A Novel Microfluidic Mixing Element for Viscoelastic Fluids

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Abstract. The flow visualizations of viscoelastic flows in the mixing-separating geometry of Cochrane *et al.* [1] showed provocative flow features that inspired the current numerical research using the upper-convected Maxwell (UCM) model. The effects of Deborah (*De*) and Reynolds (*Re*) numbers and gap size were analyzed in depth in this two-dimensional flow investigation. The normalized gap size was varied between 0 and 5, *Re* varied between 0 and 50 and *De* was varied between 0 and the maximum attainable value. The creeping flow of Newtonian fluids is always anti-symmetric, due to the anti-symmetry of the inlet conditions and the symmetry of the flow geometry. The increase in the gap size leads to an increase in the reversed flow rate ratio (R_r), here defined as the ratio between the reversed and total flow rates, an effect enhanced only for $Re \geq 5$. The creeping flow of UCM fluids however, showed two distinct flow patterns. For normalized gap sizes below a critical value the reversed flow is slightly enhanced by viscoelasticity, followed by a strong decrease in R_r towards zero as *De* further increase, whereas for a supercritical gap size viscoelasticity is responsible for a continuous increase in R_r. For near-critical flow geometries it was possible to observe a sudden jump between the two flow conditions at slightly different Deborah numbers, thus suggesting the possibility to use such geometry as a micro-mixer for viscoelastic fluids if the imposed flow rates are made time periodic to enhance an oscillation between flow patterns. At low Reynolds numbers the dependence of flow pattern on gap size and Deborah number still exhibits the described double behavior, but inertia naturally enhances the straight flow case and at *Re* = 5, R_r always decreases with Deborah number for the investigated gap sizes.

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INTRODUCTION

A simple constitutive model (UCM) is used to simulate viscoelastic flows in the mixing and separating flow geometry, schematically shown in Figure 1(a) [1]. Flow in this geometry has been investigated experimentally and numerically [1-3]. Cochrane *et al.* [1] used a finite-difference discretization with the UCM model to represent the behavior of a Boger fluid. They studied experimentally and numerically the effects of gap width (*g*) for two different flow configurations: one with equal flow rates in all channel arms and the other with unbalanced flow rates. In a sequel paper [2], consideration was given to flows using thinner insert plates with rounded edges. In both experimental works [1,2], this complex flow displayed remarkable flow features and distinct Newtonian and elastic flow behaviors, with the presence of unidirectional and reversed flows in varying degrees. Later, Baloch *et al.* [3] used the PTT model to simulate this flow using thin insert plates with rounded edges and equal flow rates in all channel arms.

This work has three main objectives: (a) to extend the limited information on the variety of viscoelastic fluid flow behavior in this geometry, clarifying and mapping flow configurations for different gap sizes under conditions of low inertia and high viscoelasticity; (b) to assess the existence and extent of purely-elastic flow instabilities, such as those observed in the cross-slot geometry [4]; (c) to explore the possibility of using this geometry as a micro-mixing device for viscoelastic fluids exploring the bifurcation pattern we observe here, to our knowledge, for the first time. In this work we focus on the problem with zero thickness insert plates with equal flow rates and fluid at all inlets.

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GOVERNING EQUATIONS AND NUMERICAL METHOD

The flow domain and problem is that of two opposed channel flows interacting through a gap of non-dimensional width ($\theta = g/H$) in the common separating wall, as shown schematically in Figure 1(a). The separating wall has a non-dimensional thickness $\alpha = a/H$. All channels have the same width (H). At the inlets fully-developed velocity and stress profiles are imposed and the inlet length ($20H$) is more than sufficient for the flow in the central region to be independent of the inlet boundary condition. No-slip conditions are imposed at all channel walls and in the outlet planes Neumann boundary conditions are applied to all variables, i.e. $\partial\varphi/\partial x = 0$, including the pressure gradient. The equations we need to solve are those of conservation of mass, $\nabla \cdot \mathbf{u} = 0$, and momentum

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \nabla \cdot \mathbf{u}\mathbf{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau} \tag{1}$$

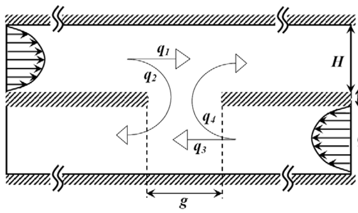
together with an appropriate constitutive equation for the extra stress, $\boldsymbol{\tau}$. Here, for reasons of rheological simplicity, we use the UCM model,

$$\boldsymbol{\tau} + \lambda \left(\frac{\partial \boldsymbol{\tau}}{\partial t} + \mathbf{u} \nabla \cdot \boldsymbol{\tau} \right) = \eta \left(\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}} \right) + \lambda \left(\boldsymbol{\tau} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}} \cdot \boldsymbol{\tau} \right) \tag{2}$$

where λ and η are the relaxation time and shear viscosity of the fluid, respectively. A fully-implicit finite-volume method was used, and details of the numerical method can be found elsewhere (e.g. [5,6]).

For all values of θ, the computational domain was mapped using six structured blocks, one in each channel and two in the central region. In the central region the meshes had different number of cells in the x direction to ensure that the minimum cell spacing remained the same. In this region, the cell spacing was uniform and the meshes were progressively refined from $\Delta x_{min} = \Delta y_{min} \approx 0.04H$ for mesh M25L to $\Delta x_{min} = \Delta y_{min} \approx 0.02H$ for mesh M51L. Figure 1(b) shows a zoomed view of the coarse mesh for $\theta = 1$. Mesh M51 contains 25 704 and 35 904 cells, for low and large non-dimensional gap widths, respectively.

(a)



(b)

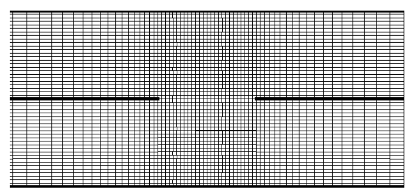


FIGURE 1. (a) Diagram of the mixing and separating geometry and (b) view of coarse mesh used for $\theta = 1$ (mesh M25L).

The Deborah number is defined as the ratio between the fluid relaxation time (λ) and a flow time scale, here taken as g/U, thus $De = \lambda U/g$. For the flows with inertia we define the Reynolds number as $Re = \rho UH/\eta$. To quantify the degree of flow reversal relative to unidirectional flow, the parameter $R_r = q_2/Q_1 = q_4/Q_2$ is used, where q_2 and q_4 correspond to the partial flow rates that reverse from each inlet arm and $Q_1 = Q_2$ are the flow rates at each inlet channel, as illustrated in Figure 1(a).

RESULTS

The influence of flow inertia on R_r as a function of the normalized gap width, θ, is presented in Figure 2(a) for Newtonian flows. For $Re \leq 1$ the variation of R_r with θ is independent of Reynolds number, but on increasing Re enhances flow reversal, especially for $Re = 10$ and 50 for $\theta \geq 1.7$ and $\theta \geq 0.035$, respectively. For all Re, the increase in the gap size leads to an increase in the reversed flow rate ratio, as was also observed experimentally [1,2]. Results for several Re using two meshes, M25 (symbols) and M51 (lines), are plotted in Figure 2(a), and are essentially indistinguishable confirming that the use of mesh M51L is adequate for accurate predictions.

The creeping flow of UCM fluids, exhibited an interesting bifurcation pattern, which depends on the gap width, as illustrated in Figure 2(b). For non-dimensional gap sizes below a critical value ($\theta \leq 1.7$) the reversed flow is slightly enhanced by viscoelasticity, followed by a strong decrease in R_r towards zero as De further increases. This flow behavior is characterized by a significant asymmetry in the gap region with the fluid tending to flow unidirectionally as opposed to a reversed flow configuration having some similarity with the streamline patterns shown in Figure 3(a) for a different flow condition and as reported in experiments [1,2].

For a supercritical non-dimensional gap size ($\theta \geq 2$) viscoelasticity is responsible for a continuous increase in R_r. However, the most interesting conditions are for intermediate gap widths ($1.7 \leq \theta \leq 2$, cf. Figure 2(b) and (c) for

$\theta = 1.74$) where a steady bi-stable bifurcation pattern is observed, with a sudden jump between the two flow configurations at slightly different Deborah numbers. This critical De for a bi-stable behavior depends on θ and is delayed with increasing flow inertia. For the critical non-dimensional gap width range the two steady stable flow fields corresponding to the two arms of R_r of Figure 2(c), are shown in the streamline plots of Figure 3 for $De = 0.351$. On the left side the flow is quasi-unidirectional ($R_r = 0.047$) and on the right side the flow is highly reversed (R_r=0.85). The bifurcation between these two flow patterns is a purely elastic instability since $Re = 0$, as also observed for a cross-slot geometry [4]. Interestingly, for $\theta = 1.74$ the critical Deborah number is 0.316, near the one reported in the cross slot geometry ($De_{crit} \approx 0.31$ [4]) even though the flow types in the center of the two geometries are quite different.

The combined effects of inertia and elasticity are also presented in Figure 2(c), where R_r is plotted as a function of De for three different gap widths ($\theta = \sqrt{2}$, 1.74 and 2) and for Re up to 10. The degree of flow reversal is almost the same for $Re \leq 1$, regardless of gap width. At higher Reynolds numbers (Re = 5 and 10) and $\theta = 2$ there is an inversion in the flow configurations: with the increase of elasticity the flow becomes less reversed, while for lower Re the increase of elasticity decreases the degree of unidirectional flow.

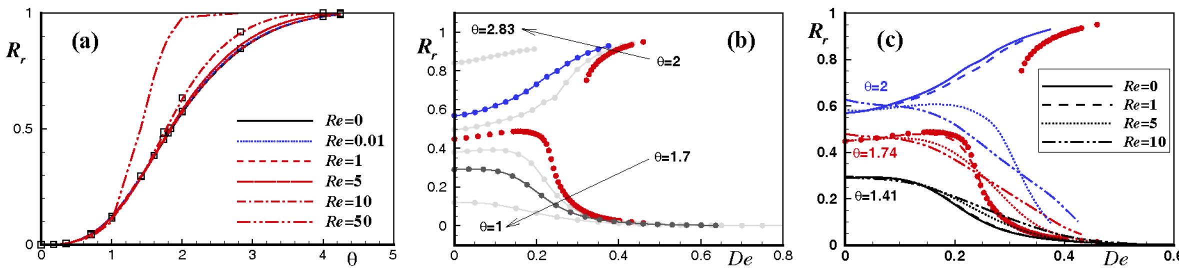


FIGURE 2. Reversed flow degree parameter, R_r (a) as function of θ, for several Re at De = 0 (o – M25, lines – M51), (b) as function of De, for several θ at Re = 0 in mesh M51 and (c) as function of De, for several θ and Re in mesh M51.

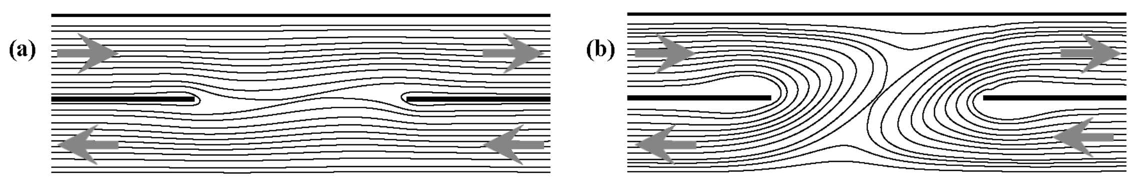


FIGURE 3. Streamlines for $\theta = 1.74$, with $\alpha = 0$, $Re = 0$ and $De = 0.351$ in mesh M51: (a) $R_r = 0.047$; (b) $R_r = 0.85$.

CONCLUSIONS

The UCM model was used to simulate viscoelastic flows in a mixing and separating flow [1]. For a combination of critical flow geometries, it was possible to identify a steady bi-stable bifurcation pattern at low inertia and high elasticity, thus suggesting the possibility to use such geometry as a micro-mixer for viscoelastic fluids if the flow is made to oscillate periodically.

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