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PARAMETRIC STUDY ON THE THREE-DIMENSIONAL DISTRIBUTION OF VELOCITY OF A FENE-CR FLUID FLOW THROUGH A CURVED CHANNEL

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OBJECTIVES

Analyse the development and distribution of velocity in the cross-sections along a curved channel, varying the retardation ratio parameter (β) and the extensibility parameter (L^2).

Numerical simulations in a 180° curved channel of square cross-section for FENE-CR viscoelastic fluid model for:

- Reynolds numbers (*Re*) ≤ 1760;
- Weissenberg numbers (*Wi*) ≤ 1.0;
- retardation ratio (β) from 1 to 0;
- extensibility parameter (L^2) from 50 to 200.

GOVERNING EQUATIONS

Assumptions:

- three-dimensional flow
- laminar
- isothermal
- steady flow
- incompressible fluid

- **Mass conservation**

$$\nabla \bullet \boldsymbol{u} = 0$$

- **Momentum conservation**

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \boldsymbol{u}.\nabla \boldsymbol{u} = -\nabla \bullet \boldsymbol{\tau}_{tot}$$

- **Constitutive**

$$\boldsymbol{\tau}_{tot} = \boldsymbol{\tau}_s + \boldsymbol{\tau}$$

Newtonian

$$\boldsymbol{\tau}_s = \eta_s \left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T \right) = 2\eta_s \boldsymbol{D}$$

FENE-CR

$$\boldsymbol{\tau} + \left(\frac{\lambda}{f(\tau)} \overset{\nabla}{\boldsymbol{\tau}} \right) = 2\eta_p \boldsymbol{D}$$

$$f(\tau) = \frac{L^2 + (\lambda/\eta_p)\, tr(\tau)}{L^2 - 3}$$

$$\boldsymbol{D} = \frac{1}{2}\left(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T \right)$$

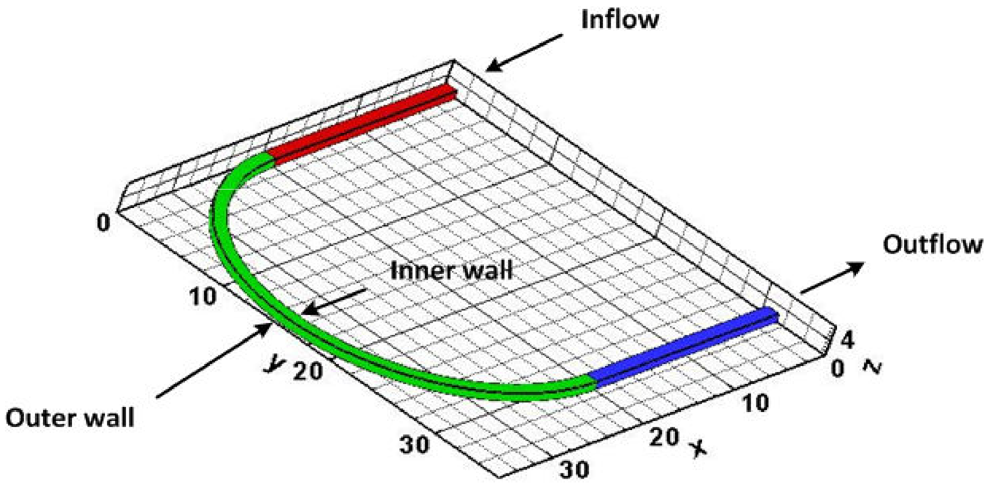
DIMENSIONLESS PARAMETERS

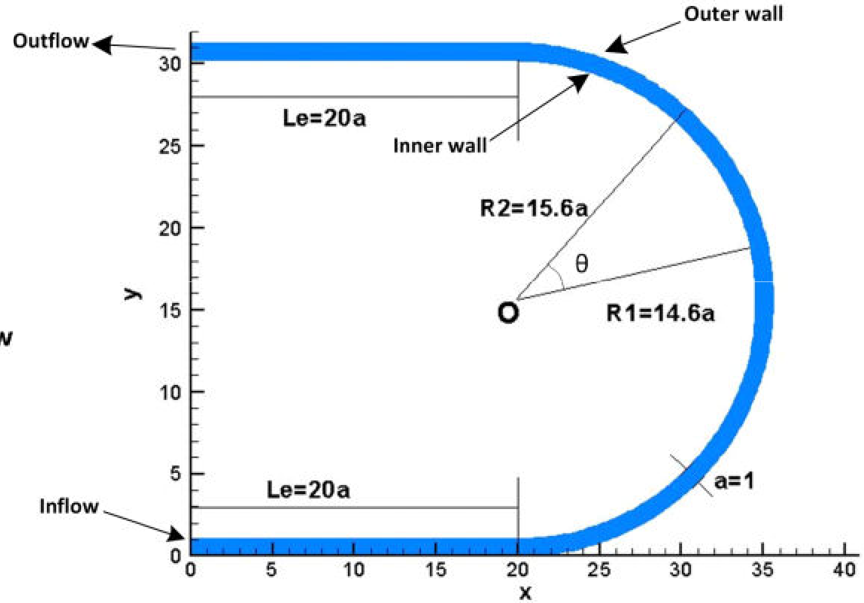
- **Reynolds Number** $\mathrm{Re} = \dfrac{\rho U_m a}{\eta}$ → $\eta = \eta_s + \eta_p$
- **Dean Number** $Dn = \dfrac{\mathrm{Re}}{\sqrt{R_c}}$ → $R_c = \dfrac{R_1 + R_2}{2a}$
- **Weissenberg Number** $Wi = \dot{\gamma}\lambda$ → $\dot{\gamma} = \dfrac{U_m}{a}$
- **Retardation ratio** $\beta = \dfrac{\lambda_r}{\lambda} = \dfrac{\eta_s}{\eta}$
- **Extensibility** L^2

ρ – fluid density
U_m – mean velocity
a – channel height/width
μ – viscosity
R_c – curvature ratio
R_1 – internal radius
R_2 – external radius
η_s – solvent viscosity
η_p – polymer viscosity
η – total shear viscosity
λ – constant zero-shear rate relaxation time
λ_r – retardation time
$\dot{\gamma}$ – characteristic rate o deformation

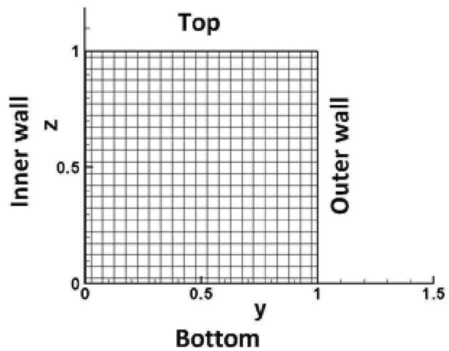
GEOMETRY & MESH

- **Channel:**
  - 180° curved duct coupled to straight ducts at entry and exit
  - Square cross-section





- **Mesh:**
  - Non-uniform at entrance and exit channels
  - Uniform at curved part and cross-section



NUMERICAL METHOD & BOUNDARY CONDITIONS

Numerical Method:

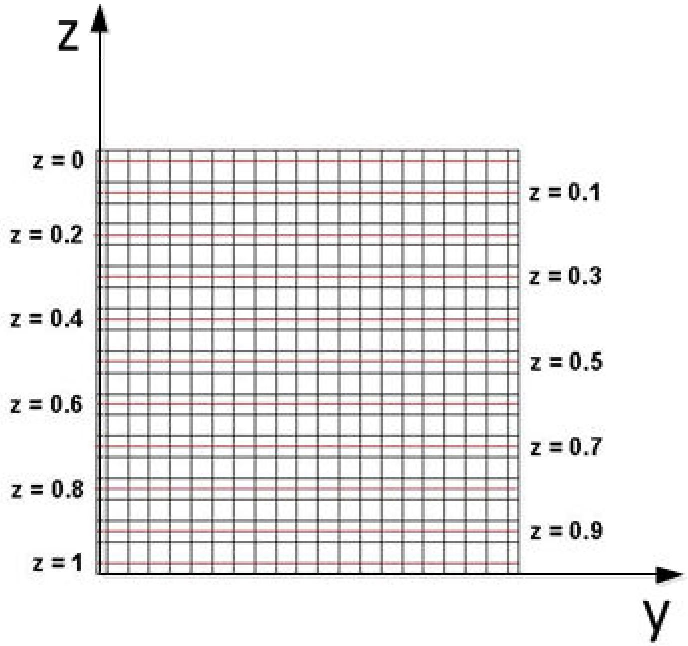
- Fully implicit finite-volume method
- General non-orthogonal coordinate system
- Collocated mesh arrangement

Boundary Conditions:

- Non-slip conditions at walls
- Fully developed flow at entrance
- Zero axial-gradient at the exit
- Entire domain was considered

RESULTS

- **Reynolds Number: *Re*= 532 and 1760;**
- **Weissenberg Number: *Wi* = 0.3, 0.5 and 0,8;**
- **Retardation ratio: β = 0.5, 0.25 and 0.1;**
- **Extensibility parameter: L^2 = 50, 100 and 200.**

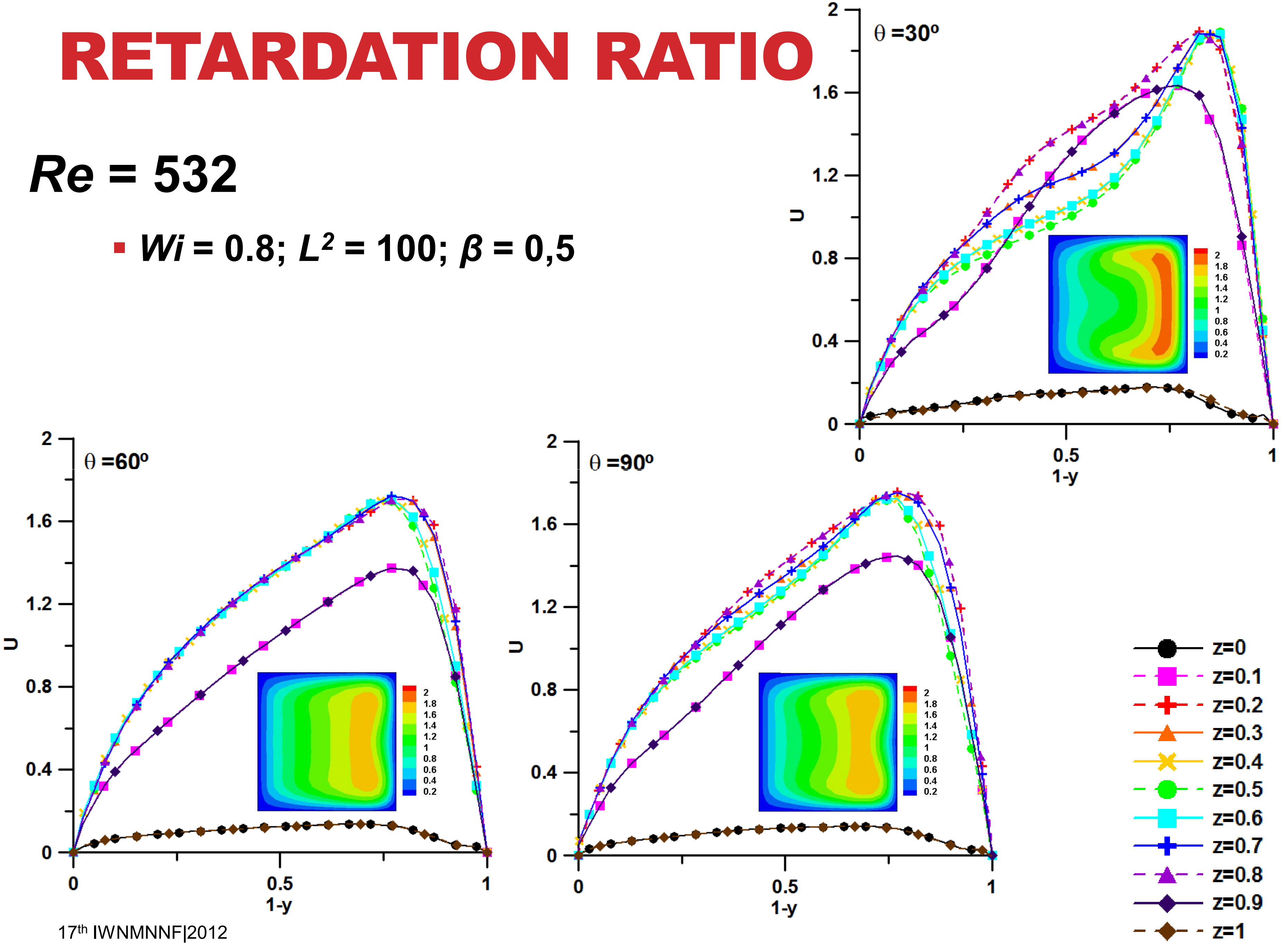


- **Velocity profiles** (along *z* axis)
- **Velocity contours** (cross-section)
- **Velocity fields** (cross-section)

RETARDATION RATIO

Re = 532

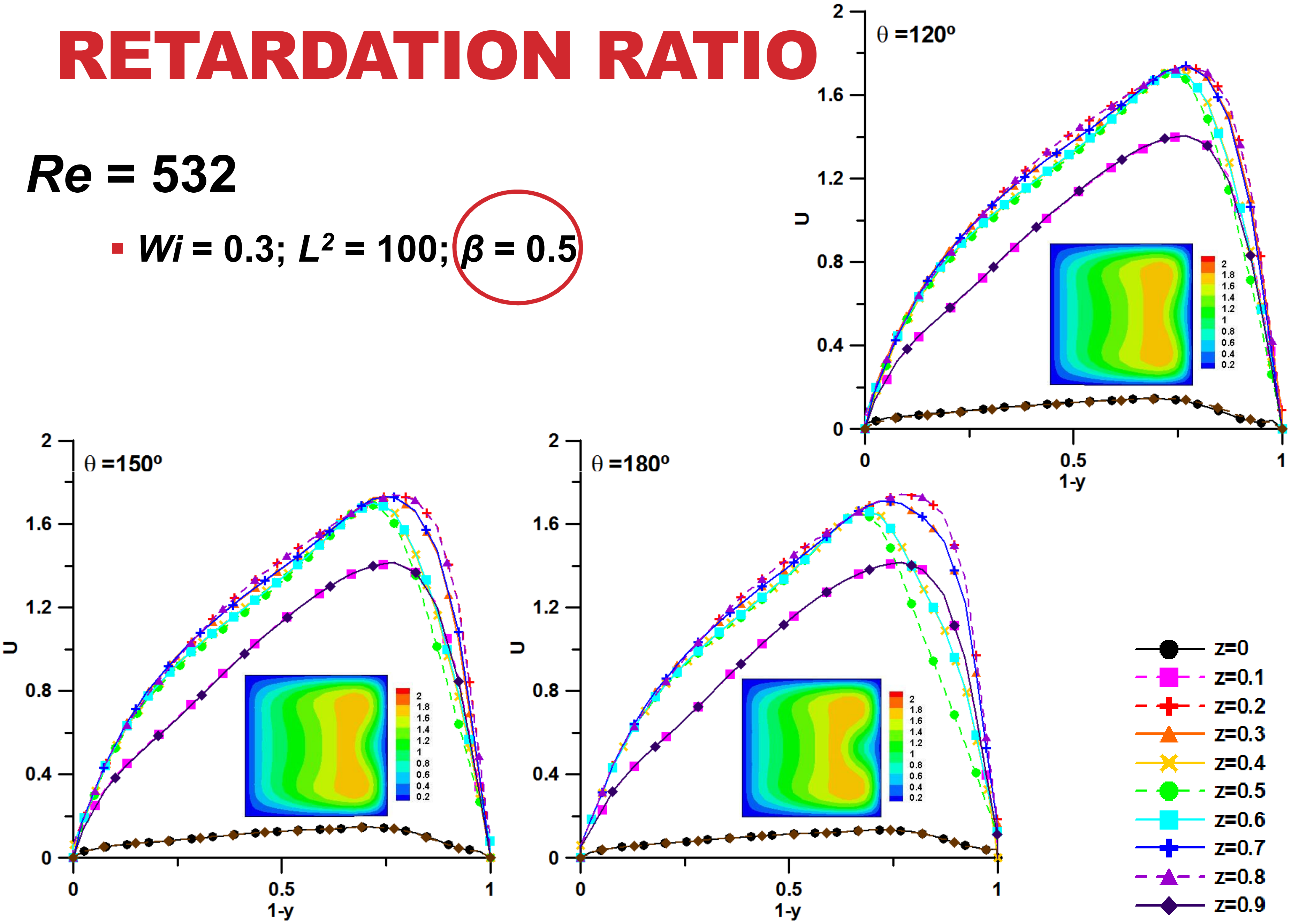
- $Wi = 0.8$; $L^2 = 100$; $\beta = 0{,}5$



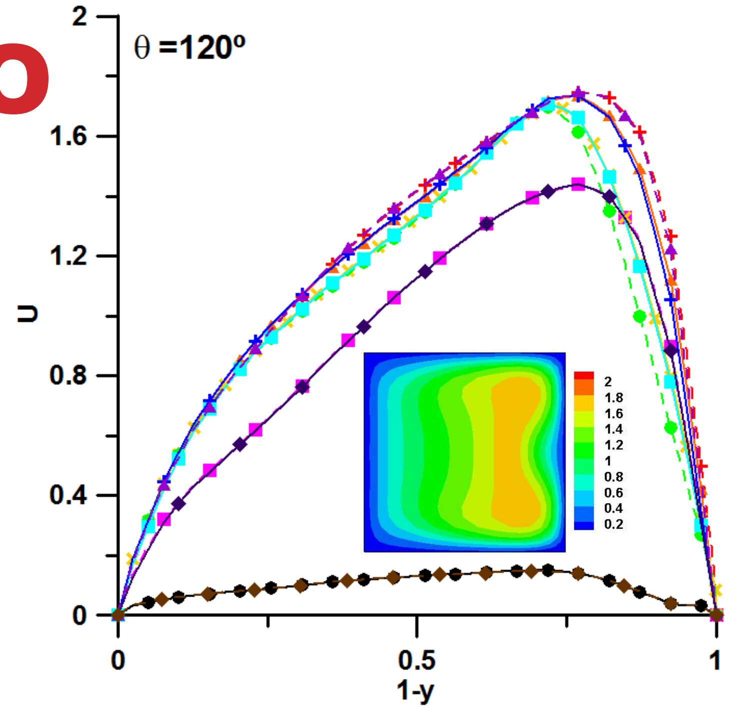
RETARDATION RATIO

Re = 532

- *Wi* = 0.3; L^2 = 100; β = 0.5

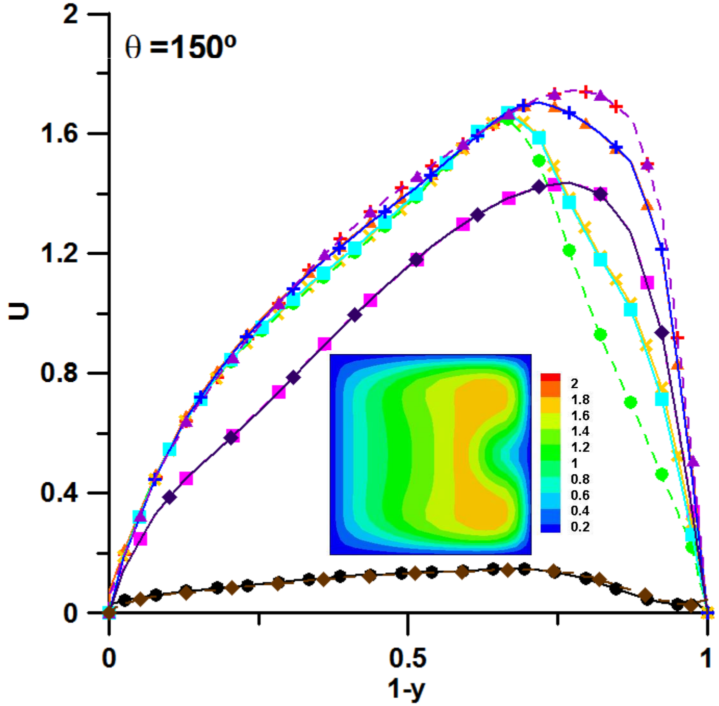


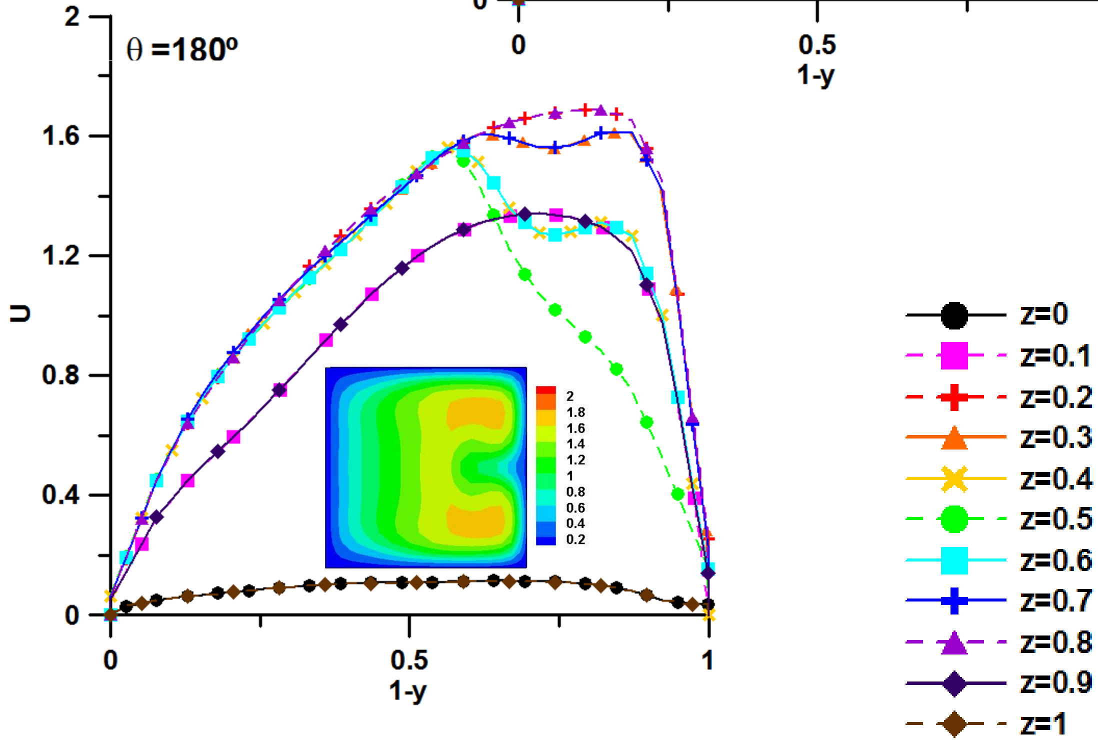
RETARDATION RATIO



Re = 532

- *Wi* = 0.3; L^2 = 100; *β* = 0.25

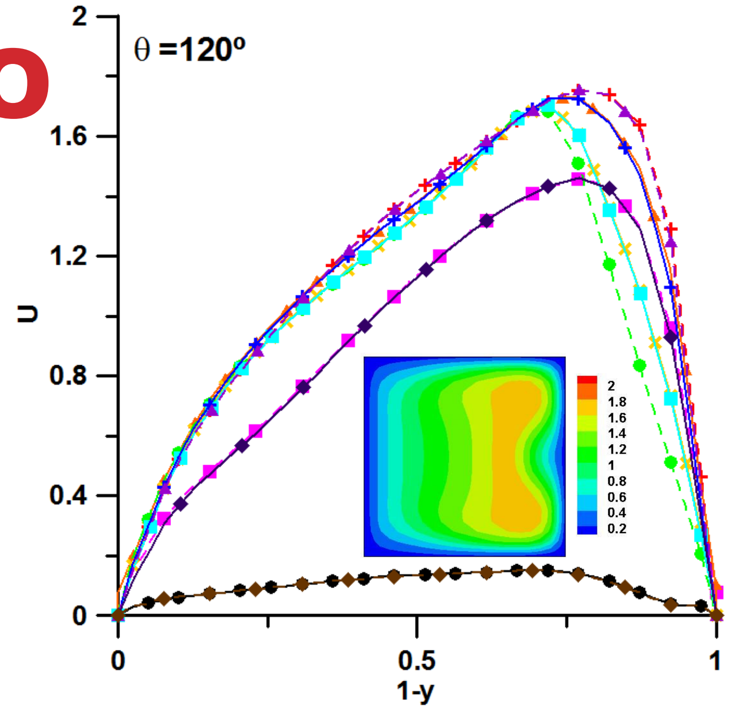


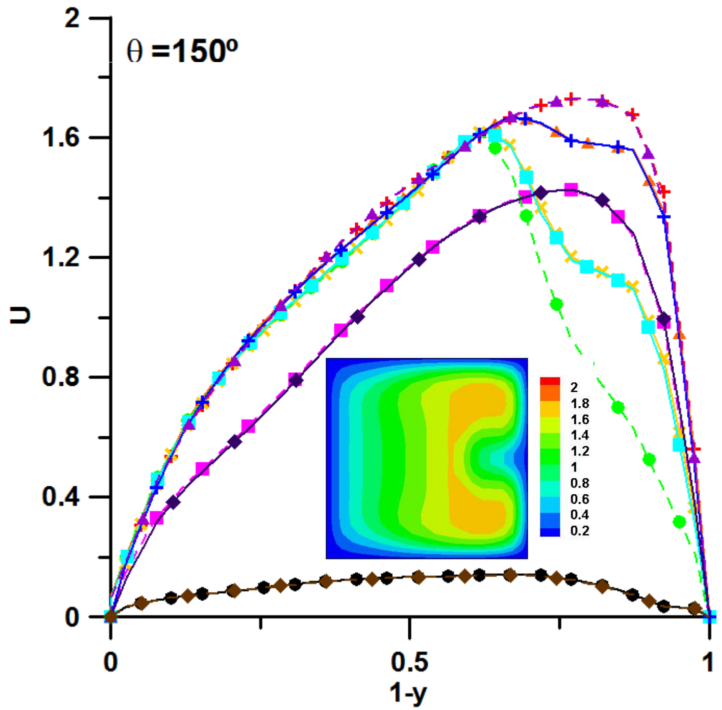


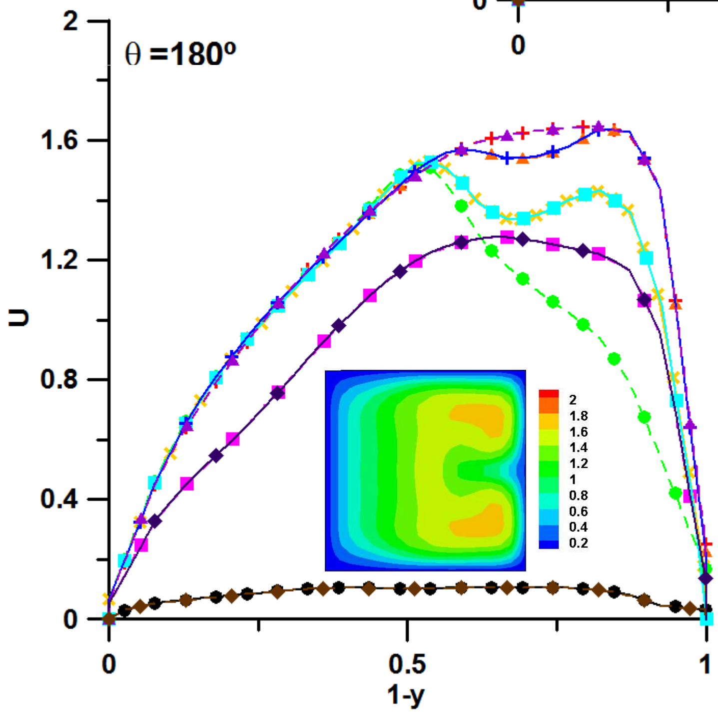
RETARDATION RATIO

Re = 532

- *Wi* = 0.3; L^2 = 100; β = 0.1



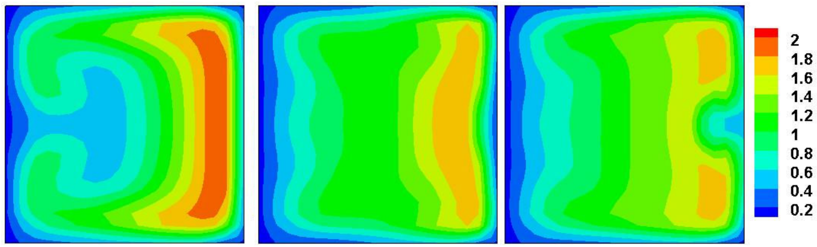


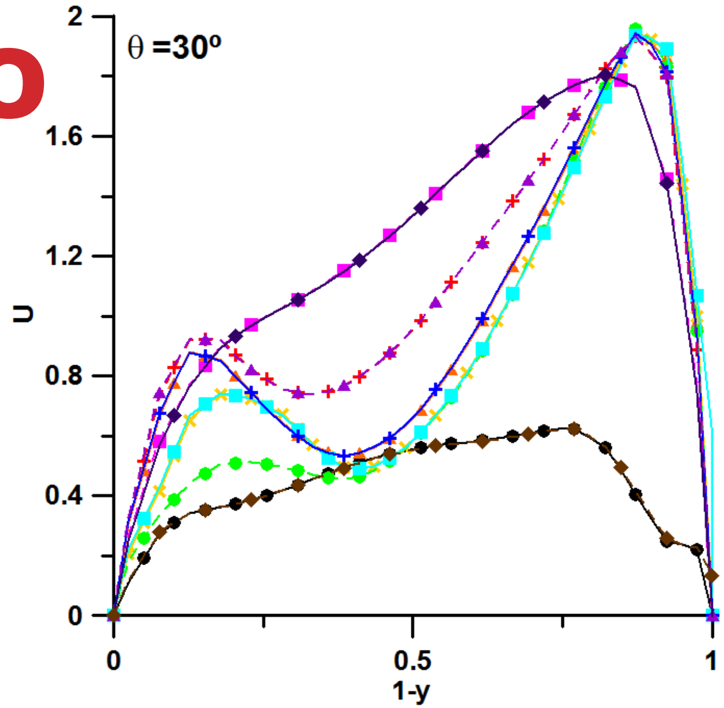


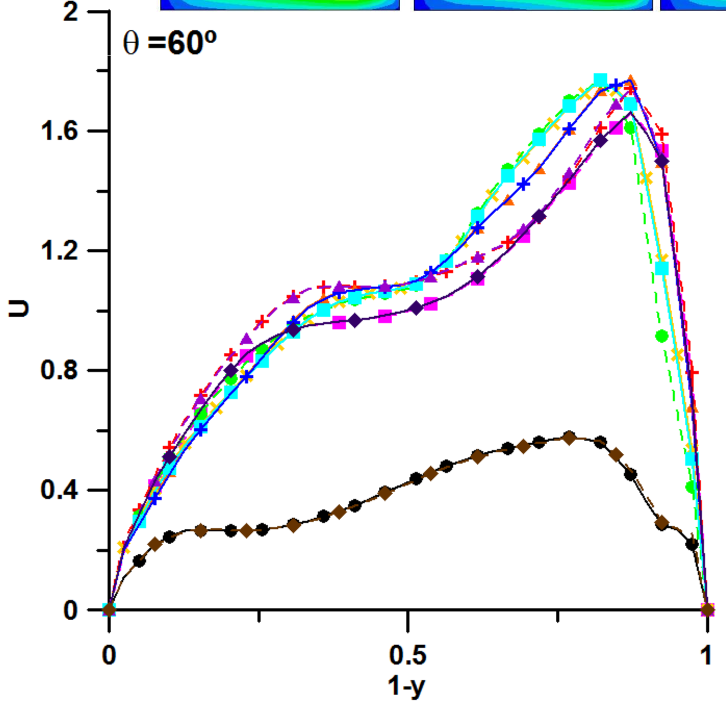
RETARDATION RATIO

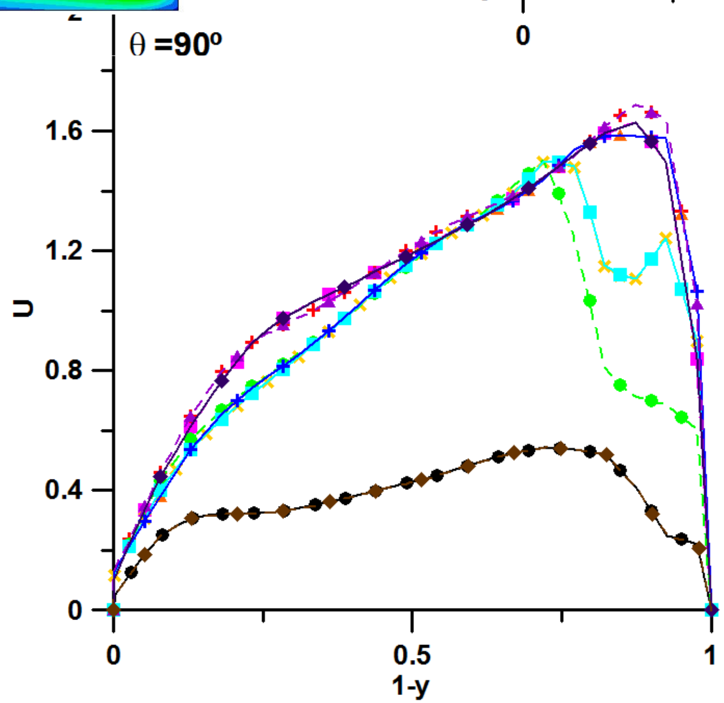
Re = 1760

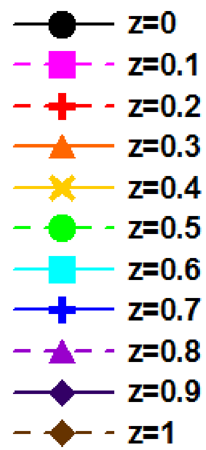
- *Wi* = 0.3; L^2 = 100; β = 0.25







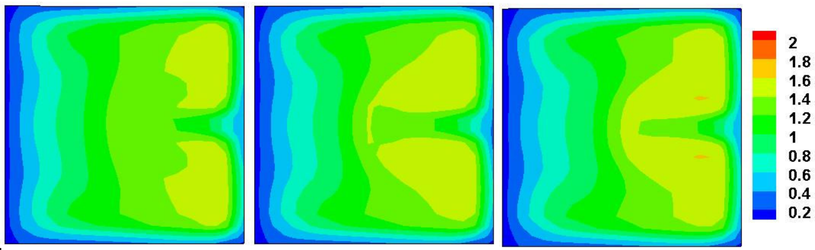


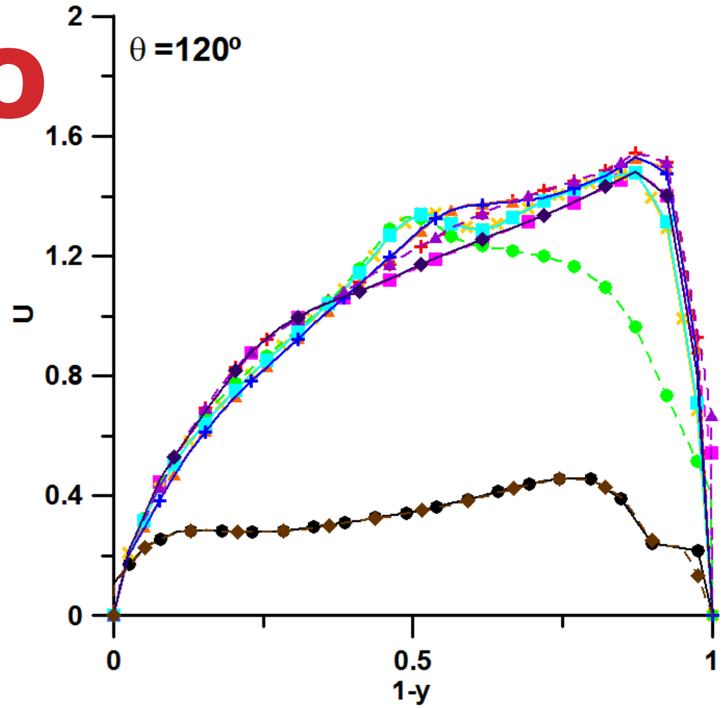


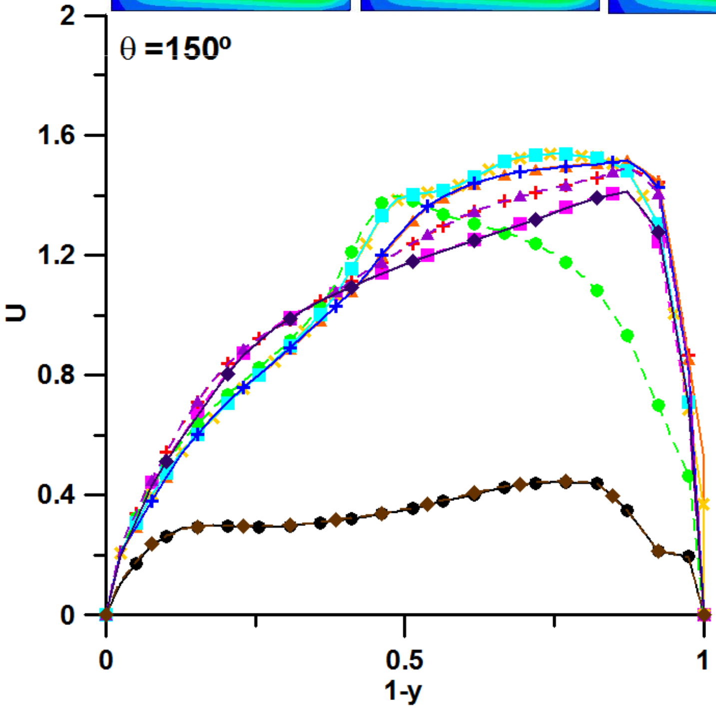
RETARDATION RATIO

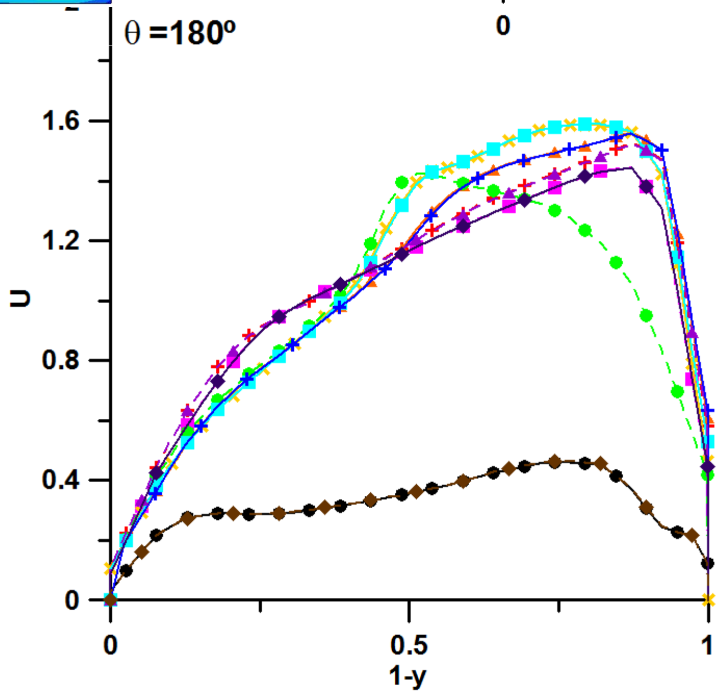
Re = 1760

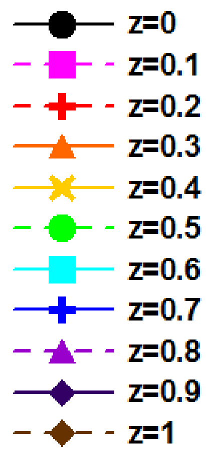
- $Wi = 0.5$; $L^2 = 100$; $\beta = 0.5$







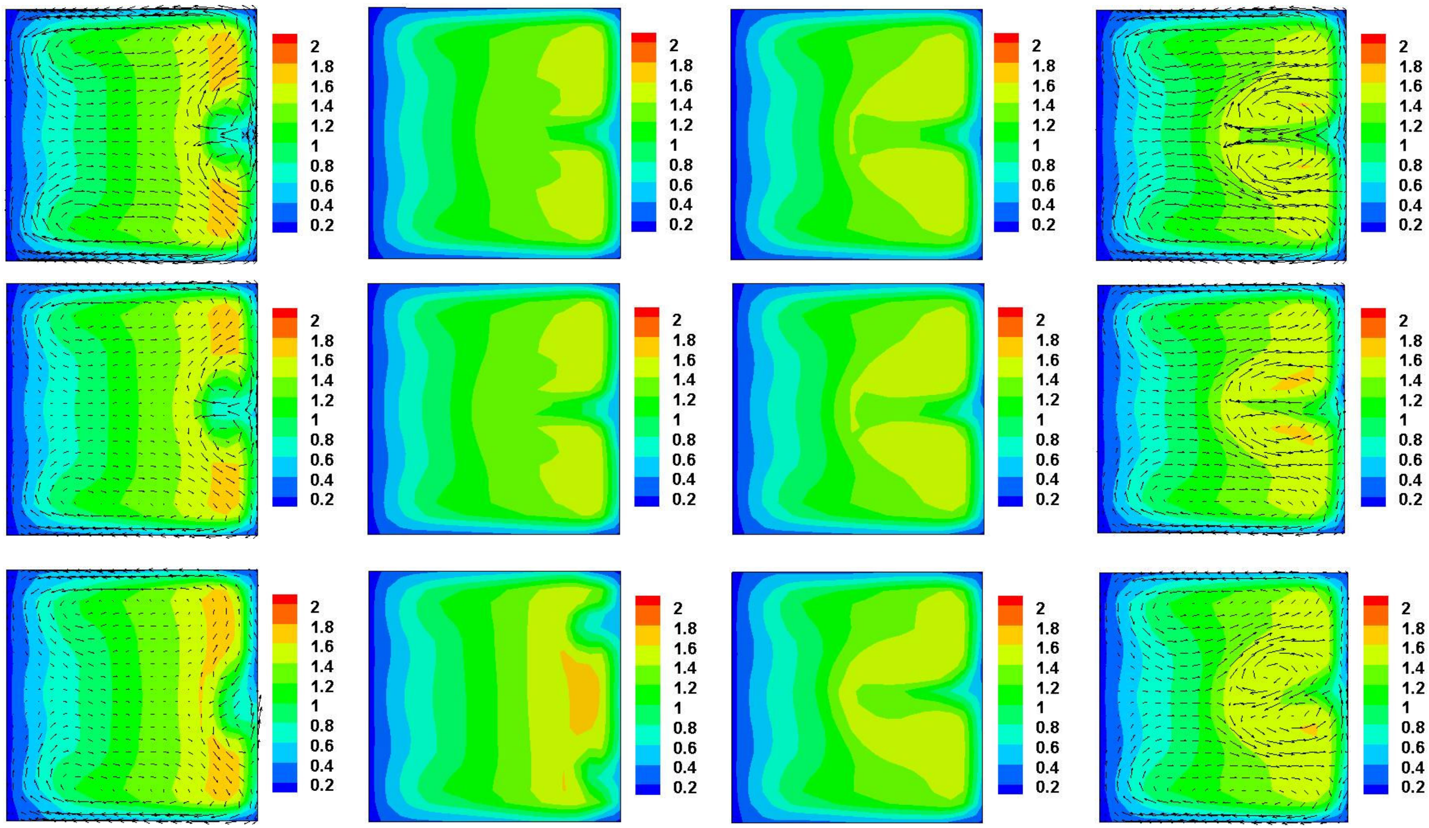




RETARDATION RATIO

Re = 1760

- $Wi = 0.5$; $L^2 = 100$



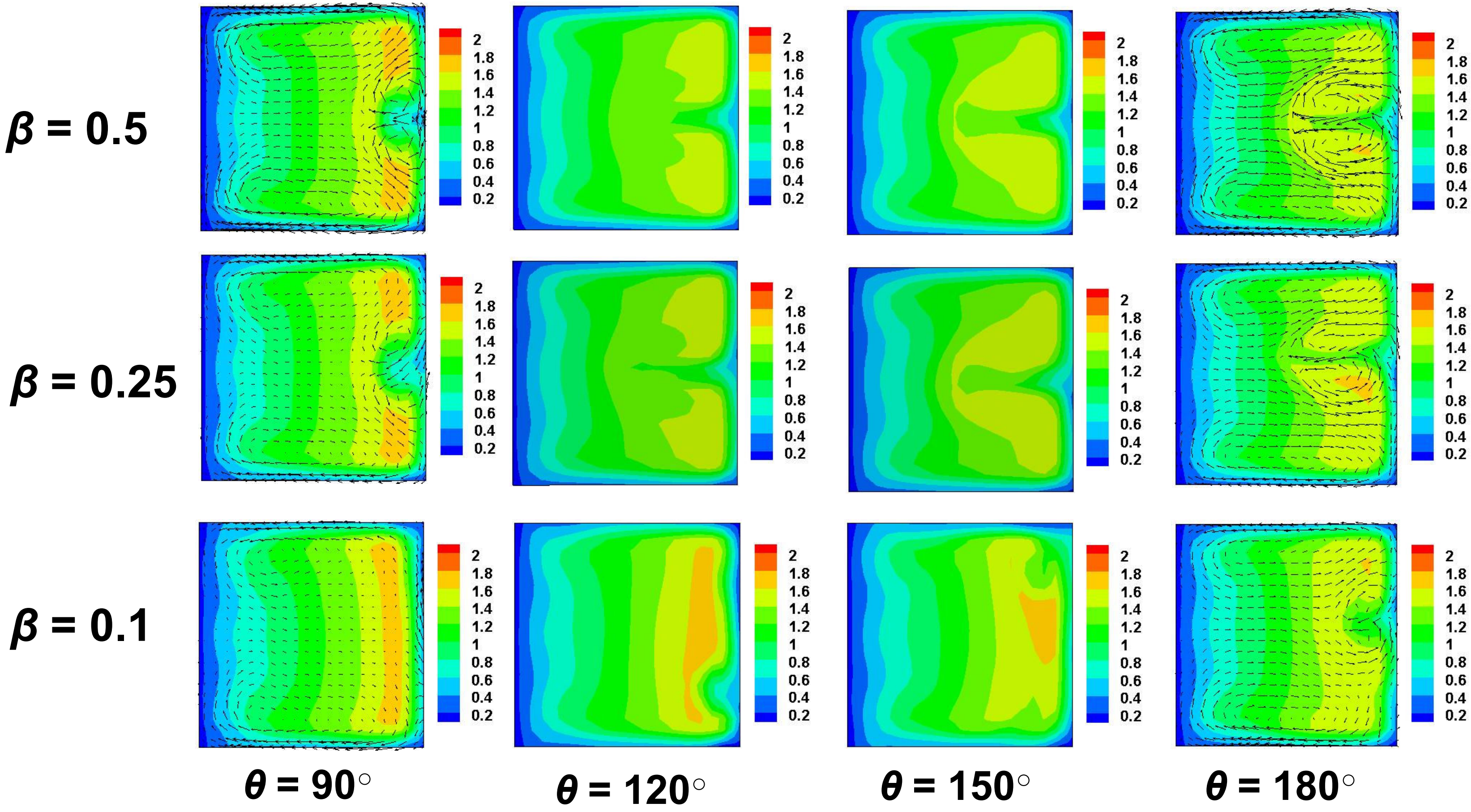
Rows: $\beta = 0.5$, $\beta = 0.25$, $\beta = 0.1$

Columns: $\theta = 90^\circ$, $\theta = 120^\circ$, $\theta = 150^\circ$, $\theta = 180^\circ$

RETARDATION RATIO

Re = 1760

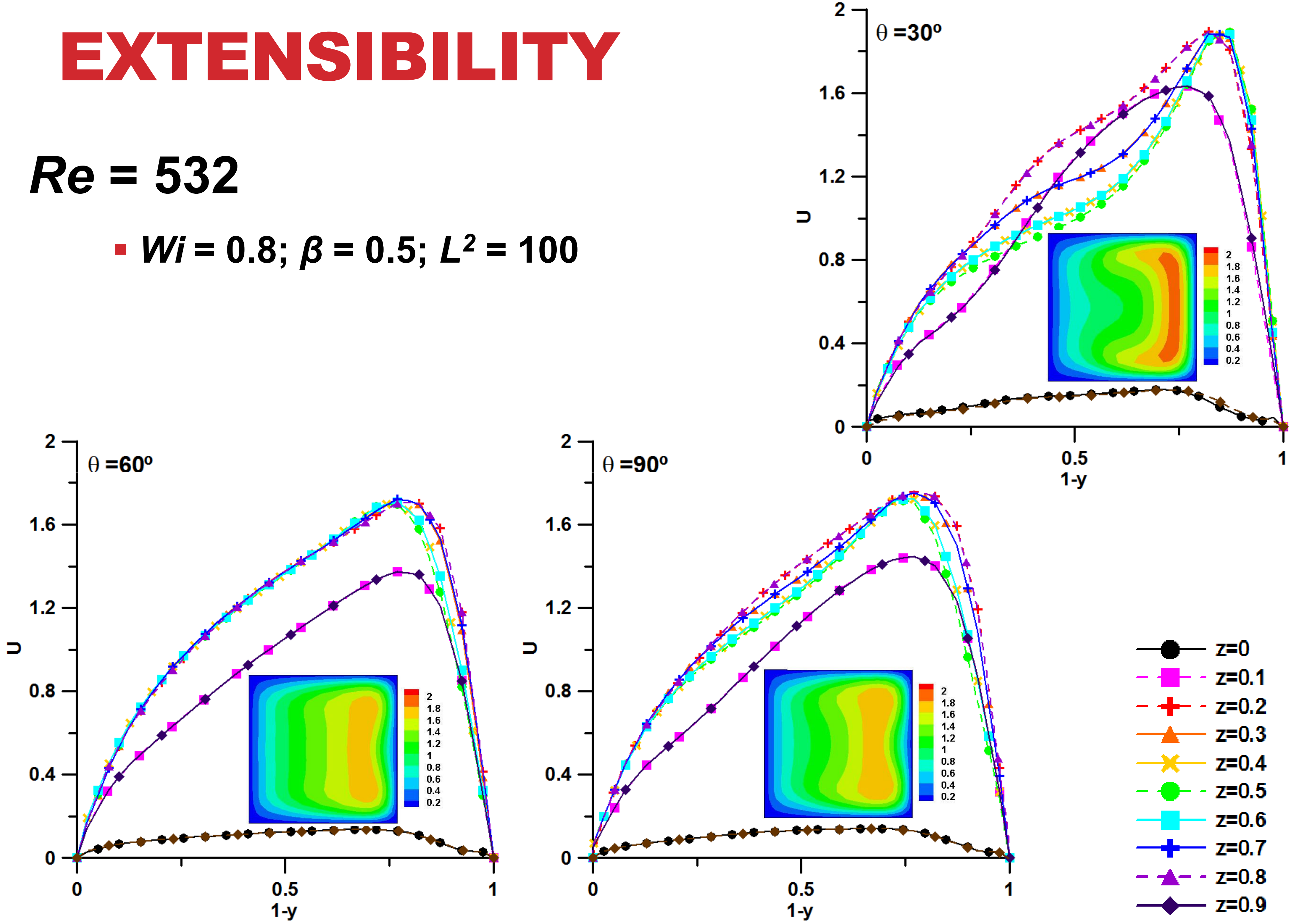
- *Wi* = 0.8; L^2 = 100



EXTENSIBILITY

Re = 532

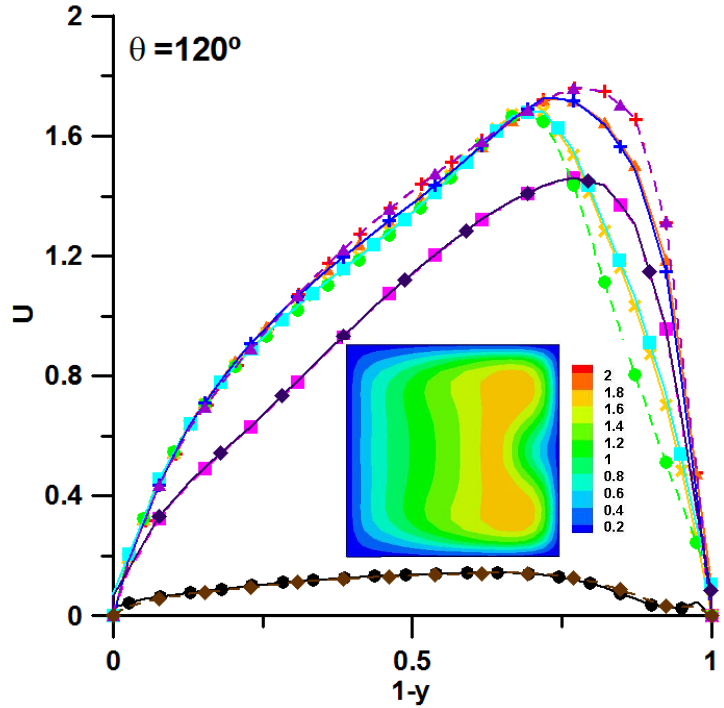
- $Wi = 0.8$; $\beta = 0.5$; $L^2 = 100$

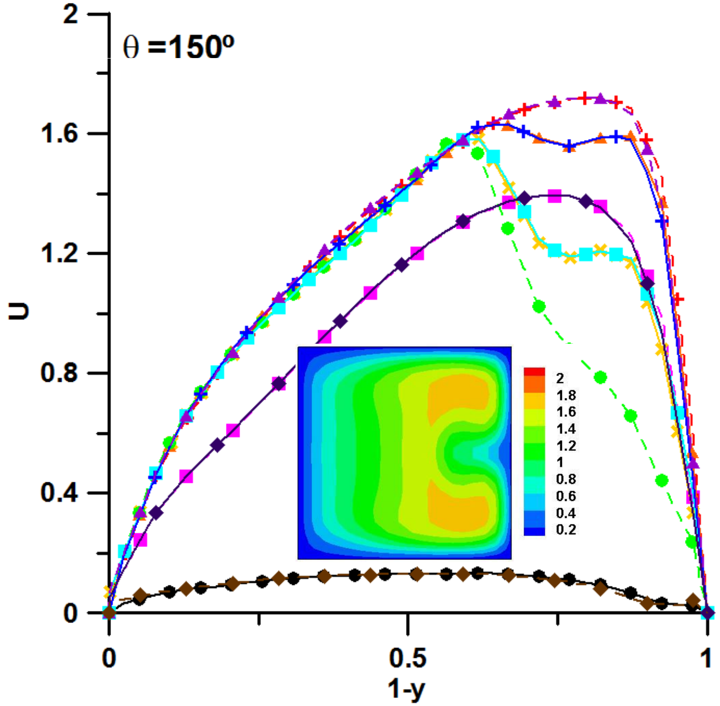


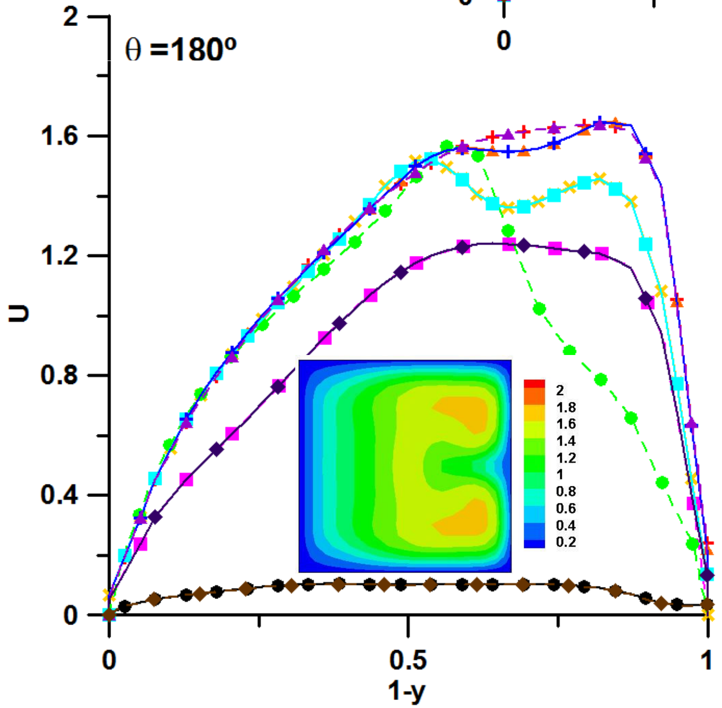
EXTENSIBILITY

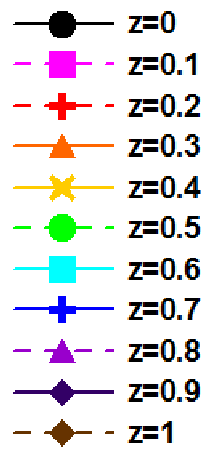
Re = 532

- $Wi = 0.8$; $\beta = 0.5$; $L^2 = 200$





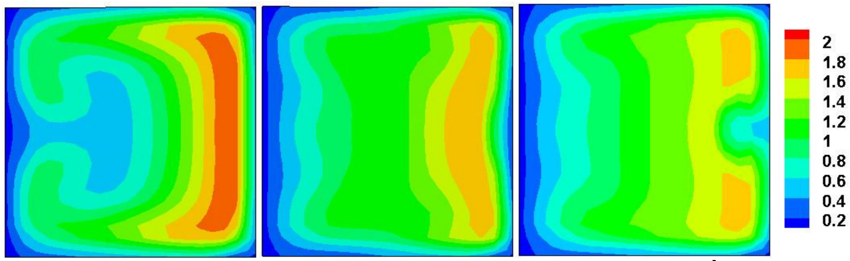


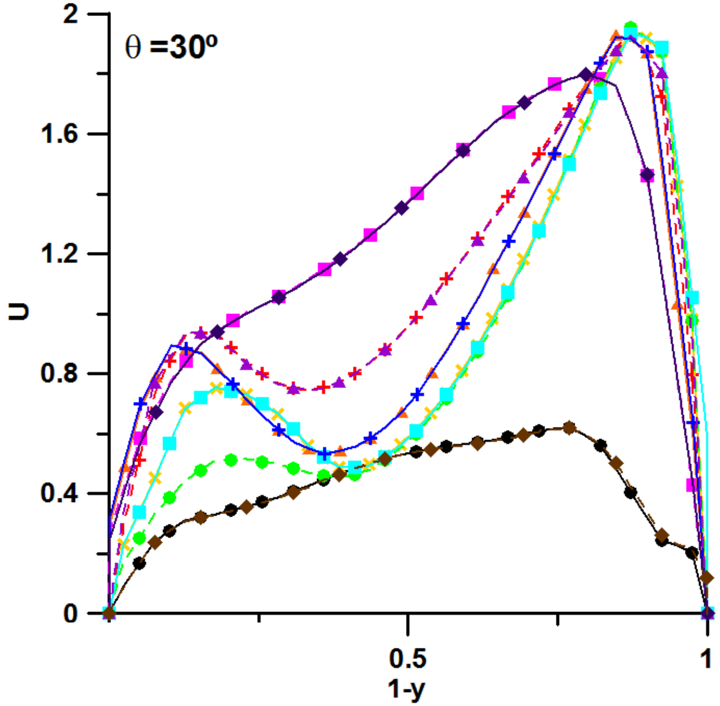


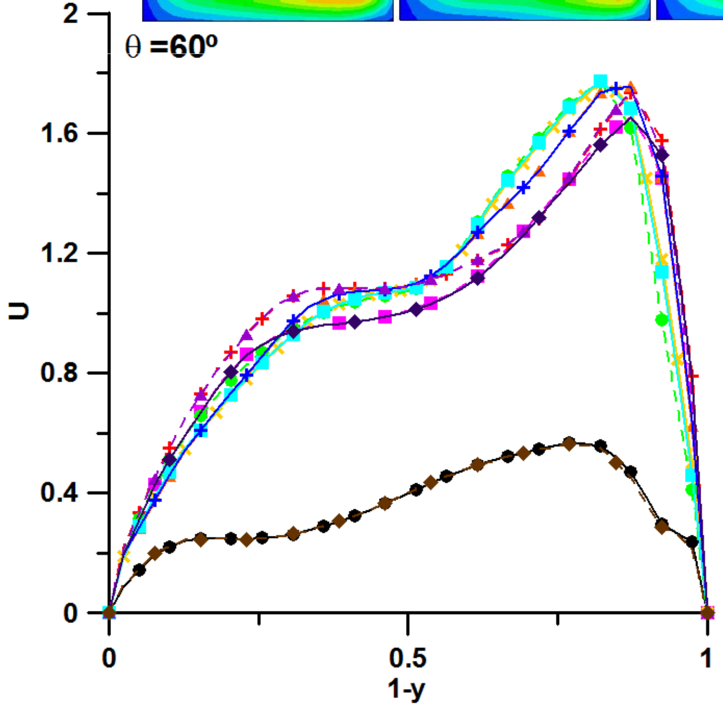
EXTENSIBILITY

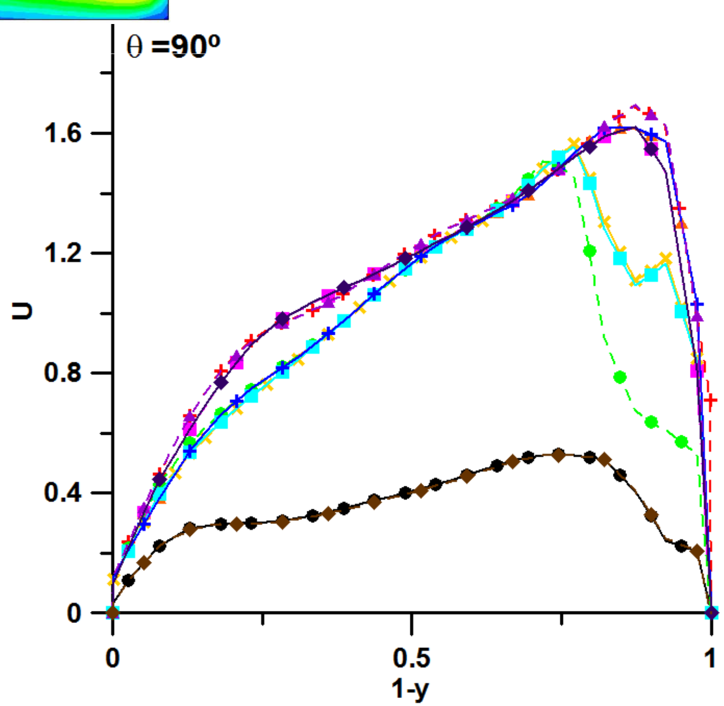
Re = 1760

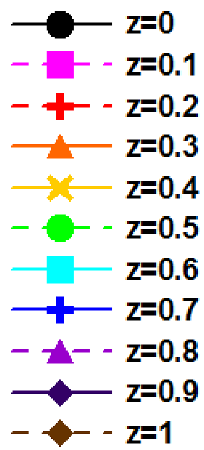
- $Wi = 0.3$; $\beta = 0.5$; $L^2 = 200$







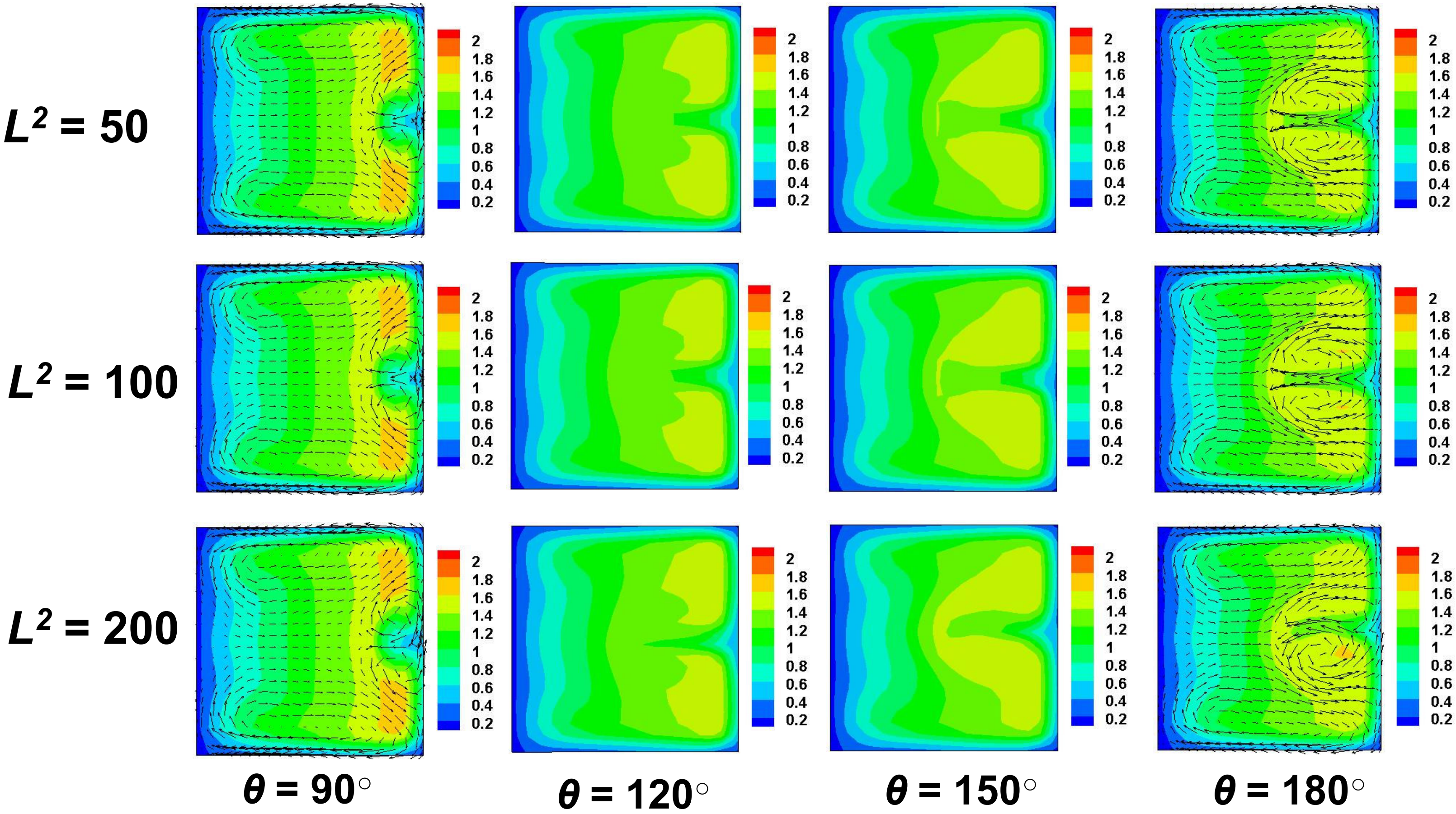




RETARDATION RATIO

Re = 1760

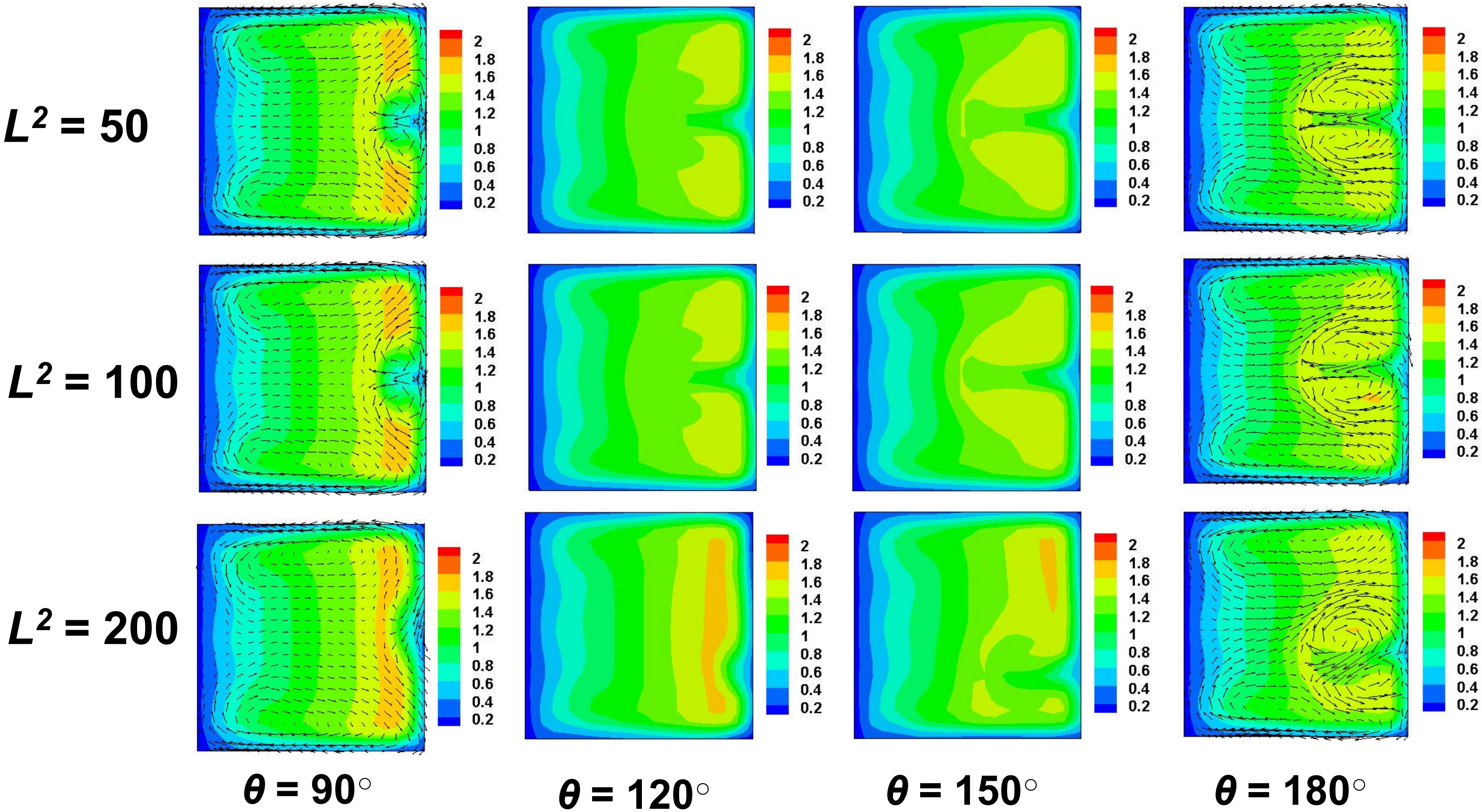
- *Wi* = 0.5; *β* = 0.5



RETARDATION RATIO

Re = 1760

- *Wi* = 0.8; β = 0.5



CONCLUSIONS

- In general, inertia dominates the flow in the first half of the curve ($\theta \leq 120^{\circ}$) for Re = 532, and for Re = 1760 in the region of $\theta < 90^{\circ}$, for all cases simulated.
- Flow development is similar when considering variation of β and L^2, along the channel and across the transverse section, for the same Re.
- Effects of decreasing β and increasing L^2 are more intensely felt for higher Re and Wi.
- Decreasing β and increasing L^2 leads to the formation of additional pair of vortices for lower Re and Wi, and to the disappearance of the additional pair of vortices for higher Re and Wi.

Variation of β:

- Symmetry is observed in all cases simulated and along the entire length of the curve, for Re = 532.
- Asymmetries are observed for Re = 1760, when Wi = 0.5 and β = 0.1, and when Wi = 0.8 and $\beta \leq 0.5$

Variation of L^2:

- Symmetry is observed in all cases simulated and along the entire length of the curve, for Re = 532.
- Asymmetries are observed for Re = 1760, when Wi = 0.5 and L^2 = 200, and when Wi = 0.8 and $L^2 \geq 100$.

ACKNOWLEDGMENT

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COMPETE

PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE

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