Elastic Bifurcation in a Modified Cross-Slot Geometry with Four Exits

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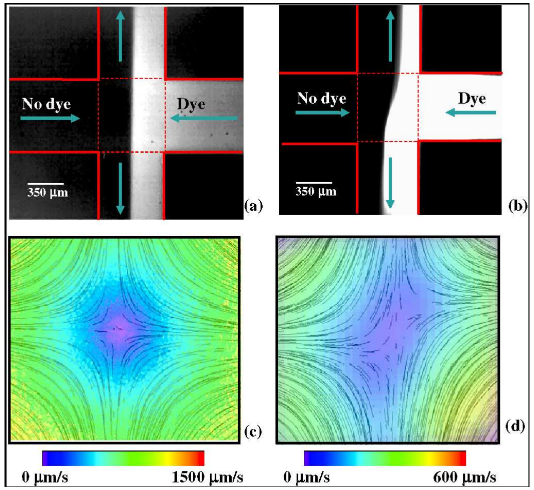
Background

Arratia et al., *Phys. Rev. Lett.* **96**, 144502 (2006)

Results: Experimental

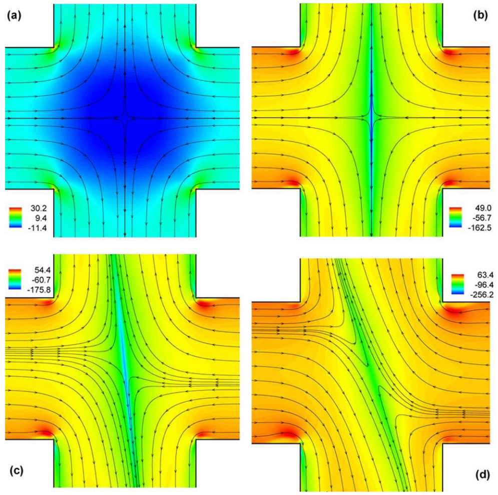


Newtonian PAA solution

$Re < 10^{-2}$ $De = 4.5$; $Re < 10^{-2}$

Poole et al., *Phys. Rev. Lett.* **99**, 164503 (2007)

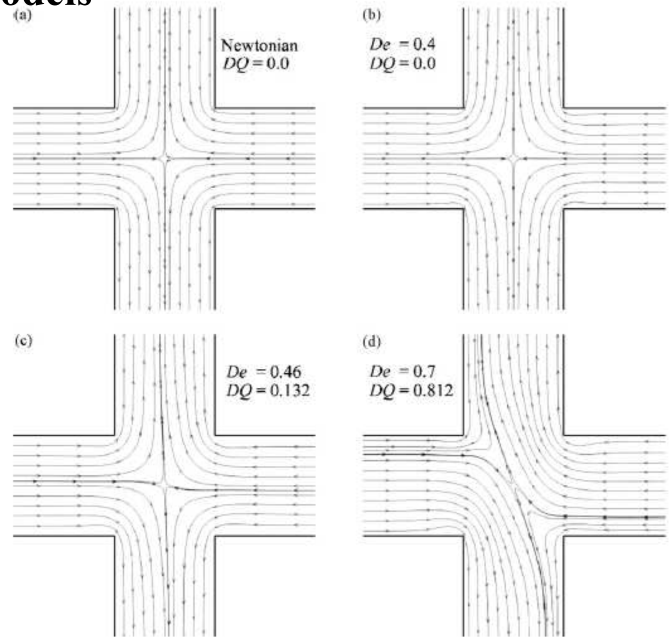
Numerical: UCM model



Streamlines superimposed onto contour lines of N_1 (τ_{xx}-τ_{yy}), for: (a) Newtonian fluid, (b) $De = 0.3$, (c) $De = 0.32$, and (d) $De = 0.4$

Rocha et al., *J. Non-Newtonian Fluid Mech.* **156**, 58-69 (2009)

Numerical: FENE-P and FENE-CR models



Streamline maps for: (a) Newtonian fluid, (b)$De = 0.40$, (c) $De = 0.46$, and (d) $De = 0.70$

(FENE-CR, $L^2 = 100$ and $\beta = 0.1$)

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General Objectives

- Investigate origin of purely elastic instabilities: consider a modified cross-slot geometry;

- Use a constant viscosity rheological model (Finite Extensibility Nonlinear Elastic model, FENE-CR): avoid complications due to shear-thinning;

- Study influence of limiting the extent of birefringence strand (very thin normal stress layer formed along outlet arms).

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Governing Equations

Assume flow is 2D, steady, incompressible, isothermal and laminar

(1) Mass: $$\frac{\partial u_i}{\partial x_i} = 0$$

(2) Momentum (creeping flow): $$-\frac{\partial p}{\partial x_i} + \frac{\partial \boxed{\tau_{ij}}}{\partial x_j} = 0$$

$$\boxed{\tau_{ij}} = \underbrace{\tau_{ij,s}}_{\text{Newtonian solvent}} + \underbrace{\tau_{ij,p}}_{\text{Polymeric solute}}$$

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(3) Constitutive equation: **Chilcott and Rallison, JNNFM (1988)**

FENE-CR Model

$$\tau_{ij}\left[1+\lambda\left(\frac{\partial\left(1/f\right)}{\partial t}+u_k\frac{\partial\left(1/f\right)}{\partial x_k}\right)\right]+\frac{\lambda}{\boxed{f}}\left(\frac{\partial \tau_{ij}}{\partial t}+u_k\frac{\partial \tau_{ij}}{\partial x_k}\right)=\eta_p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\frac{\lambda}{f}\left(\tau_{ik}\frac{\partial u_j}{\partial x_k}+\tau_{jk}\frac{\partial u_i}{\partial x_k}\right)$$

$$\boxed{f(\tau)=\frac{L^2+\lambda/\eta_p\,(\tau_{kk})}{L^2-3}}$$

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Dimensionless Parameters

Relevant dimensionless parameters to be varied in a parametric way are:

(1) Extensibility parameter of FENE-CR model:

$$L^2 = 50,\ 100 \text{ and } 200$$

(2) Deborah number: $De = {}^{\lambda U}\!/_{d}$ $\quad (0.0 \rightarrow 1.0)$

(3) Solvent viscosity ratio: $\beta = {}^{\eta_s}\!/_{\eta_0}$ $\quad (\beta = 0.1)$

where the total shear viscosity is $\eta_0 = \eta_s + \eta_p$ (constant)

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Geometry and boundary conditions

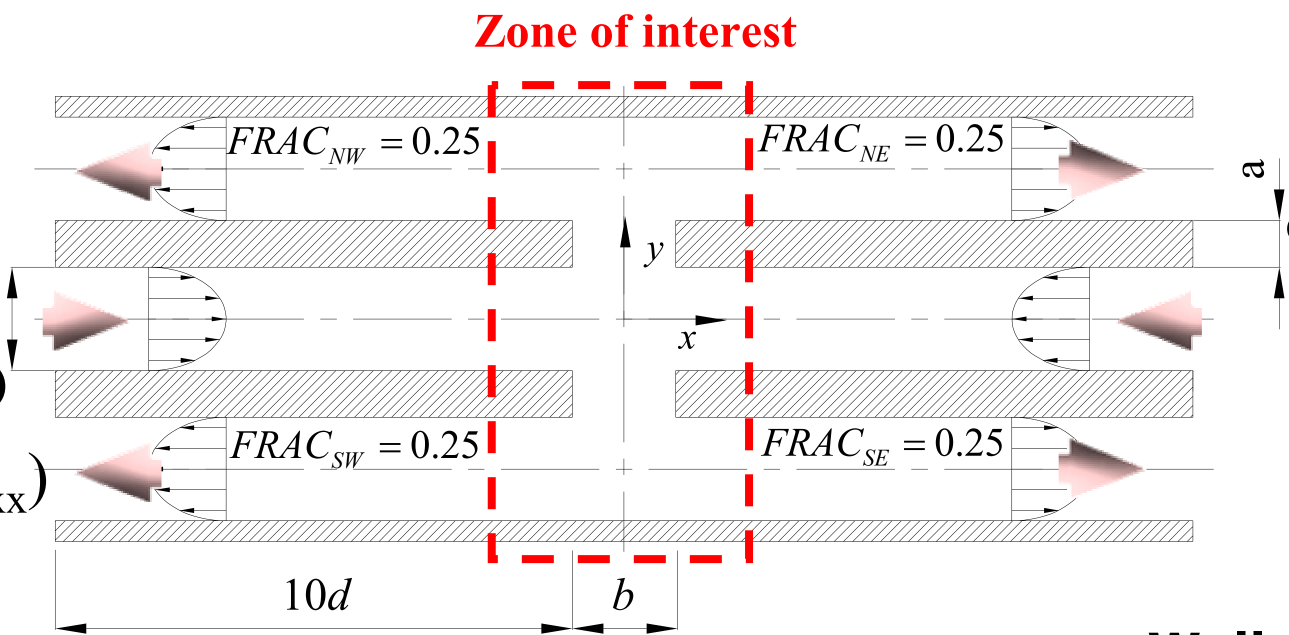
Inlet conditions

Fully developed profile (steady flow)

$(u(y);\ \tau_{xy};\ \tau_{xx})$

Outlet conditions

Neumann conditions



Adimensional geometric parameters

Thickness	Gap width
$\Lambda = a/d = 0.1$	$\Theta = b/d = 0.5,\ 1.0$ and 1.5

Wall conditions

No slip conditions: $u = v = 0$

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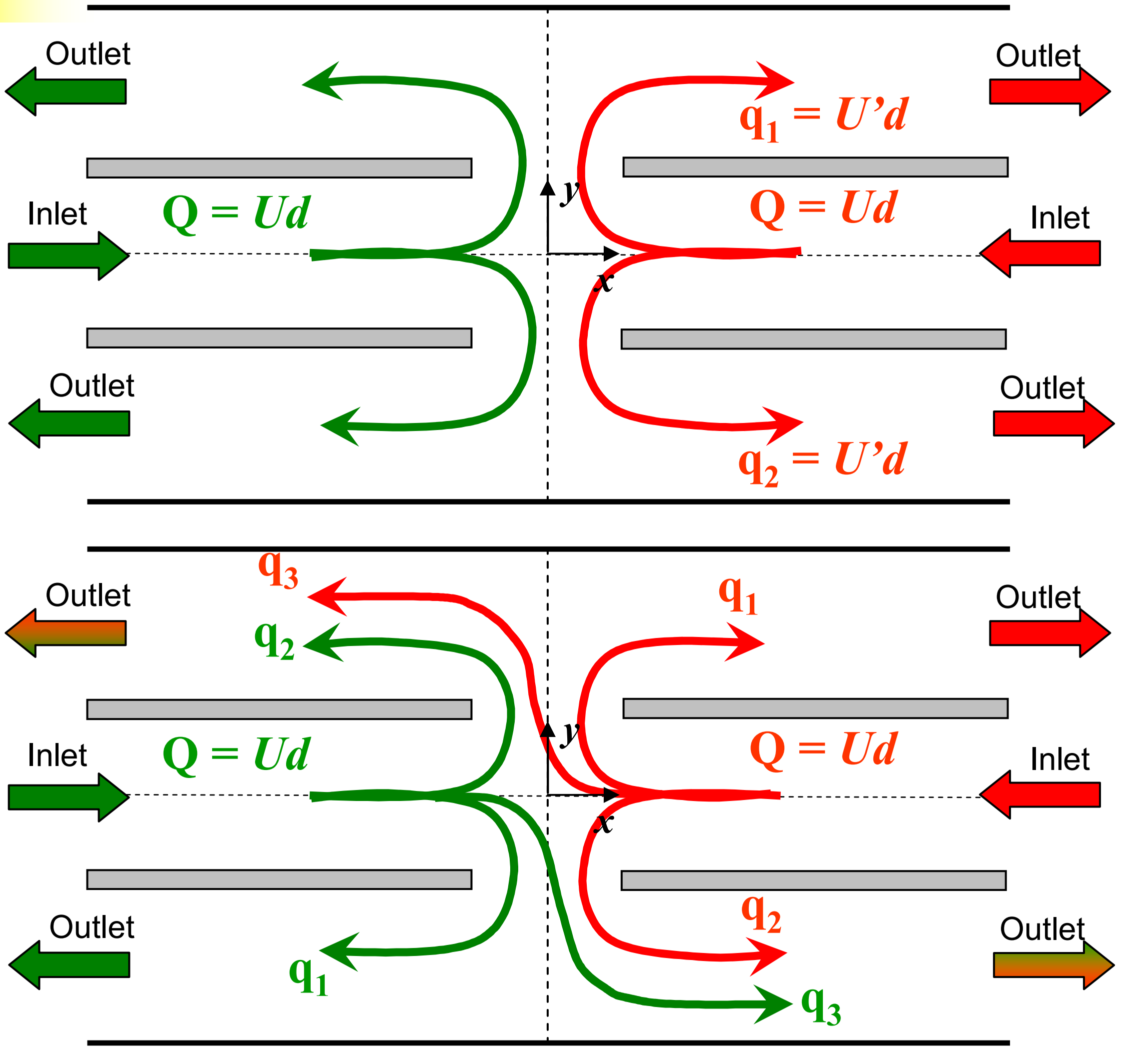
Geometry and boundary conditions



Symmetric flow

Total flow rate

$$Q = q_1 + q_2$$

$\mathbf{q_1}$ and $\mathbf{q_2}$ – flow rate divided into either the upper outflow arm and the lower outflow arm.

***U* – Mean velocity**

$$U' = 0.5U$$

Asymmetric flow

Total flow rate

$$Q = q_1 + q_2 + q_3$$

$$q_1 = q_2 + q_3$$

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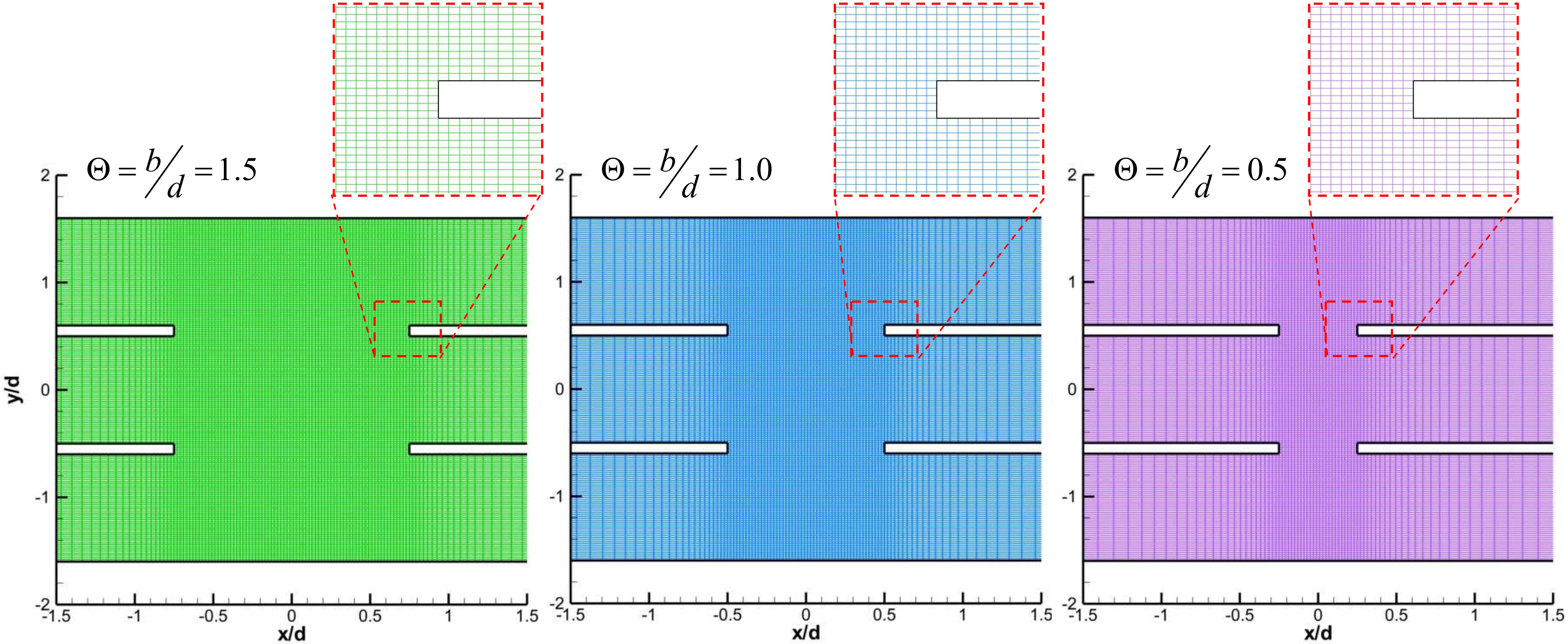
Numerical Method

- **Finite-volume method** (Oliveira *et al.*, JNNFM (1998));
  - Non-uniform orthogonal mesh: $\Delta x_{\min} = \Delta y_{\min} \approx 0.02d$



Number of control volumes **27525** **23613** **19375**

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Mesh refinement (accuracy)

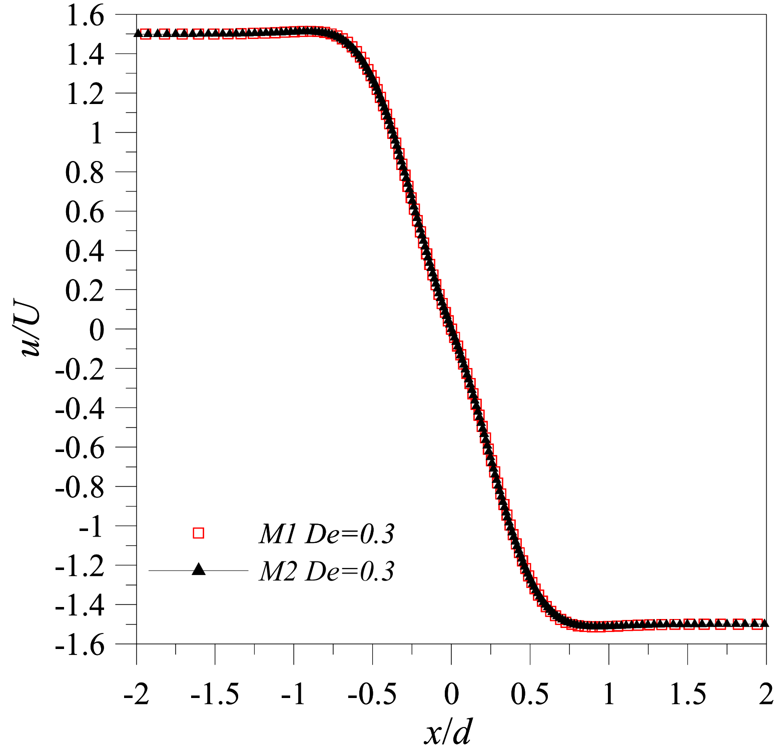
Velocity and normal stress profiles along the central line ($y = 0$)

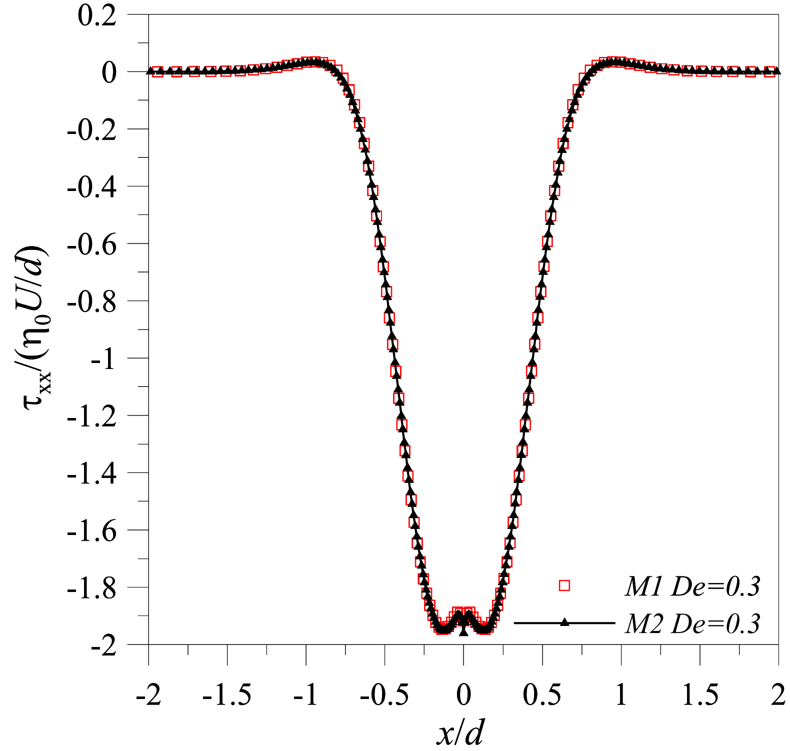
$b/d = 1.0$

$De = 0.30$

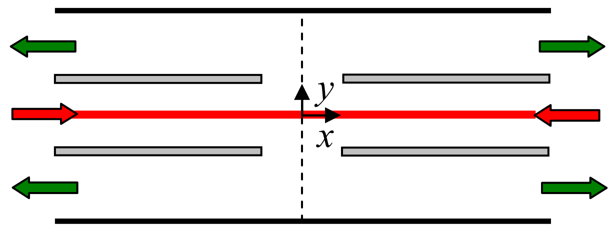
$L^2 = 100$

$\beta = 0.10$





	Control volumes	Size of smallest cell
Mesh 1	23613	$\Delta x_{\min} = \Delta y_{\min} \approx 0.02d$
Mesh 2	93223	$\Delta x_{\min} = \Delta y_{\min} \approx 0.01d$



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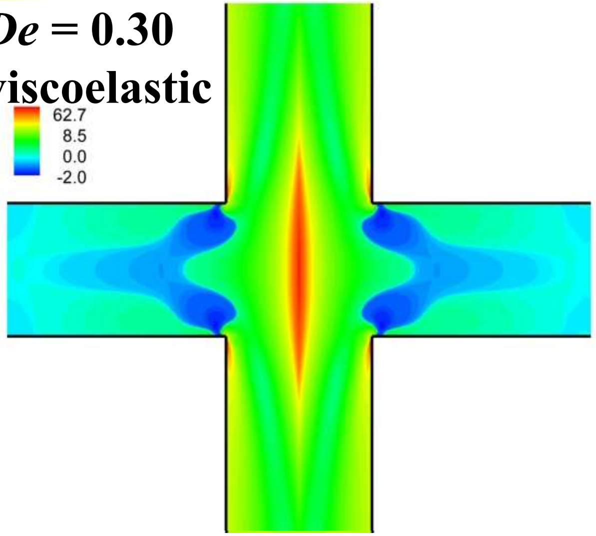
Contour plots of $\tau_{yy}/(\eta_0 U/d)$

$L^2 = 100$ and $\beta = 0.1$

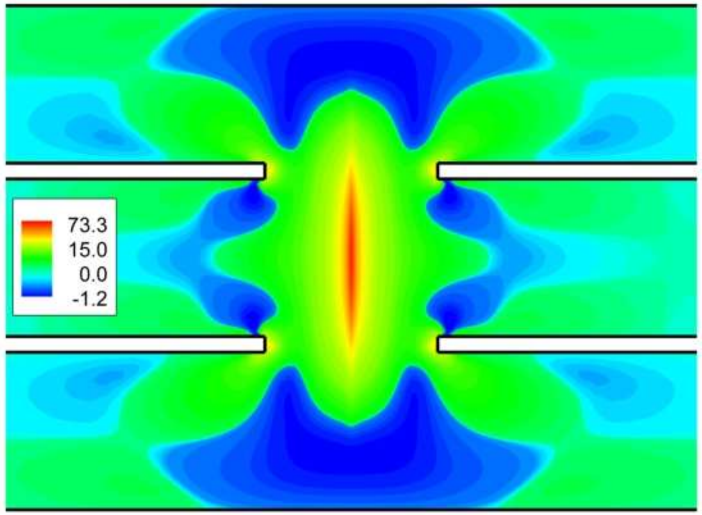
Standard Cross-slot

De = 0.30
viscoelastic

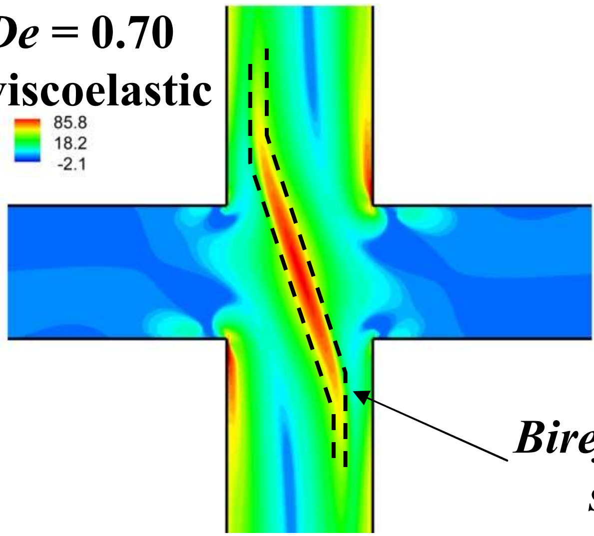


De = 0.0
Newtonian

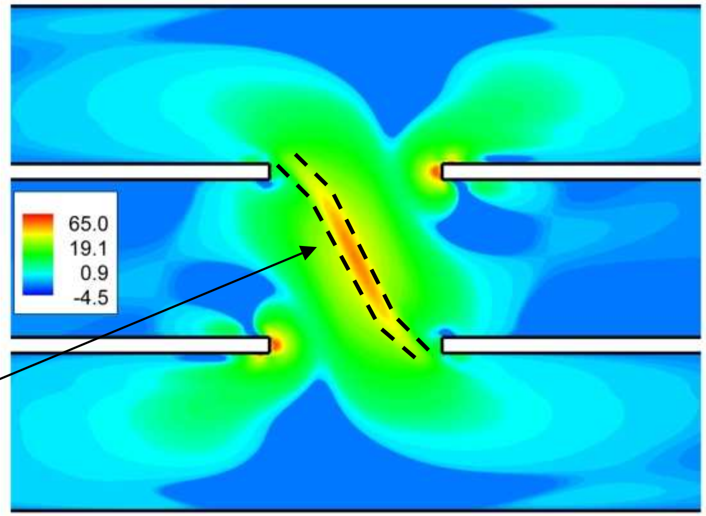
De = 0.30
viscoelastic



De = 0.70
viscoelastic



De = 0.30
viscoelastic



Birefringence strand

Modified 4-exits configuration

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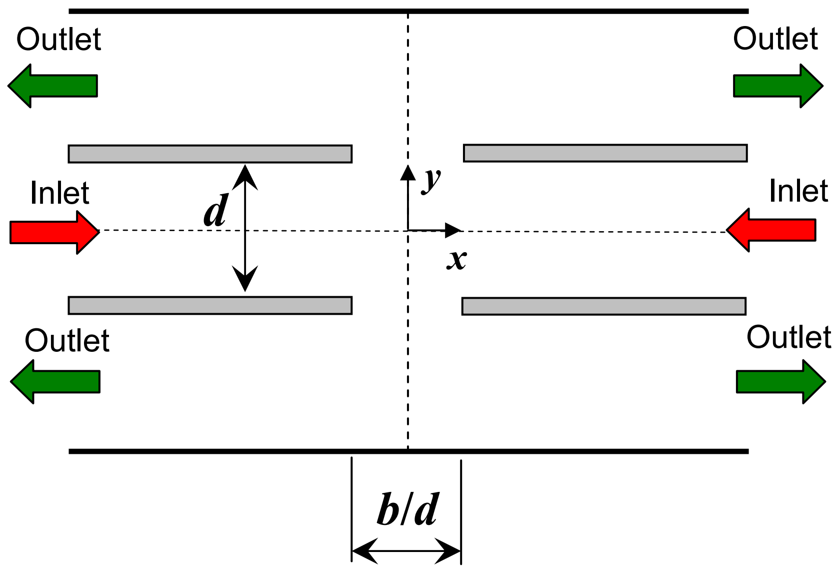
Streamlines (Flow patterns)

$L^2 = 100$ and $\beta = 0.1$

$b/d = 0.5$

$b/d = 1.0$

$b/d = 1.5$



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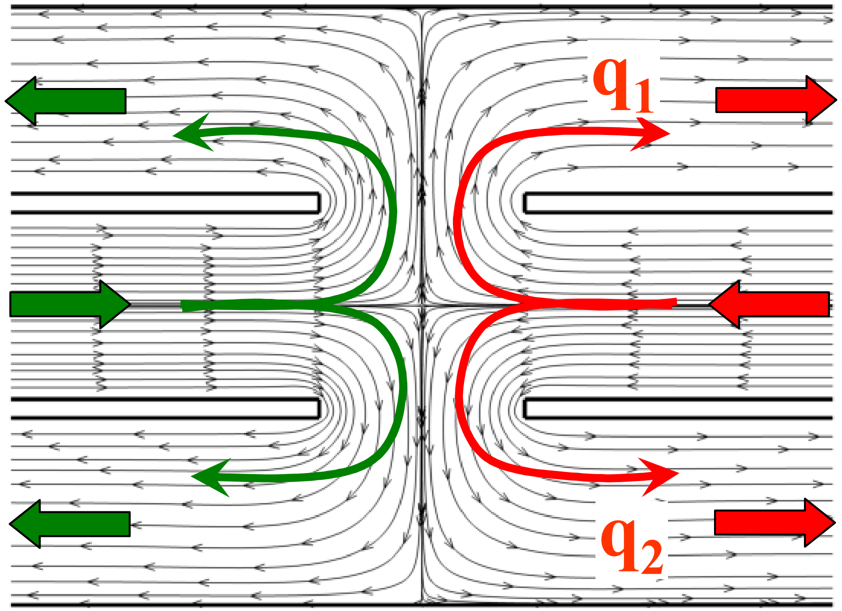
Quantifying the degree of asymmetry

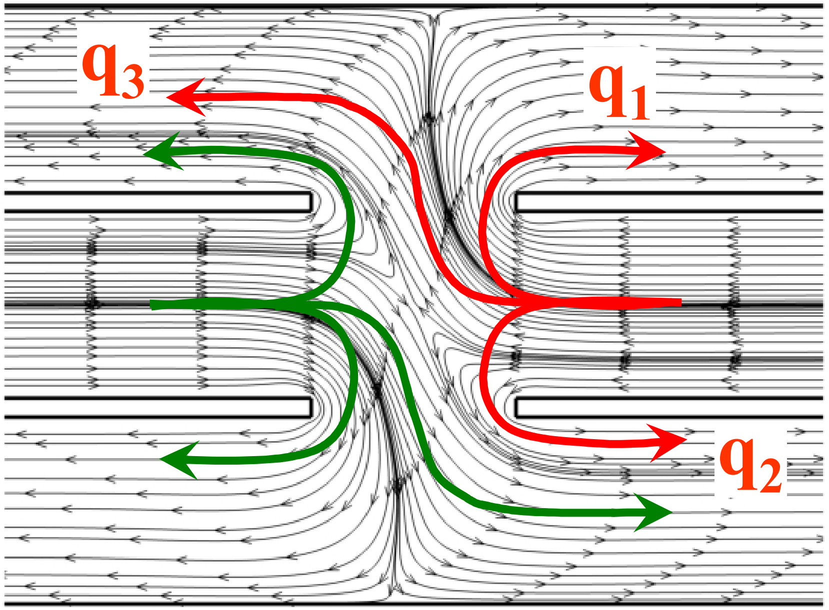
$L^2 = 100$, $\beta = 0.1$ and $b/d = 1.0$

$$DQ = \frac{q_{up} - q_{down}}{q_{in}} = \frac{(q_1 + q_3) - q_2}{Q}$$



Symmetric flow:

$DQ = 0 \quad (q_1 = q_2)$



Asymmetric flow:

$DQ \neq 0 \quad (q_1 \neq q_2)$

(completely asymmetric flow $DQ = \pm 1.0$)

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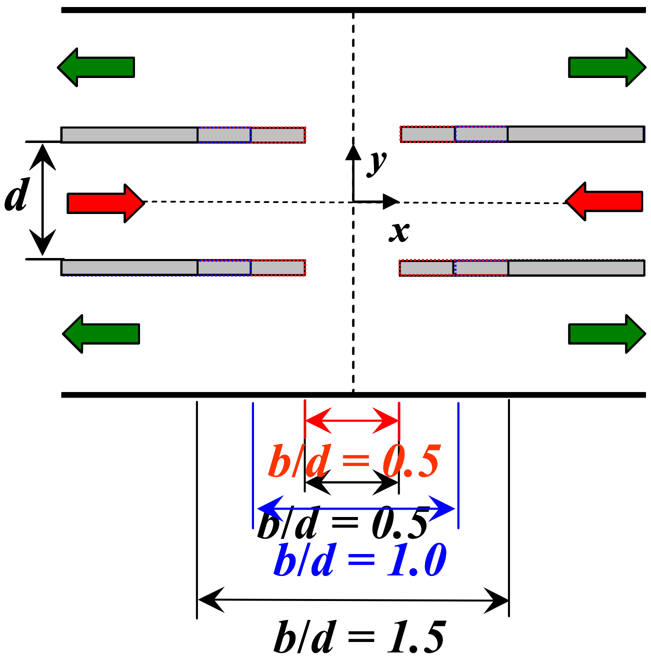
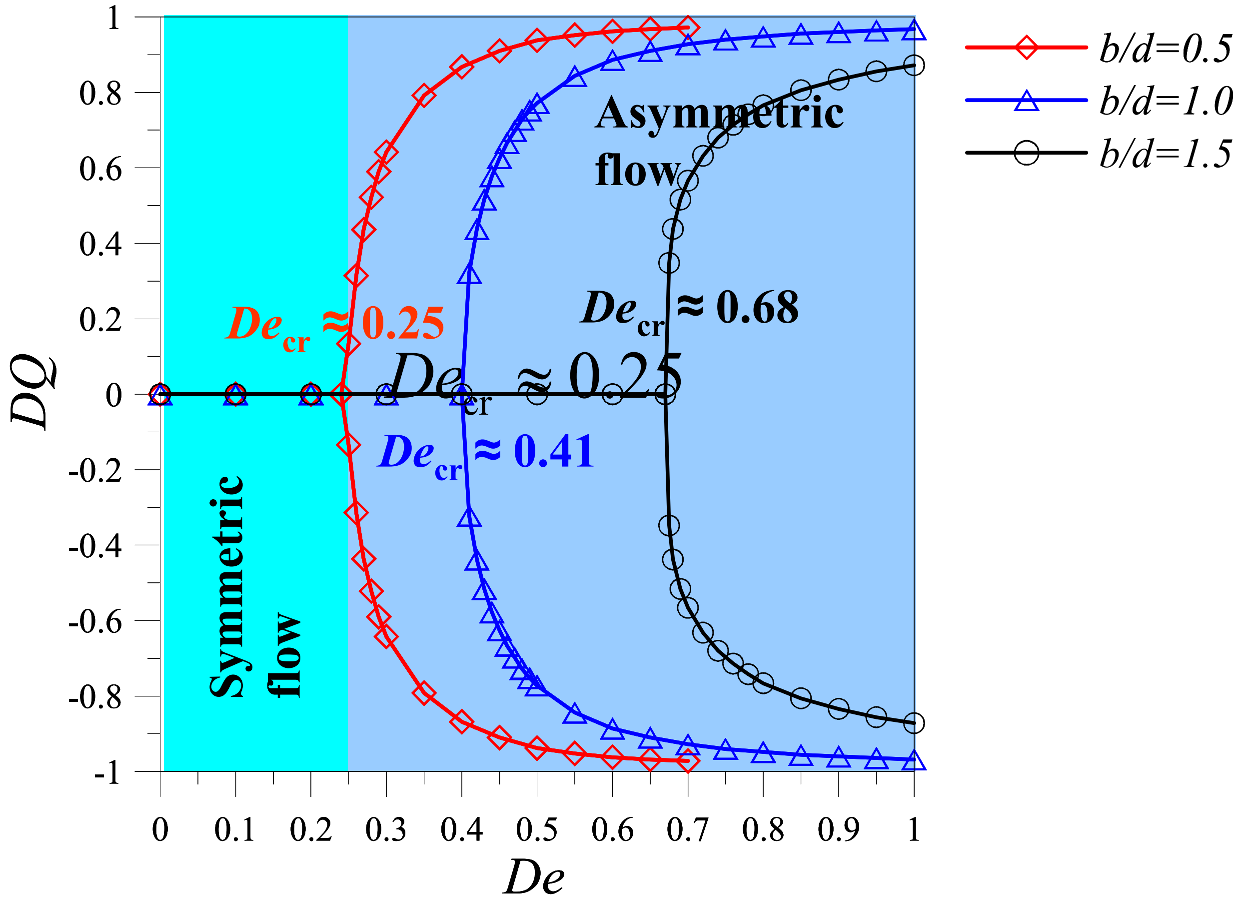
Influence of elasticity (*De*)

Variation of gap width ***b*/*d***



$L^2 = 100$ and $\beta = 0.1$



The critical Deborah number (De_{cr}) defines the transition point from symmetric to asymmetric flow.

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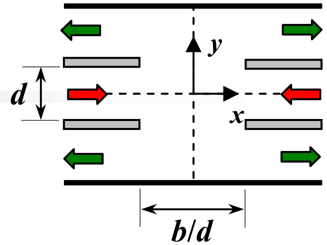
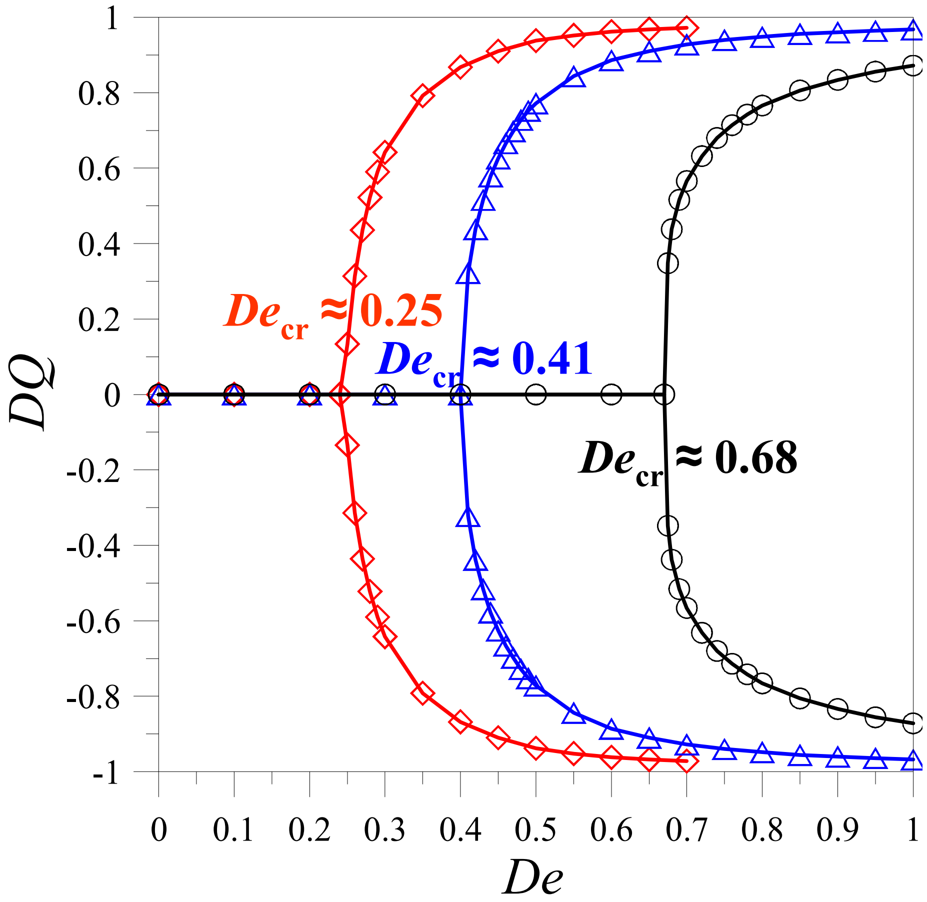
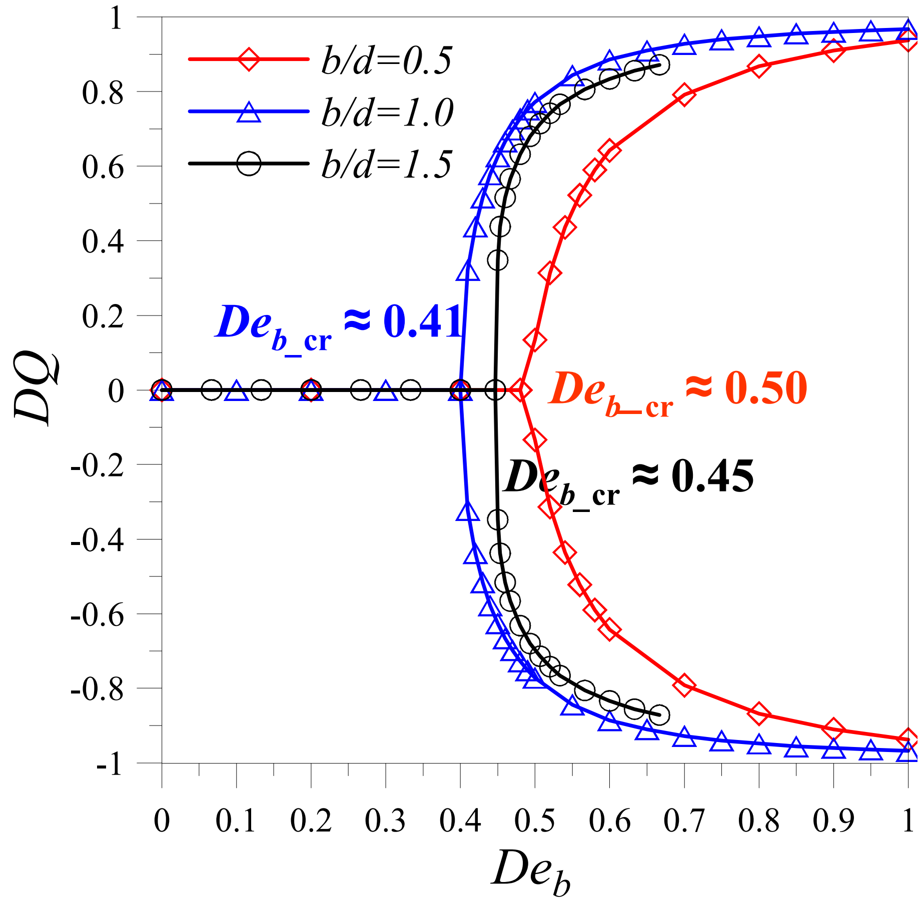
Influence of elasticity (*De*)



$L^2 = 100$ and $\beta = 0.1$

$$\begin{cases} De = \lambda U / d \\ De_b = \lambda U / b \end{cases} \Rightarrow De_b = \frac{De}{(b/d)}$$





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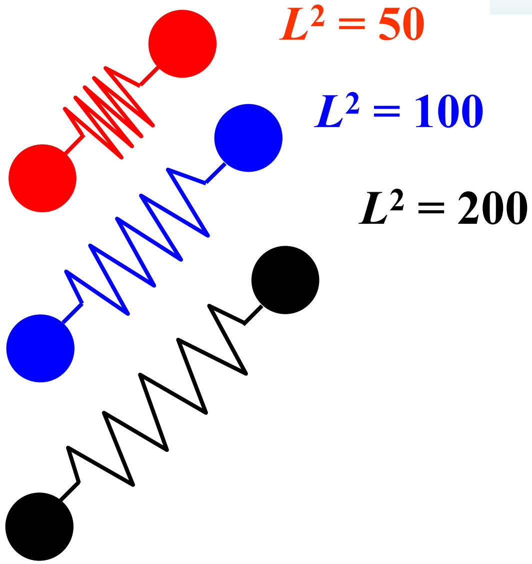
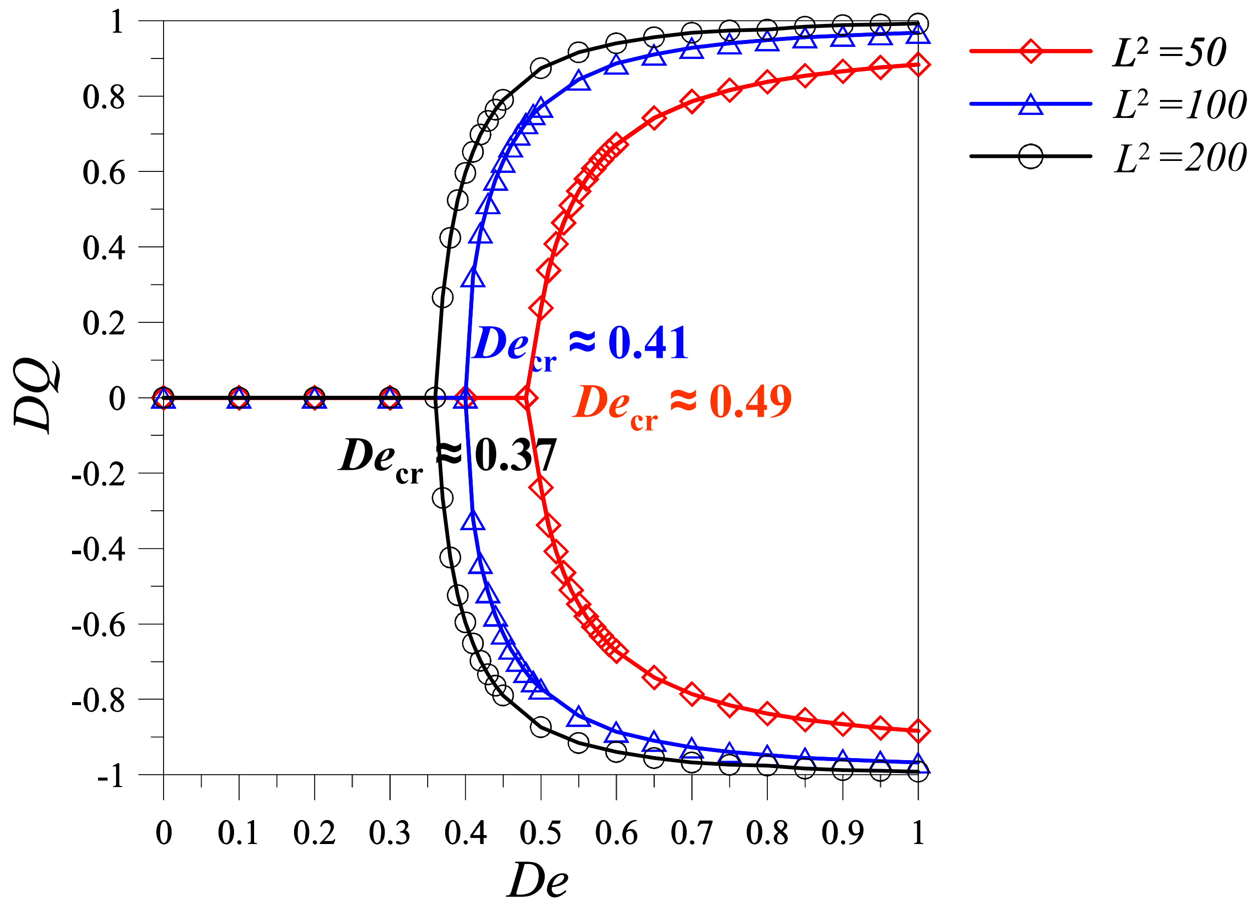
Influence of extensibility (L^2)

Dumbbell model



$L^2 = 100$, $\beta = 0.1$ and $b/d = 1.0$



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Conclusions

- **Elasticity** directly drives the instability: for $\boldsymbol{De} < \boldsymbol{De}_{\mathbf{cr}}$ the flow is **symmetric and steady**, while for $\boldsymbol{De} \geq \boldsymbol{De}_{\mathbf{cr}}$ it becomes **asymmetric**, remaining steady till $De = 1.0$.

- The **Extensibility parameter** (L^2), which controls the extensional viscosity, directly influences the asymmetry: De_{cr} decreases with increasing L^2.

- By increasing the gap, the asymmetry is delayed.

- The triggering mechanism for the bifurcation is not linked to the size of birefringence strand: when limiting its extent, the instability remains.

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