Numerical Study of non-Newtonian Inelastic Fluid Flow in a 2D Bifurcation at Ninety Degrees

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1 - T-JUNCTION FLOWS

- The human circulatory system (many bifurcations)
2 - OBJECTIVES

- Evaluate the effect of inertia and flow rate ratio in flow through a T-junction (dividing flow arrangement) for Newtonian and non-Newtonian inelastic fluids (fixed n).

\[
Re = \frac{\rho u_1 H}{\eta_0}
\]

\[
50 \leq Re \leq 1000
\]

\[
\beta = \frac{Q_3}{Q_1}
\]

\[
0.1 \leq \beta \leq 0.9
\]
2 - OBJECTIVES

- Evaluate the effect of inertia and flow rate ratio in flow through a T-junction (dividing flow arrangement) for Newtonian and non-Newtonian inelastic fluids (fixed n).

- Evaluate the effect of power law exponent $n$ variation in the Carreau-Yasuda model for this kind of flow.
3 - EQUATIONS

- Conservation of mass
  \[ \nabla \cdot \mathbf{u} = 0 \]

- Conservation of linear momentum
  \[ \rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \cdot \mathbf{\tau} \]

  - Newtonian
    \[ \mathbf{\tau} = \eta \mathbf{\gamma} \]

  - non-Newtonian
    \[ \mathbf{\tau} = \eta (|\mathbf{\gamma}|) \mathbf{\gamma} \]
3 - EQUATIONS

- Conservation of mass
  \[ \nabla \cdot \mathbf{u} = 0 \]

- Conservation of linear momentum
  \[ \rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \cdot \mathbf{\tau} \]

- GNF fluid
- Carreau-Yasuda model
- non-Newtonian

\[ \mathbf{\tau} = \eta(|\mathbf{\gamma}|) \dot{\mathbf{\gamma}} \]
3 - EQUATIONS

- Carreau-Yasuda model

\[ \eta = \eta_\infty + (\eta_0 - \eta_\infty) \left[ 1 + \left( \frac{\lambda |\dot{\gamma}|}{a} \right)^a \right]^{n-1} \frac{1}{a} \]

- Blood parameters (Banerjee et al. 1997)
  - Power law exponent, \( n=0.3568 \) (for results with fixed \( n \));
  - Carreau parameter, \( a=2 \);
  - Zero shear rate viscosity, \( \eta_0=0.056 \text{ Pa.s} \);
  - Infinite shear rate viscosity, \( \eta_\infty =0.00345 \text{ Pa.s} \);
  - Time constant \( \lambda=3.313 \text{ s} \).
4 - NUMERICAL METHOD

- Finite-volume method for discretization of equations.
- Nonstaggered mesh arrangement.
  - Pressure-velocity coupling: Rhie e Chow, 1983.
- Convective terms: CUBISTA scheme (Alves et al. 2003).
- Pressure-correction SIMPLEC algorithm with time-marching.
- Convergence tolerance for iterative process when norm of residuals less than $10^{-8}$. 
Flow rate
\[ \beta = \frac{Q_3}{Q_1} \]

Fully developed upstream flow

Vanishing axial variation for all quantities
\[ \left( \frac{\partial}{\partial x} = 0 \text{ or } \frac{\partial}{\partial y} = 0 \right) \]
except pressure (linear extrapolation)

Outlet
\[ Y_{\text{tot}} = 21 \, H \]

Walls
No slip condition \( u = v = 0 \)
Orthogonal but non uniform meshes

- MESH M1, 12800 VC ($X_{tot}=26H$ $Y_{tot}=21H$)
  \[ \Delta x_{\text{min}} = \Delta y_{\text{min}} = 2.5 \times 10^{-2} \]

- MESH M2, 22400 VC ($X_{tot}=66.5H$ $Y_{tot}=60.5H$)
  \[ \Delta x_{\text{min}} = \Delta y_{\text{min}} = 2.5 \times 10^{-2} \]
MESH M1 (12800 VC)

\[ \Delta x_{\text{min}} = \Delta y_{\text{min}} = 2.5 \times 10^{-2} \]
7 - RESULTS

- Streamlines for increasing Reynolds number

Graphs:
- Newtonian:
  $\beta = 0.3 \quad Re = 50$
- GNF:
  $\beta = 0.3 \quad Re = 50$
7 - RESULTS

- Vertical recirculation length

**Graphs:**
- **Newtonian**
  - Legend: \( \beta = 0.1 \), \( \beta = 0.2 \), \( \beta = 0.3 \), \( \beta = 0.4 \), \( \beta = 0.5 \), \( \beta = 0.6 \), \( \beta = 0.7 \), \( \beta = 0.8 \), \( \beta = 0.9 \).

- **GNF**
7 - RESULTS

- Streamlines for increasing Reynolds number

![Streamlines Diagram](image)

GNF (n=0.3568)

$\beta=0.8 \quad Re=50$
7 - RESULTS

- Vortex strength of vertical recirculation

**Newtonian**

**GNF**

![Graphs showing vortex strength vs. Reynolds number for Newtonian and GNF fluids at different values of β.](image-url)
7 - RESULTS

- Vortex strength of vertical recirculation

![Graph showing vortex strength for different Re values and β values]
Streamlines (secondary branch)

\[ \beta = 0.8 \text{ (GNF) } \quad \text{Re} = 400 \]

\[ \beta = 0.8 \text{ (GNF) } \quad \text{Re} = 400 \]
7 - RESULTS

- Horizontal recirculation length

![Graphs showing horizontal recirculation length for Newtonian and GNF fluids with different values of \( \beta \).]
Vortex strength of horizontal recirculation

\[ \beta = 0.1 \quad \beta = 0.2 \quad \beta = 0.3 \quad \beta = 0.4 \quad \beta = 0.5 \quad \beta = 0.6 \quad \beta = 0.7 \quad \beta = 0.8 \quad \beta = 0.9 \]
Shear stress field: variation with Re
7 - RESULTS

- Maximum values of shear stress
7 - RESULTS (variation with $n$)

- Streamlines for increasing power law index

![Streamlines diagram with annotations](image)
7 - RESULTS (variation with $n$)

- Recirculation length (fixed $\beta$)

**Vertical**

- $\beta = 0.5$
- $n = 0.10$
- $n = 0.20$
- $n = 0.30$
- $n = 0.40$
- $n = 0.50$
- $n = 0.60$

**Horizontal**

- $\beta = 0.5$
- $n = 0.10$
- $n = 0.20$
- $n = 0.30$
- $n = 0.40$
- $n = 0.50$
- $n = 0.60$
7 - RESULTS (variation with $n$)

- Recirculation length (fixed $Re$)

**Vertical**

![Graph showing recirculation length for different $n$ values for vertical flow.]

**Horizontal**

![Graph showing recirculation length for different $n$ values for horizontal flow.]

Legend:
- $n=0.10$
- $n=0.20$
- $n=0.30$
- $n=0.40$
- $n=0.50$
- $n=0.60$

$Re = 102$
7 - RESULTS (variation with $n$)

- Vortex strength of recirculations (fixed $\beta$)

**Vertical**

![Graph showing vortex strength of recirculations for vertical orientation.](image)

**Horizontal**

![Graph showing vortex strength of recirculations for horizontal orientation.](image)
7 - RESULTS (variation with $n$)

- Shear stress field: variation with $n$
8 - CONCLUSIONS

- The length and intensity of the recirculations increase with Re for both fluids.

- The recirculation lengths $Y_L$ and $X_L$ increase with $\beta$ for $\beta \leq 0.6$ and decrease for $\beta > 0.6$, for both fluids.

- Non-Newtonian fluids show larger recirculation lengths.

- Reduction of growth rates of $X_L$ and $Y_L$ with Re after a new recirculation is formed at the opposite wall (distal and top walls).

- Sudden increase of $\psi_V$ with Re due to the division of the vertical recirculation into two vortices.
8 - CONCLUSIONS

- Decrease of $\psi_V$ with $\beta$, for low Re numbers and vice-versa. Increase of $\psi_H$ with $\beta$.

- Low stresses inside recirculating zones and high stresses in the re-entrant corners of the bifurcation.

- Larger stress maxima with Newtonian fluid.

- Stress increase (in modulus) with Re and $\beta$.

- Length and intensity of recirculations decrease with increase of power law exponent (n).

- Stress increase (in modulus) with increase of power law exponent (n).
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