PARAMETRIC STUDY ON THE THREE-DIMENSIONAL DISTRIBUTION OF VELOCITY OF A FENE-CR FLUID FLOW THROUGH A CURVED CHANNEL

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OBJECTIVES

Analyse the development and distribution of velocity in the cross-sections along a curved channel, varying the retardation ratio parameter ($\beta$) and the extensibility parameter ($L^2$).

Numerical simulations in a $180^\circ$ curved channel of square cross-section for FENE-CR viscoelastic fluid model for:

- Reynolds numbers ($Re$) $\leq 1760$;
- Weissenberg numbers ($Wi$) $\leq 1.0$;
- retardation ratio ($\beta$) from 1 to 0;
- extensibility parameter ($L^2$) from 50 to 200.
GOVERNING EQUATIONS

- Mass conservation
  \[ \nabla \cdot \mathbf{u} = 0 \]

- Momentum conservation
  \[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \nabla \mathbf{u} = -\nabla \cdot \mathbf{\tau}_{\text{tot}} \]

- Constitutive
  \[ \mathbf{\tau}_{\text{tot}} = \mathbf{\tau}_s + \mathbf{\tau} \]
  
  Newtonian
  \[ \mathbf{\tau}_s = \eta_s \left( \nabla \mathbf{u} + \left( \nabla \mathbf{u} \right)^T \right) = 2\eta_s \mathbf{D} \]
  
  FENE-CR
  \[ \nabla \mathbf{\tau} + \left( \frac{\lambda}{f(\tau)} \right) \mathbf{\tau} = 2\eta' \mathbf{D} \]
  \[ f(\tau) = \frac{L^2 + \left( \frac{\lambda}{\eta_p} \right) tr(\tau)}{L^2 - 3} \]
  \[ D = \frac{1}{2} \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \]

Assumptions:
- three-dimensional flow
- laminar
- isothermal
- steady flow
- incompressible fluid
**DIMENSIONLESS PARAMETERS**

- **Reynolds Number**
  \[ \text{Re} = \frac{\rho U_m a}{\eta} \]
  \[ \eta = \eta_s + \eta_p \]

- **Dean Number**
  \[ Dn = \frac{\text{Re}}{\sqrt{R_c}} \]
  \[ R_c = \frac{R_1 + R_2}{2a} \]

- **Weissenberg Number**
  \[ \text{Wi} = \dot{\gamma} \]
  \[ \dot{\gamma} = \frac{U_m}{a} \]

- **Retardation ratio**
  \[ \beta = \frac{\lambda_c}{\lambda} = \frac{\eta_s}{\eta} \]

- **Extensibility**
  \[ L^2 \]

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\( \rho \) – fluid density
\( U_m \) – mean velocity
\( a \) – channel height/width
\( \mu \) – viscosity
\( R_c \) – curvature ratio
\( R_i \) – internal radius
\( R_2 \) – external radius
\( \eta_s \) – solvent viscosity
\( \eta_p \) – polymer viscosity
\( \eta \) – total shear viscosity
\( \lambda \) – constant zero-shear rate relaxation time
\( \dot{\lambda}_r \) – retardation time
\( \dot{\gamma} \) – characteristic rate of deformation
GEOMETRY & MESH

- **Channel:**
  - $180^\circ$ curved duct coupled to straight ducts at entry and exit
  - Square cross-section

- **Mesh:**
  - Non-uniform at entrance and exit channels
  - Uniform at curved part and cross-section
NUMERICAL METHOD & BOUNDARY CONDITIONS

Numerical Method:

- Fully implicit finite-volume method
- General non-orthogonal coordinate system
- Collocated mesh arrangement

Boundary Conditions:

- Non-slip conditions at walls
- Fully developed flow at entrance
- Zero axial-gradient at the exit
- Entire domain was considered
RESULTS

- Reynolds Number: Re = 532 and 1760;
- Weissenberg Number: Wi = 0.3, 0.5 and 0.8;
- Retardation ratio: \( \beta = 0.5, 0.25 \) and 0.1;
- Extensibility parameter: \( L^2 = 50, 100 \) and 200.

- Velocity profiles (along z axis)
- Velocity contours (cross-section)
- Velocity fields (cross-section)
$\text{RETARDATION RATIO}$

$Re = 532$

- $Wi = 0.8$; $L^2 = 100$; $\beta = 0.5$

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RETARDATION RATIO

$Re = 532$

- $Wi = 0.3$; $L^2 = 100$; $\beta = 0.5$
RETARDATION RATIO

$Re = 532$

- $Wi = 0.3; \quad L^2 = 100; \quad \beta = 0.25$
RETIARDATION RATIO

Re = 532

- Wi = 0.3; $L^2 = 100$; $\beta = 0.1$

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RETARDATION RATIO

Re = 1760

- $Wi = 0.3$; $L^2 = 100$; $\beta = 0.25$
RETARDATION RATIO

Re = 1760

- $Wi = 0.5; \quad L^2 = 100; \quad \beta = 0.5$
RETNADATION RATIO

\[ Re = 1760 \]

- \( Wi = 0.5; L^2 = 100 \)

\( \beta = 0.5 \)

\( \beta = 0.25 \)

\( \beta = 0.1 \)

\( \theta = 90^\circ \)

\( \theta = 120^\circ \)

\( \theta = 150^\circ \)

\( \theta = 180^\circ \)

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RETARDATION RATIO

\[ \text{Re} = 1760 \]

- \( Wi = 0.8; \ L^2 = 100 \)

\( \beta = 0.5 \)

\( \theta = 90^\circ \)

\( \theta = 120^\circ \)

\( \theta = 150^\circ \)

\( \theta = 180^\circ \)

\( \beta = 0.25 \)

\( \theta = 90^\circ \)

\( \theta = 120^\circ \)

\( \theta = 150^\circ \)

\( \theta = 180^\circ \)

\( \beta = 0.1 \)

\( \theta = 90^\circ \)

\( \theta = 120^\circ \)

\( \theta = 150^\circ \)

\( \theta = 180^\circ \)
EXTENSIBILITY

Re = 532

- $Wi = 0.8; \quad \beta = 0.5; \quad L^2 = 100$
EXTENSIBILITY

Re = 532

- Wi = 0.8; β = 0.5; L^2 = 200
EXTENSIBILITY

\( Re = 1760 \)

- \( Wi = 0.3; \beta = 0.5; L^2 = 200 \)

\[ \theta = 30^\circ \]

\[ \theta = 60^\circ \]

\[ \theta = 90^\circ \]

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**RETARDATION RATIO**

\[ Re = 1760 \]

- \( Wi = 0.5; \beta = 0.5 \)

\( L^2 = 50 \)

\( L^2 = 100 \)

\( L^2 = 200 \)

\( \theta = 90^\circ \)  \( \theta = 120^\circ \)  \( \theta = 150^\circ \)  \( \theta = 180^\circ \)

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RETRADATION RATIO

$Re = 1760$

- $Wi = 0.8; \beta = 0.5$

$L^2 = 50$

$L^2 = 100$

$L^2 = 200$

$\theta = 90^\circ$  $\theta = 120^\circ$  $\theta = 150^\circ$  $\theta = 180^\circ$

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CONCLUSIONS

- In general, inertia dominates the flow in the first half of the curve \( \theta \leq 120^\circ \) for \( Re = 532 \), and for \( Re = 1760 \) in the region of \( \theta < 90^\circ \), for all cases simulated.
- Flow development is similar when considering variation of \( \beta \) and \( L^2 \), along the channel and across the transverse section, for the same \( Re \).
- Effects of decreasing \( \beta \) and increasing \( L^2 \) are more intensely felt for higher \( Re \) and \( Wi \).
- Decreasing \( \beta \) and increasing \( L^2 \) leads to the formation of additional pair of vortices for lower \( Re \) and \( Wi \), and to the disappearance of the additional pair of vortices for higher \( Re \) and \( Wi \).

Variation of \( \beta \):
- Symmetry is observed in all cases simulated and along the entire length of the curve, for \( Re = 532 \).
- Asymmetries are observed for \( Re = 1760 \), when \( Wi = 0.5 \) and \( \beta = 0.1 \), and when \( Wi = 0.8 \) and \( \beta \leq 0.5 \)

Variation of \( L^2 \):
- Symmetry is observed in all cases simulated and along the entire length of the curve, for \( Re = 532 \).
- Asymmetries are observed for \( Re = 1760 \), when \( Wi = 0.5 \) and \( L^2 = 200 \), and when \( Wi = 0.8 \) and \( L^2 \geq 100 \).
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