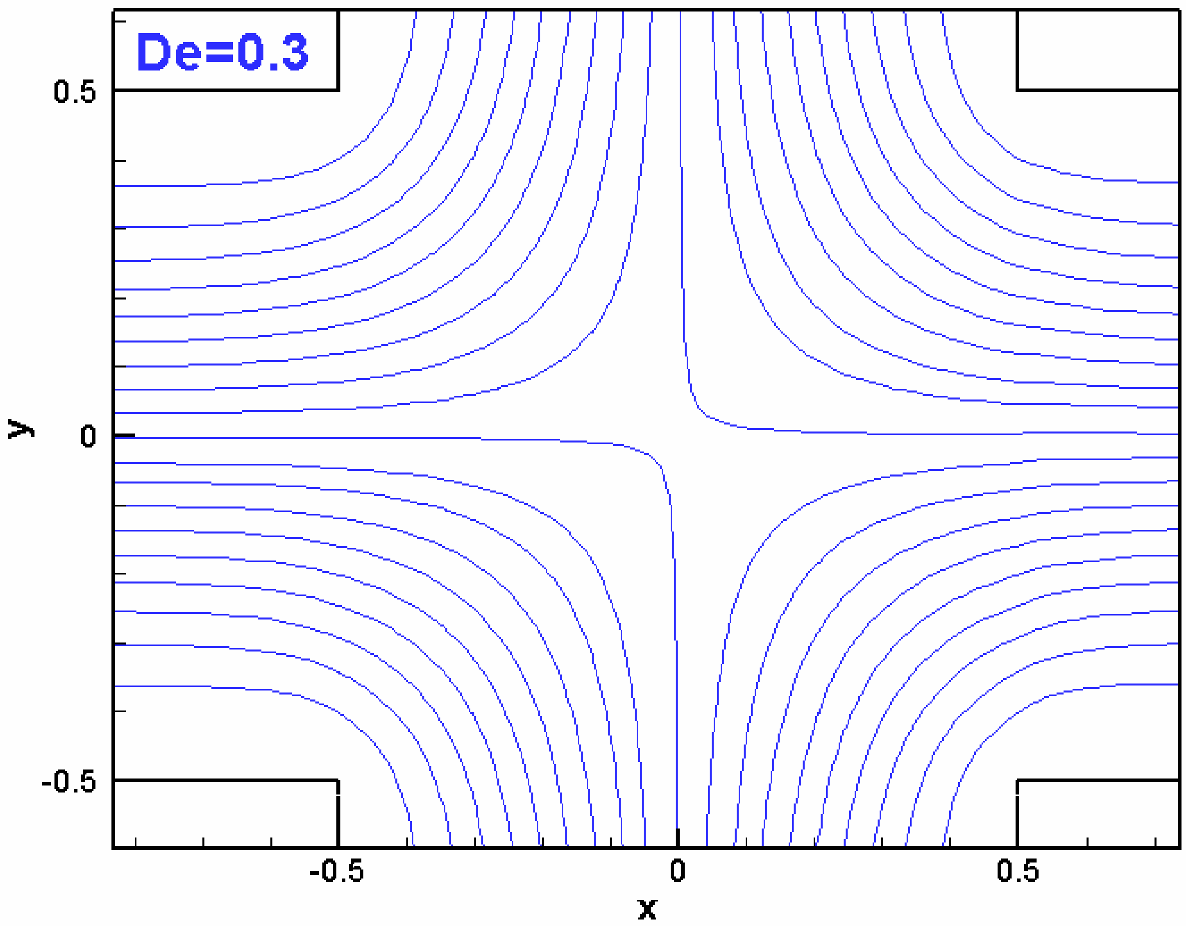
For ε>0.08, asymmetric stable flow disappears



- ε .vs.*De* stability map (Creeping flow and β=1/9)



- Streamlines (creeping flow, ε=0.02 and β=1/9)



Conclusions

- *Increasing the solvent viscosity (increasing β) increases the* critical *De. For β>2/9 the first steady instability disappears and the flow becomes unsteady (but still asymmetric);*

- *Increasing the level of inertia* (*increasing Re*) *shifts the onset of the instability to higher De and decreases the degree of asymmetry. Essentially inertia appears to stabilize the flow.*

- *For the PTT model, decreasing the extensional viscosity (increasing ε) increases the* critical *De.*

- *We propose that this flow would make a good "benchmark" case for purely-elastic instabilities*

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