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New Formulation for Stress Calculation: Application to Flow in a T - Junction with Viscoelastic Fluids

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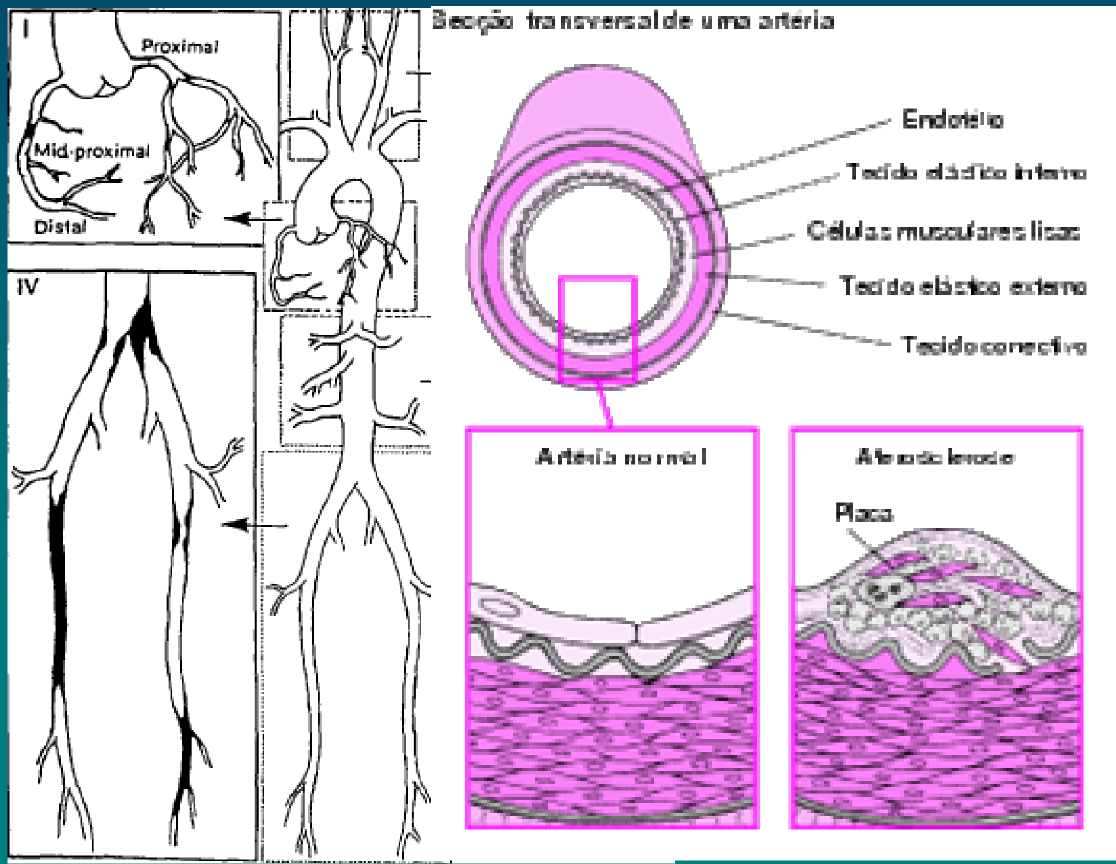
FEUP

1 - Influence of time-step

- Problem: different steady-state numerical solutions with different time-steps (inconsistent !).
- In addition: robustness and optimal choice of time-step (convergence or divergence of iterations).
  - Small time-step values
    - Easier to obtain a numerical solution.
    - CPU-times unnecessary longer.
  - Large time-step values
    - CPU-times are reduced.
    - More difficult to obtain a numerical solution.

1 - T-Junction flows (test case)

- The human circulatory system (many bifurcations)



2 - OBJECTIVES

- Devise better formulations to interpolate stress to cell faces (finite volume method).
- Study dependence of numerical solution on time-step values for those stress formulations.
- Evaluate the effect of elasticity in flow through a T-junction (dividing flow arrangement).

3 - EQUATIONS (1)

- Conservation of mass

$$\nabla \bullet \boldsymbol{u} = 0$$

- Conservation of linear momentum

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \nabla \bullet \boxed{\boldsymbol{\tau}} + \nabla \bullet (\eta_s \boldsymbol{D})$$

- Constitutive Equations
  - FENE-CR Model
  - FENE-MCR Model

3 - EQUATIONS (2)

- Constitutive equations
  - FENE-CR Model
    (Chilcott e Rallison, 1988)
  - FENE-MCR Model
    (Coates et al.,1992)

$$\boldsymbol{\tau} + \lambda \left(\frac{\overset{\nabla}{\boldsymbol{\tau}}}{f(\boldsymbol{\tau})} \right) = 2\eta_p \boldsymbol{D}$$

$$\boldsymbol{\tau} + \frac{\lambda}{f(\boldsymbol{\tau})} \overset{\nabla}{\boldsymbol{\tau}} = 2\eta_p \boldsymbol{D}$$

$$f(\boldsymbol{\tau}) = \frac{L^2 + (\lambda/\eta_p)\,\mathrm{tr}(\boldsymbol{\tau})}{L^2 - 3}$$

$$\frac{Df(\boldsymbol{\tau})}{Dt} \approx 0$$

4 - NONDIMENSIONAL PARAMETERS

- Extensibility parameter (in FENE models)

$$L^2 = 100$$

- Reynolds number

$$\mathrm{Re} = \frac{\rho u_1 H}{\eta_0} \simeq 102$$

- Deborah number

$$De = \frac{\lambda u_1}{H} \qquad (0 - \ldots)$$

- Viscosity ratio

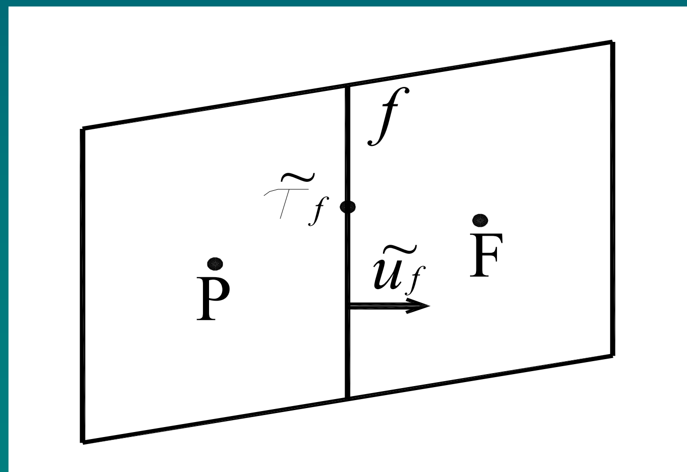
$$\beta' = \frac{\eta_S}{\eta_0} = 0.8$$

5 - NUMERICAL METHOD

- Finite-volume method for discretization of equations.
- Nonstaggered mesh arrangement.
  - Pressure-velocity coupling: Rhie e Chow, 1983.
  - Stress-velocity coupling: Oliveira et al., 1999.
- Convective terms: CUBISTA scheme (Alves et al. 2003).
- Pressure-correction SIMPLEC algorithm with time-marching.
- Convergence tolerance for iterative process when norm of residuals less than 10^{-8}.

5 - TESTED FORMULATIONS

- Formulations tested are related with two issues:
  - Calculation of convective fluxes at control-volume faces (formulation type ***F***).
  - Calculation of stresses at control-volume faces (formulation type ***T***)



5.1 - CALCULATION OF FLUXES

- Formulation ***F1*** type

$$\boxed{a_p = a_0 + \frac{\rho V}{\Delta t}}$$

$$\widetilde{u_f} = \frac{\overline{a_p u_p} - \overline{B_f}\,\widetilde{\Delta p_f} + \overline{B \Delta p}}{\boxed{\overline{a_p}}}$$

- Formulation ***F2*** type

$$\widetilde{u_f} = \frac{\overline{a_p u_p} - \overline{B_f}\,\widetilde{\Delta p_f} + \overline{B \Delta p} - \overline{\frac{\rho V}{\Delta t} u^0} + \overline{\frac{\rho V}{\Delta t}}\,\widetilde{u^0}}{\overline{\left(a_0 + \frac{\rho V}{\Delta t}\right)}}$$

5.2 - CALCULATION OF STRESSES (1)

- Formulation ***T1*** type

$$\left(\tilde{\tau}_{ij}\right)=\left(\overline{\tau_{ij}}\right)_f-\overline{\left(b'_{fi}[\Delta u_j]_f+b'_{fj}[\Delta u_i]_f-\frac{2}{3}\sum_k \eta_P B'_{fk}[\Delta u_k]_f\,\delta_{ij}\right)}$$

$$+\left(\tilde{b}'_{fi}[\Delta u_j]_f+\tilde{b}'_{fj}[\Delta u_i]_f-\frac{2}{3}\sum_k \overline{\eta_P}\tilde{B}'_{fk}[\Delta u_k]_f\,\delta_{ij}\right)$$

$$a_P^\tau=\frac{\lambda V}{\Delta t}+V+\lambda\sum_{F=1}^{6}a_F^\tau$$

$$b'_{fi}=\frac{\eta_p B_{fi}+\lambda\sum_k B_{fk}\tau_{ki}}{a_P^\tau}\quad;\quad \tilde{b}'_{fi}=\frac{\left(\eta_p B_{fi}+\lambda\sum_k B_{fk}\overline{\tau_{ki}}\right)_f}{V_f\overline{\left(a_P^\tau/V_P\right)}}$$

5.2- CALCULATION OF STRESSES (2)

- Formulation ***T2*** type **(NEW FORMULATION PROPOSED)**

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta_p \mathbf{D}$$

$$\left(\tilde{\tau}_{ij}\right)_f = \left(\overline{\tau_{ij}}\right)_f \pm \frac{1}{\left(1 + p a_0^p / V_P\right)}$$

$$\overset{\nabla}{\boldsymbol{\tau}} = \frac{D\boldsymbol{\tau}}{Dt} - \left[\nabla\mathbf{u}^T \cdot \boldsymbol{\tau} + \boldsymbol{\tau}\cdot\nabla\mathbf{u}\right]$$

$$\tilde{\boldsymbol{\tau}} + \lambda \overset{\nabla}{\tilde{\boldsymbol{\tau}}} = 2\eta_p \tilde{\mathbf{D}}$$

$$\left[-\frac{1}{V_P}\left(\eta_p B_{fi}[\Delta u_j]_f + \eta_p B_{fj}[\Delta u_i]_f - \frac{2}{3}\eta_p \sum_k B_{fk}[\Delta u_k]_f \delta_{ij}\right)\right.$$

$$\left. + \frac{1}{V_f}\left(\eta_p \left\{B_{fi}[\Delta u_j]\right\}_f + \eta_p B_{fj}[\Delta u_i]_f - \frac{2}{3}\eta_p \sum_k B_{fk}[\Delta u_k]_f \delta_{ij}\right)\right]$$

$$\tilde{\boldsymbol{\tau}} - \boldsymbol{\tau} = \lambda\left(\overset{\nabla}{\boldsymbol{\tau}} - \overset{\nabla}{\tilde{\boldsymbol{\tau}}}\right) + 2\eta_p\left(\tilde{\mathbf{D}} - \mathbf{D}\right)$$

5.2 - CALCULATION OF STRESSES (3)

- Formulation ***T3*** type

$$\left(\tilde{\tau}_{ij}\right) = \left(\overline{\tau_{ij}}\right)$$

5.3 - FINAL FORMULATIONS

- The final formulations tested are a combination of type ***F*** and ***T*** formulations

Designation of the final Formulation	Formulation for ***Fluxes***	Formulation for ***Stress***	Dependency on time-step (Δt).
General Formulation (MCR)	***F2***	***T1***	Yes
Tau E Formulation (CR and MCR)	***F2***	***T2***	No
Formulation with Linear Interpolations (MCR)	***F2***	***T3***	No

6 - FLOW GEOMETRY

Flow rate

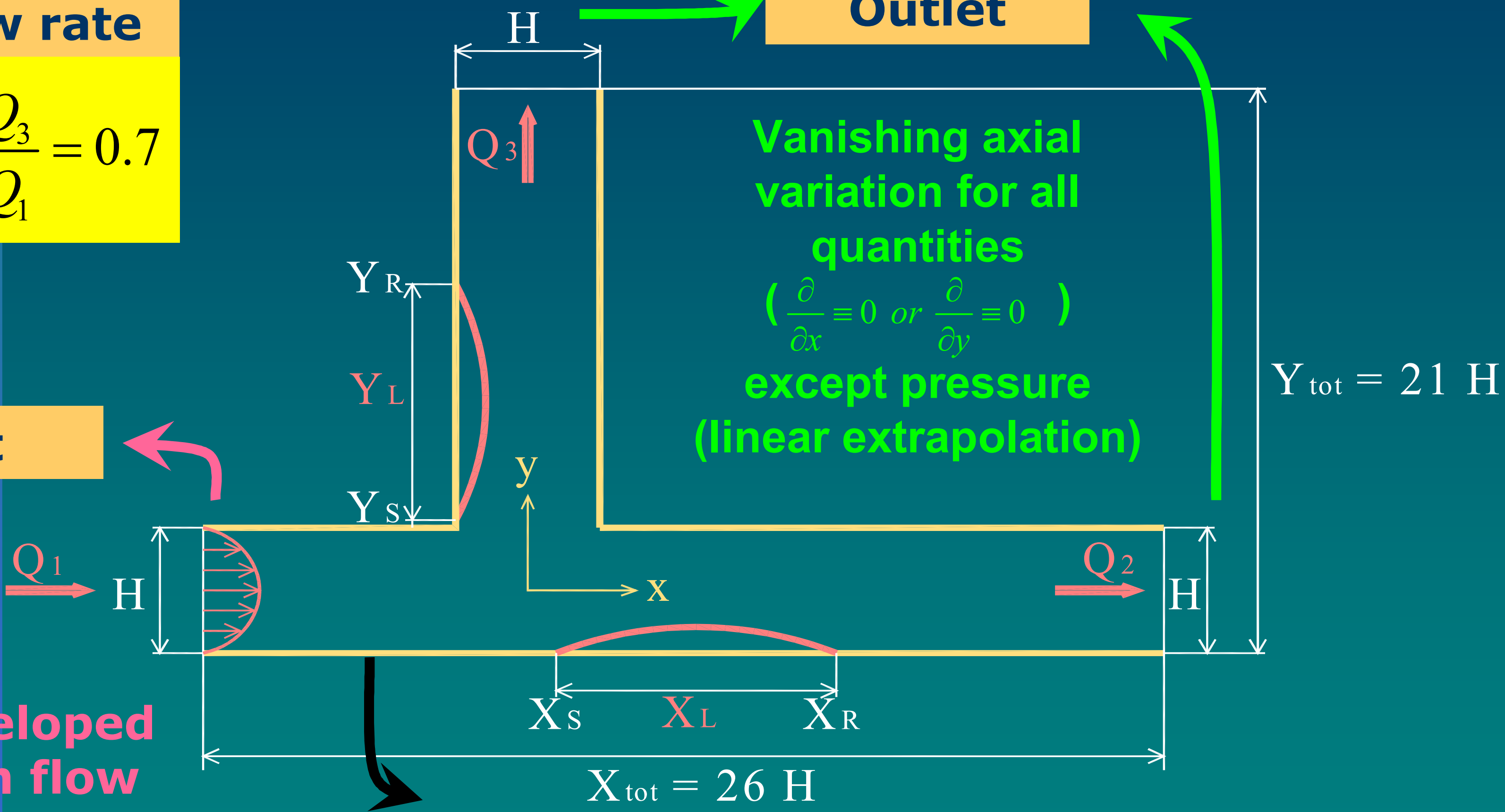
$$\beta = \frac{Q_3}{Q_1} = 0.7$$

Outlet

Vanishing axial variation for all quantities ($\frac{\partial}{\partial x} \equiv 0$ or $\frac{\partial}{\partial y} \equiv 0$) except pressure (linear extrapolation)

Inlet

Fully developed upstream flow



Walls

No slip condition u = v = 0

7 - MESH REFINEMENT (1)

- Orthogonal but non-uniform mesh.
- Three meshes progressively refined.
  - MESH M1 (6,400 control-V

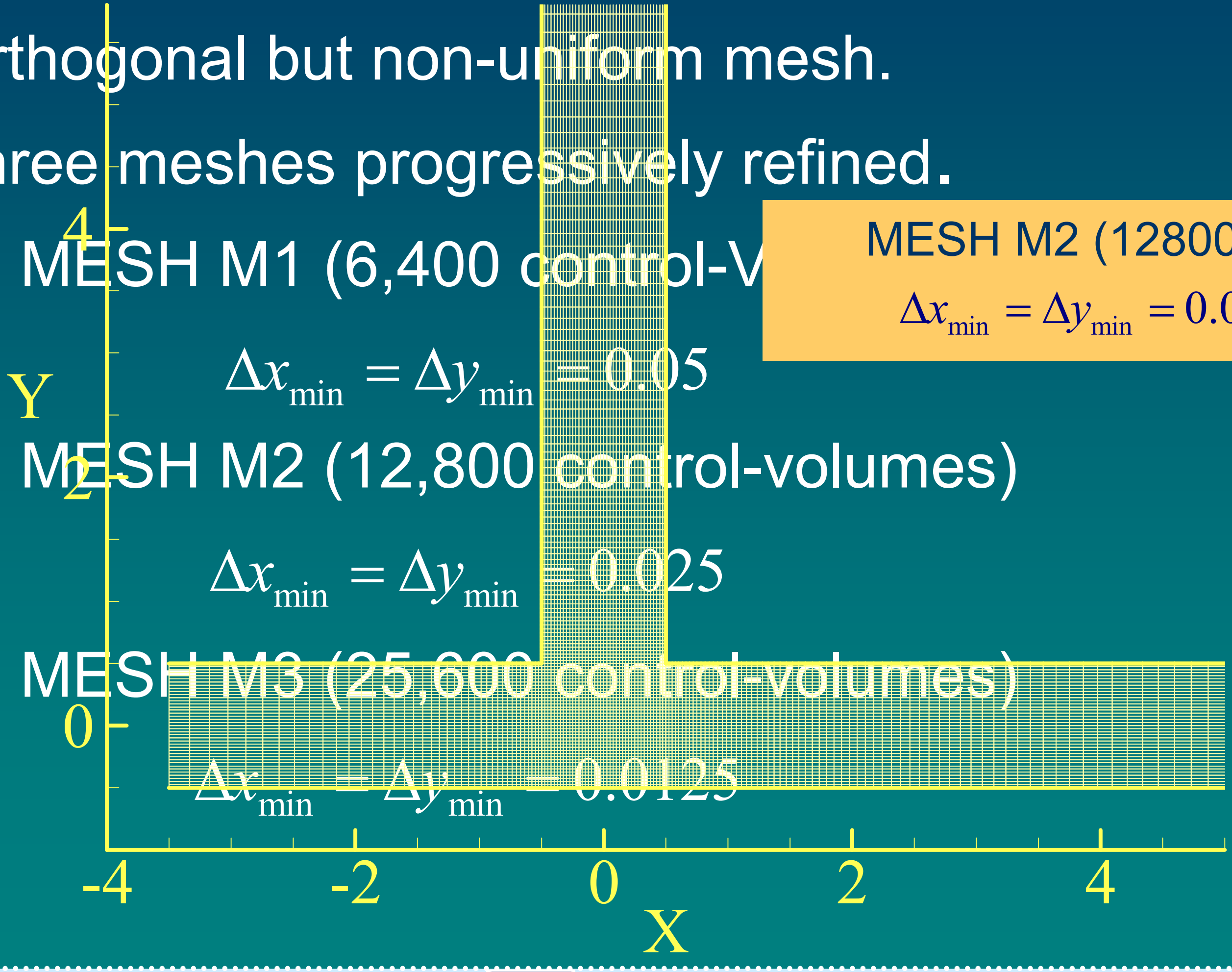
    $$\Delta x_{\min} = \Delta y_{\min} = 0.05$$

  - MESH M2 (12,800 control-volumes)

    $$\Delta x_{\min} = \Delta y_{\min} = 0.025$$

  - MESH M3 (25,600 control-volumes)

    $$\Delta x_{\min} = \Delta y_{\min} = 0.0125$$



MESH M2 (12800 VC)

$$\Delta x_{\min} = \Delta y_{\min} = 0.025$$

7 - MESH REFINEMENT (2)

- Mesh independency

Viscoelastic case
***Re* = 102, *De* = 2.5 and *β'* = 0.8**

1,E+00

Error

	X_L
Mesh M1	1.4974
Mesh M2	1.4746
Mesh M3	1.4696
Extrap. Richardson	1.468

Mesh M1	1.99%
Mesh M2	0.44%
Mesh M3	0.097%

$$\log(Error(Y_L)) = 1.36\log(\Delta y_{\min}) + 3.58$$

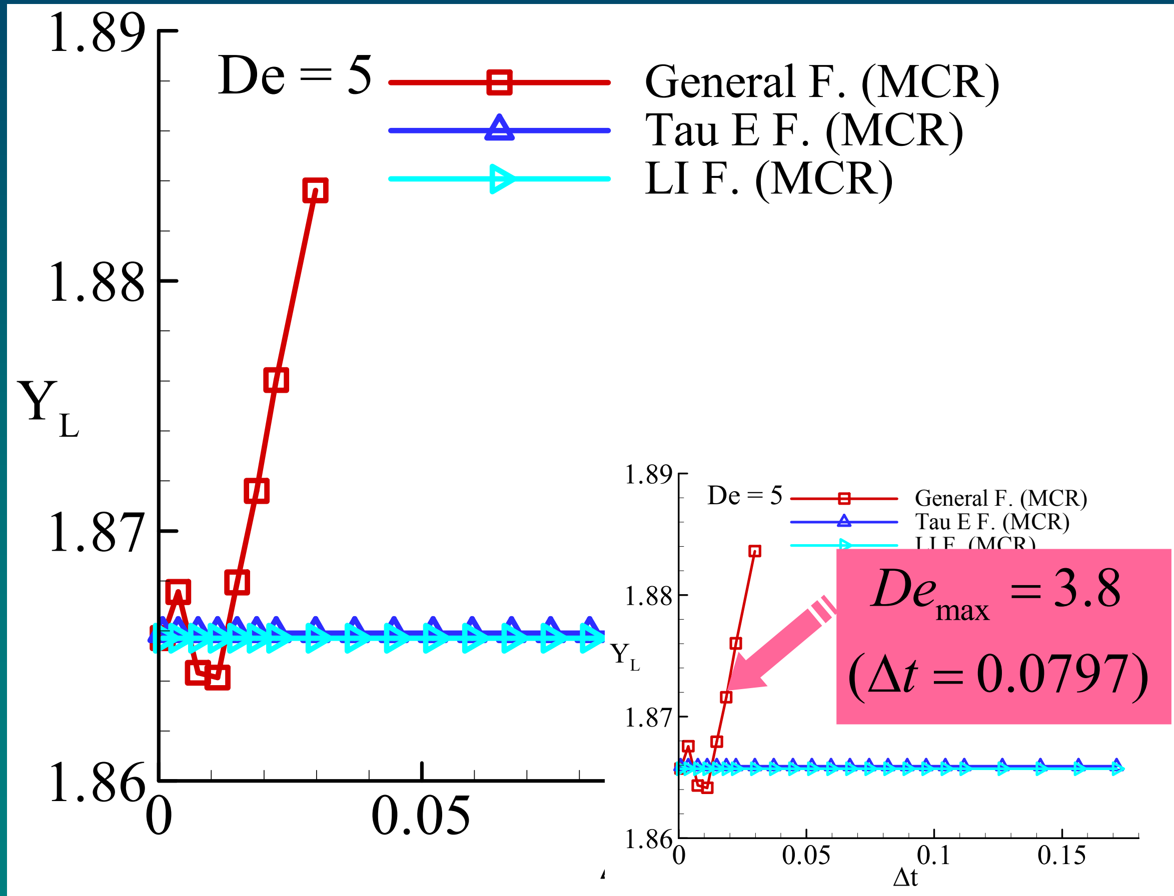
$$\log(Error(X_L)) = 2.18\log(\Delta x_{\min}) + 5.49$$

	Y_L
Mesh M1	1.7379
Mesh M2	1.8865
Mesh M3	1.9445
Extrap. Richardson	1.9816

	$Error\ (Y_L)$
Mesh M1	12.3%
Mesh M2	4.8%
Mesh M3	1.87%

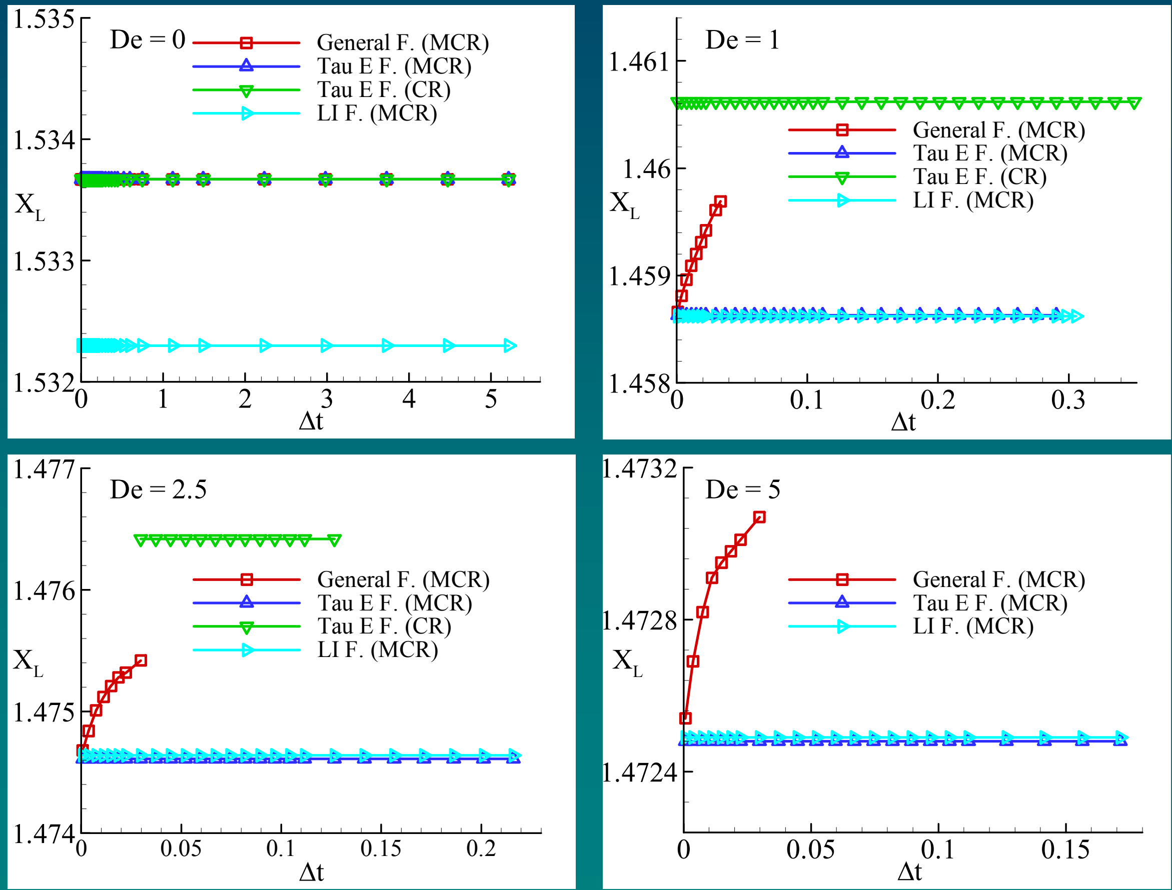
7 - RESULTS (1) (Independence with Δt)

- Vertical recirculation length



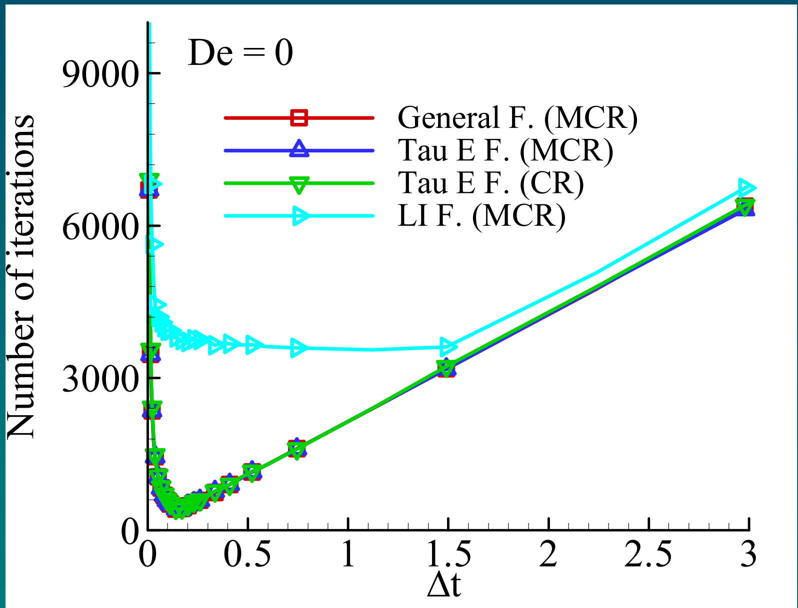
7 - RESULTS (2) (Independence with Δt)

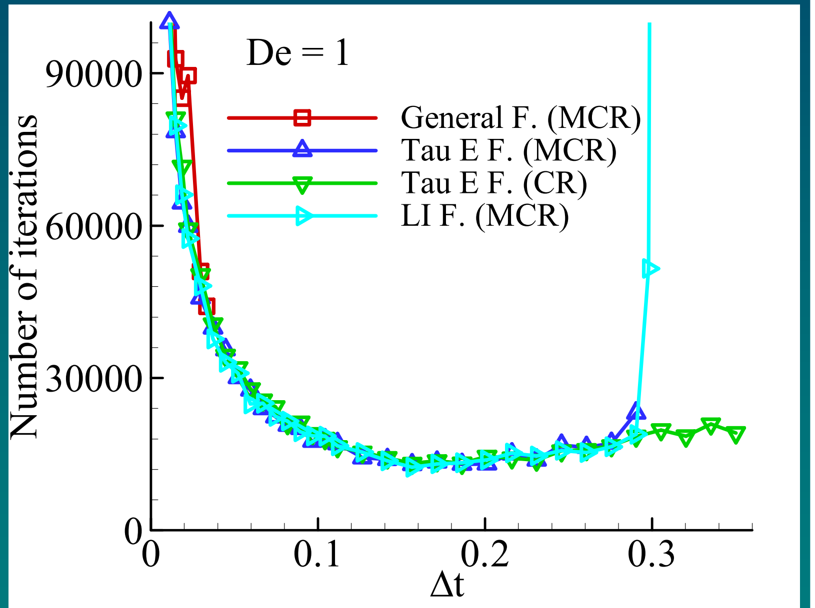
- Horizontal recirculation length



7 - RESULTS (3) (Independence with Δt)

- Number of iterations as a function of time-step

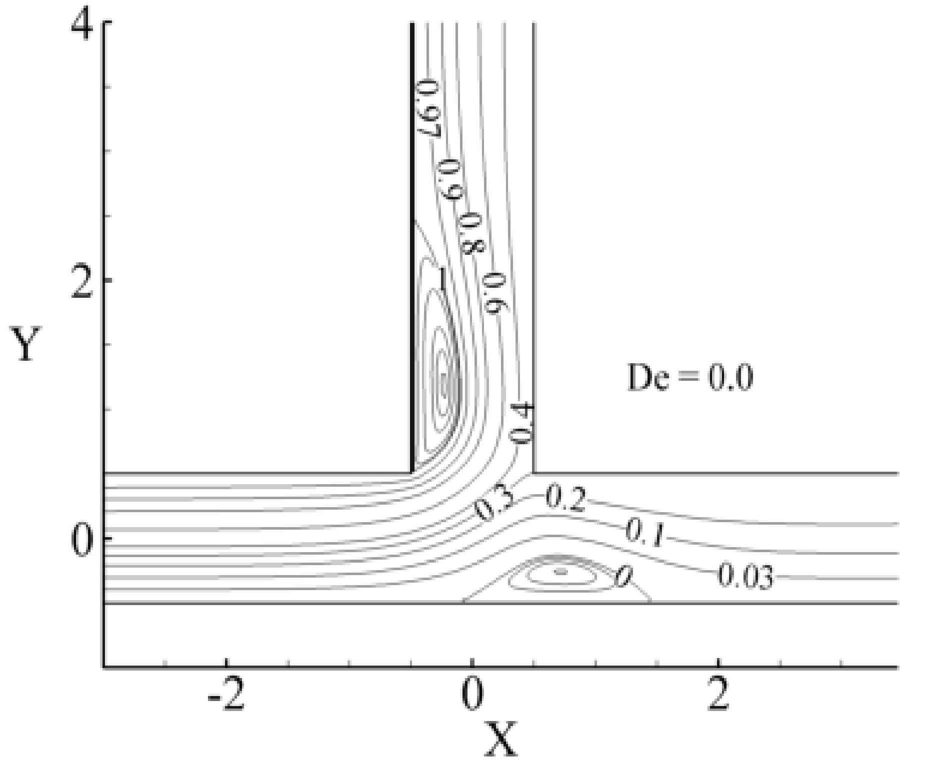


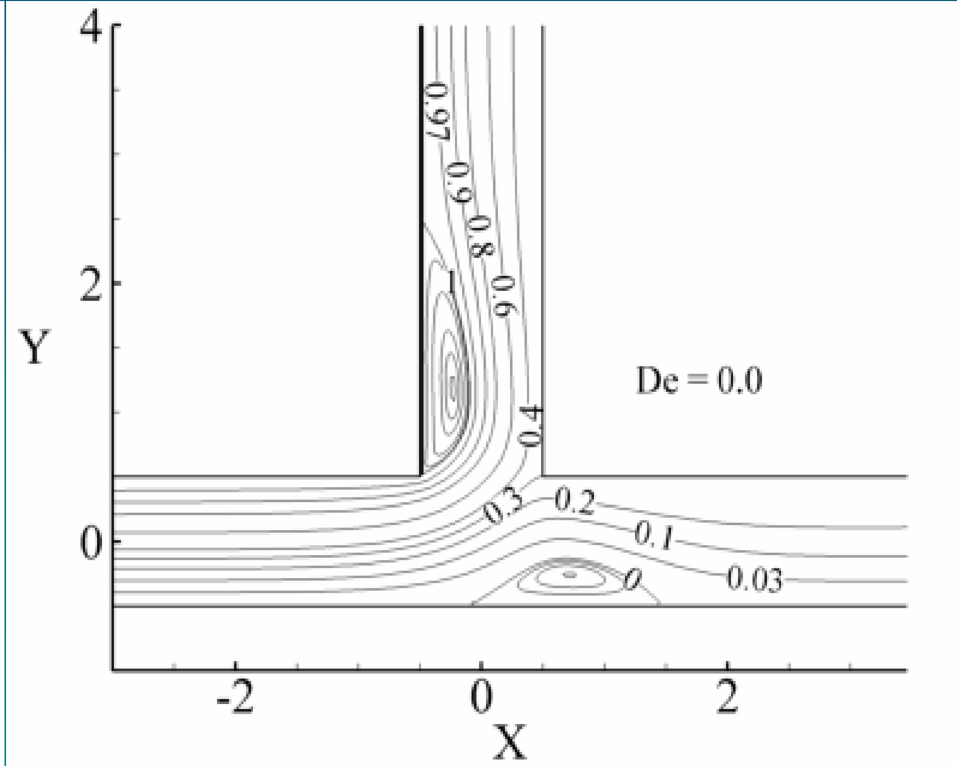


7 - RESULTS (4) (Variation with De)

- Streamlines for increasing Deborah number

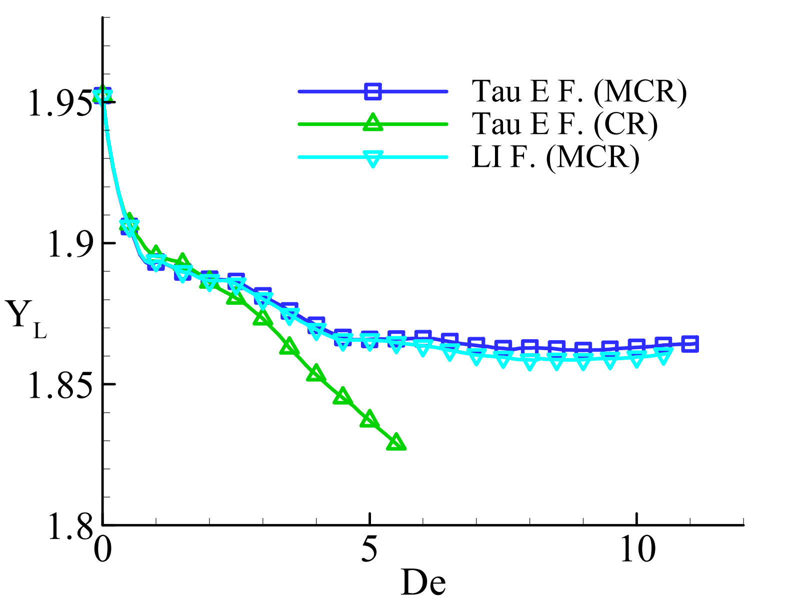
Tau E (MCR) ***Tau E (CR)***

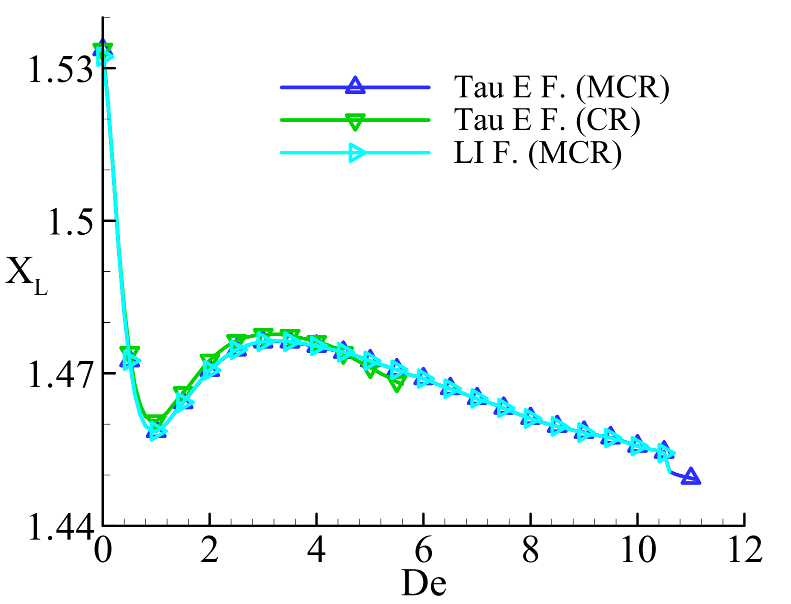




7 - RESULTS (5) (Variation with De)

- Variation of recirculation lengths with *De*

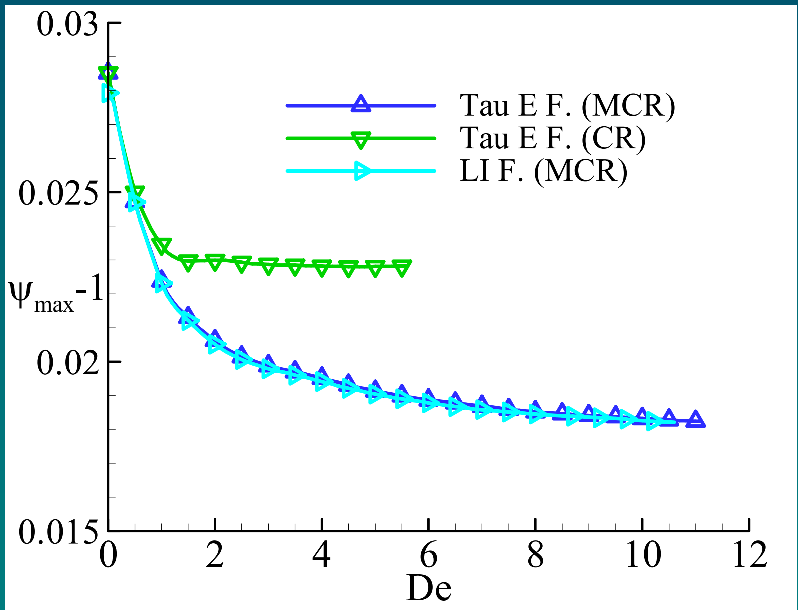




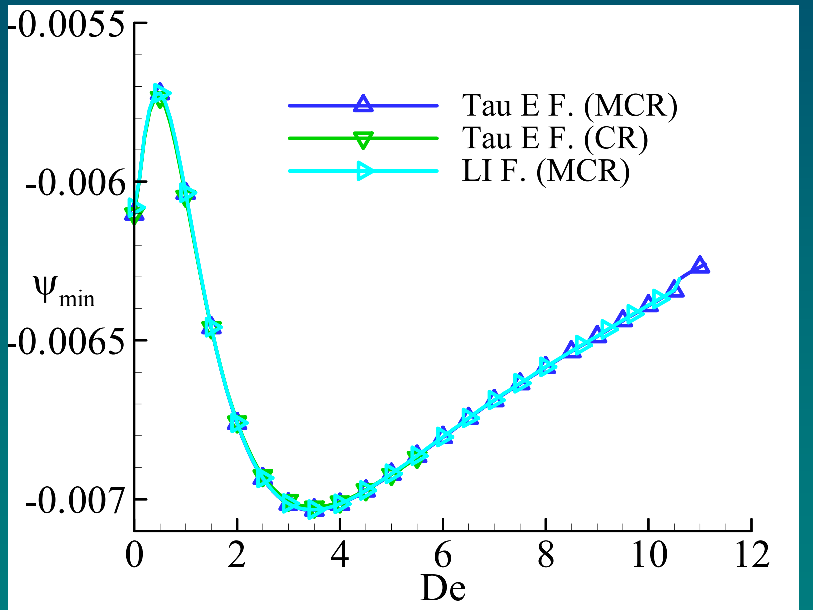
7 - RESULTS (6) (Variation with De)

- Vortex strength of recirculating zones

Vertical recirculation



Horizontal recirculation



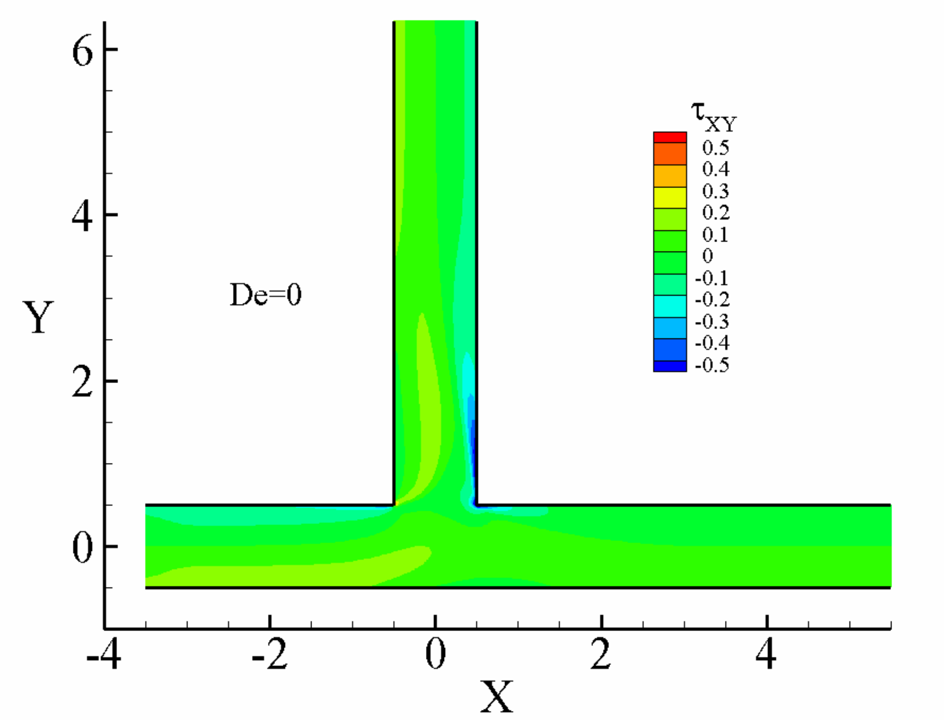
7 - RESULTS (7) (Variation with De)

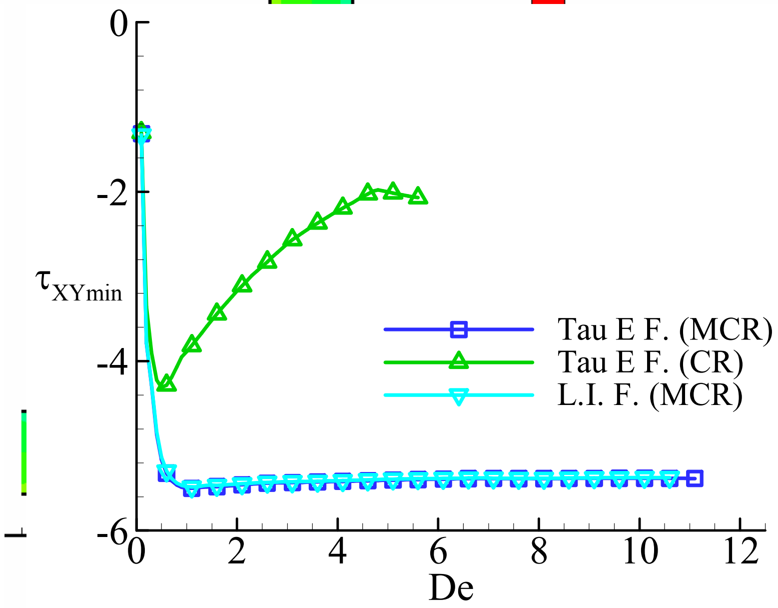
- Stress field variation

Tau E (MCR)

Tau E (CR)

on Tau E (MCR), De=1





8 - CONCLUSIONS

- The formulation proposed (Tau E) is the most versatile.
- The results obtained with the two constitutive models (FENE) are slightly different (maximum variation of 2.1%).
- The existence of low stresses in the recirculating zones and high at the re-entrant corners of the bifurcation was confirmed
- The length and intensity of the vertical recirculation decreased with increase of De. The length and intensity of the horizontal recirculation have a non-monotone behavior.

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