AERC 2009
5th Annual European Rheology Conference
April 15-17, Cardiff - United Kingdom

Numerical Study of a Laminar Pulsating Flow Through a Cross-slot Channel

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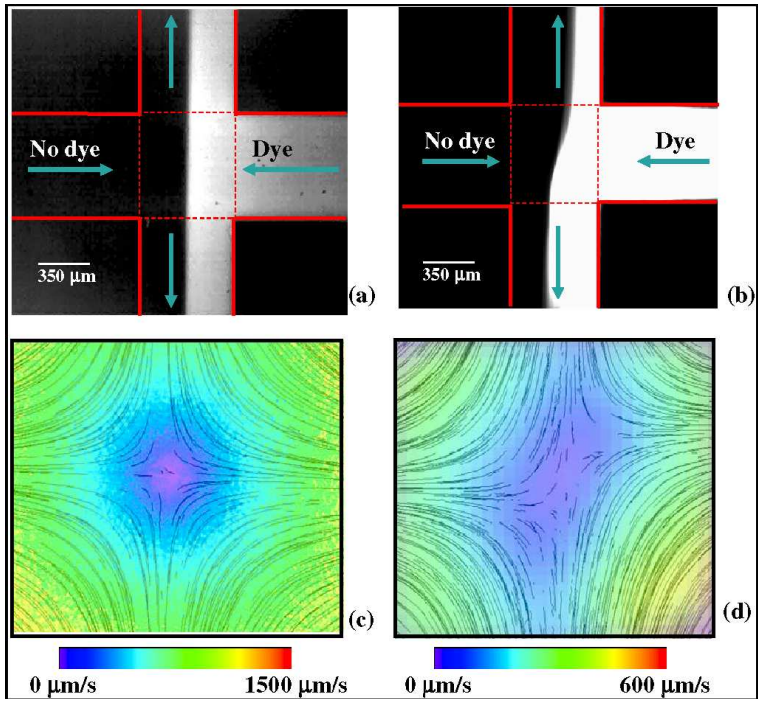
Background: Steady flow

Arratia et al., *Phys. Rev. Lett.* **96**, 144502 (2006)

Results: Experimental

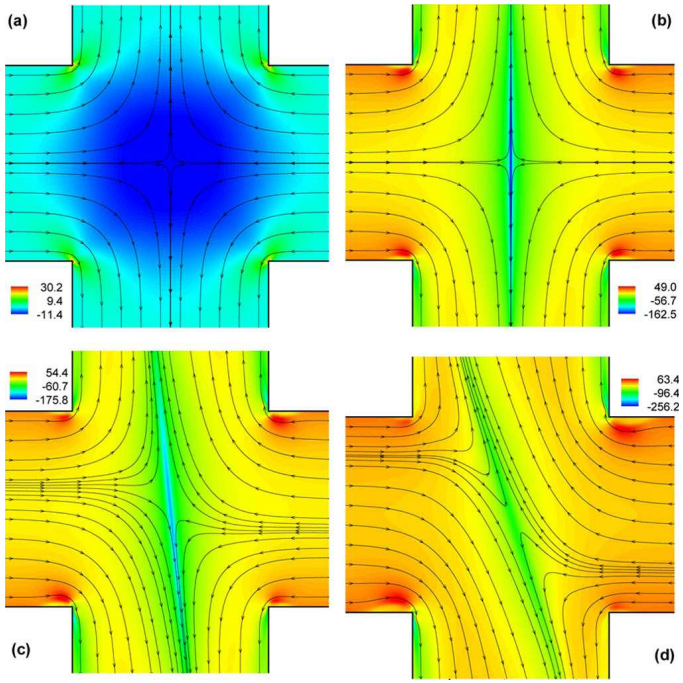


Newtonian $Re < 10^{-2}$

PAA solution $De = 4.5$; $Re < 10^{-2}$

Poole et al., *Phys. Rev. Lett.* **99**, 164503 (2007)

Numerical: UCM model



Streamlines superimposed onto contour lines of N_1 (τ_{xx}-τ_{yy}), for: (a) Newtonian fluid, (b) $De = 0.3$, (c) $De = 0.32$, and (d) $De = 0.4$

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Motivation: Pulsating flow

Phelan et al., *Phys. of Fluids* **20**, 023101 (2008)

Numerical: Navier-Stokes equations

Newtonian fluid

Chaotic mixing in CFM: cross-flow mixer

Generating chaotic flow by temporal variation and "crossing streamlines" principle (Ottino)

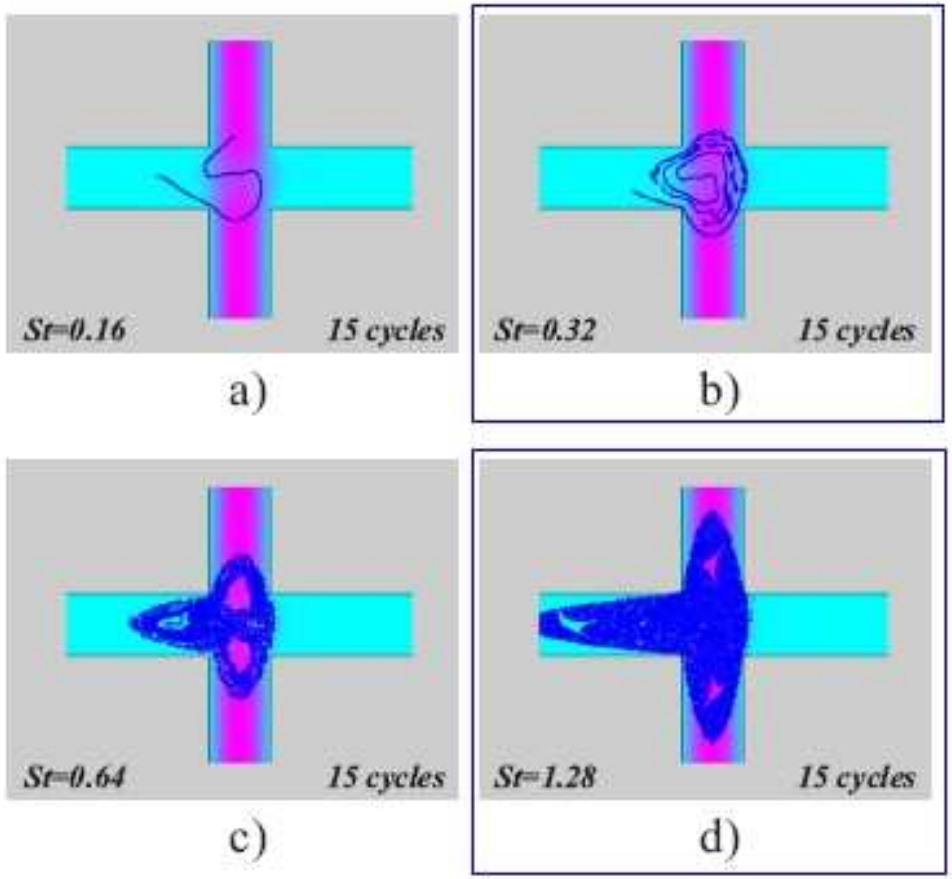

FIG. 2. (Color online) Stretching of a material line in the CFM as a function of Strouhal number after 15 cycles for the initial condition shown in Fig. 1(b). (a) St=0.16, (b) St=0.32, (c) St=0.64, (d) St=1.28 (enhanced online).

General Objectives:

- Continue numerical study of elastic instabilities
- Cross-slot at Re=0, see how elasticity affects mixing capability. Will chaotic mixing play a role?
- How will the degree of asymmetry (bifurcation) vary with pulsation?

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Governing Equations:

Assume flow is two-dimensional, incompressible, isothermal and laminar

(1) Equation of motion.

(2) Rheological equation of state: FENE-CR model.

Solution method: Finite Volume Method

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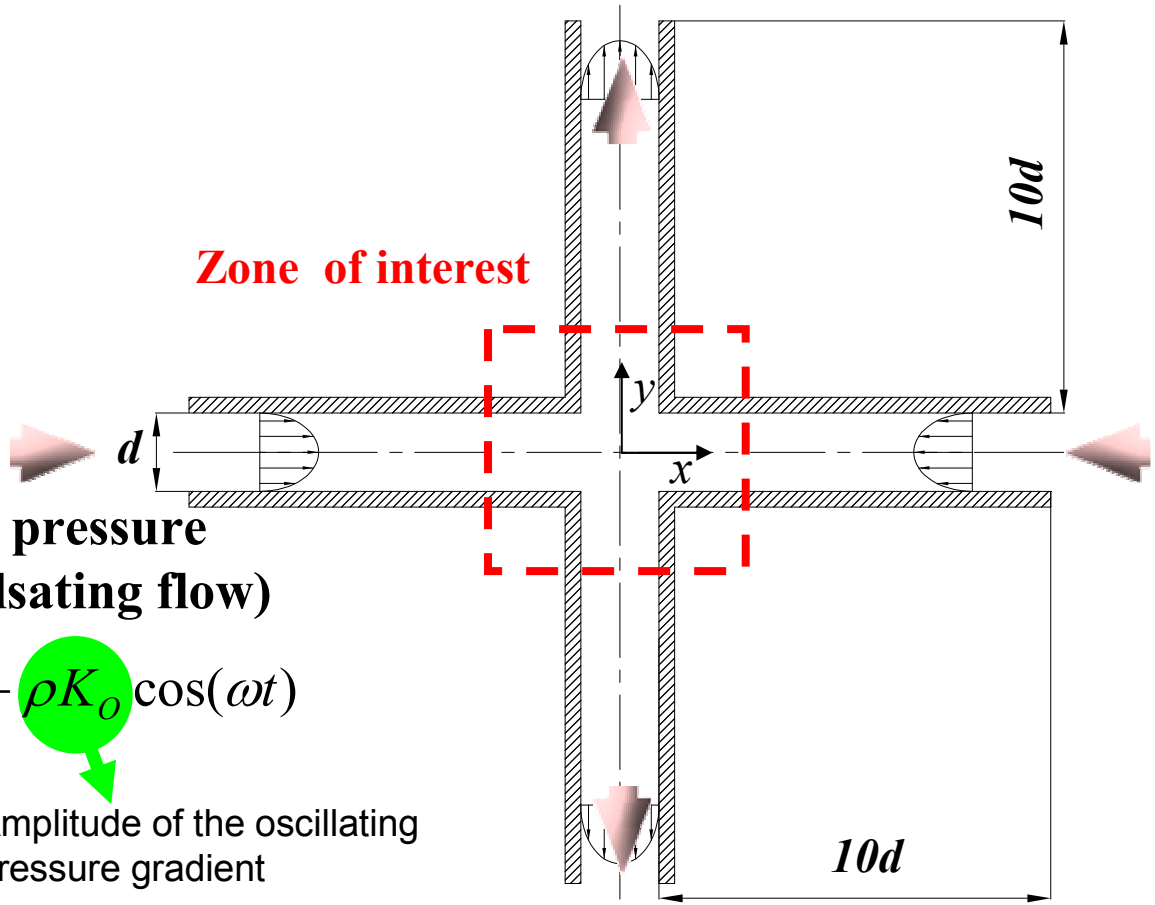
Geometry and inlet conditions:



Sinusoidal pressure gradient (pulsating flow)

$$-\frac{dp}{dx} = \rho K_s + \rho K_O \cos(\omega t)$$

Amplitude of the oscillating pressure gradient

Amplitude of the steady pressure gradient

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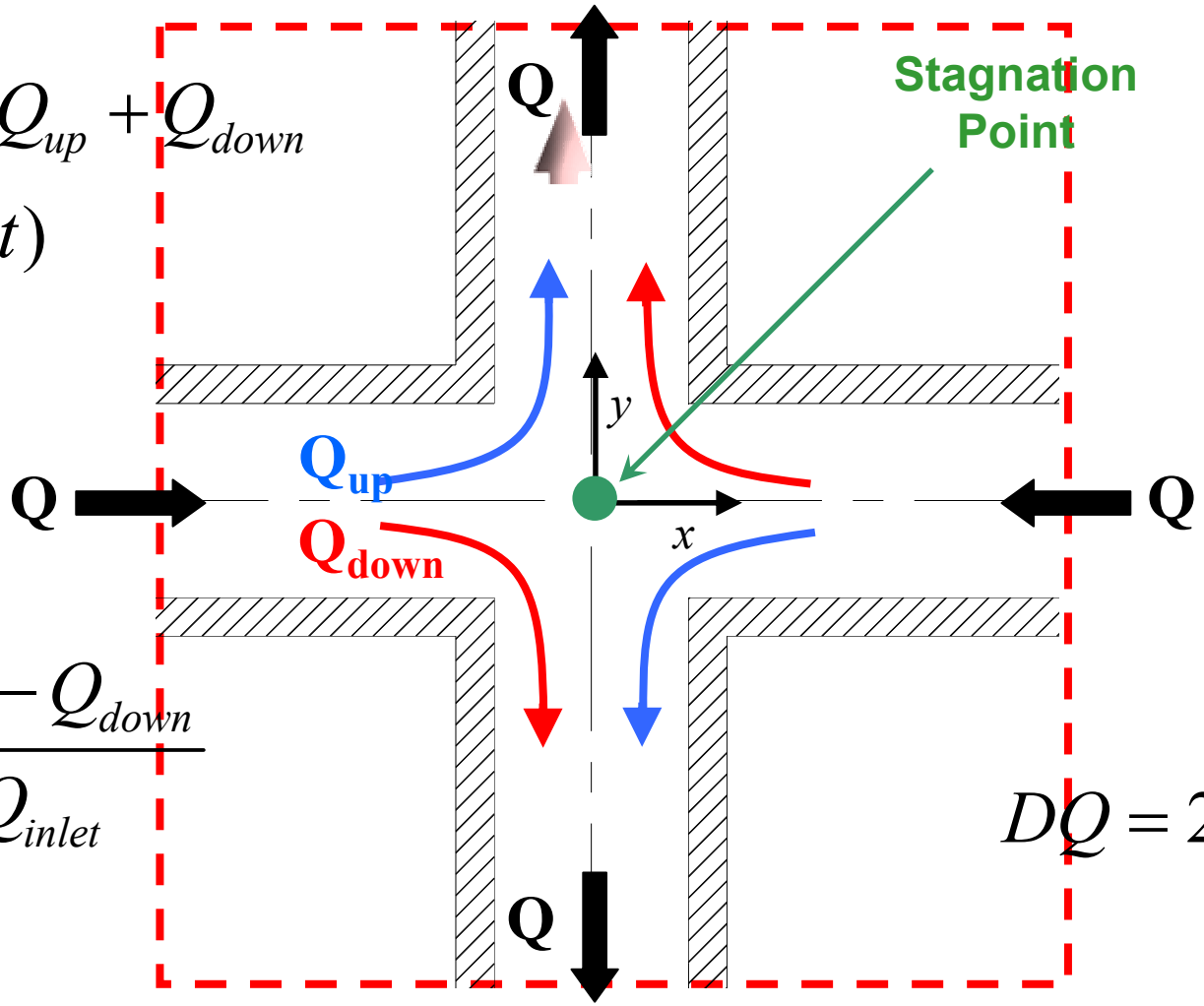
Geometry and boundary conditions:

$$Q = Ud = Q_{up} + Q_{down}$$

$$Q_{inlet} = Q(t)$$



$$\psi_c$$

$$DQ = \frac{Q_{up} - Q_{down}}{Q_{inlet}}$$

$$DQ = 2(\psi_c - 0.5)$$

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Dimensionless Parameters:

Increased number of independent parameters to be varied:

(1) Deborah number: $De = \lambda U / d$

(2) Imposed frequency, Womersely number

$\alpha = \frac{1}{2} d / \sqrt{\eta_0 / \rho\omega}$ or Strouhal number: $St = UT / d$

(3) Oscillating-to-steady pressure gradient:

K_o / K_s

$\beta = \eta_s / \eta_0 = 0.1$ $L^2 = 100$

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Steady Flow: Influence of elasticity (*De*)

Quantifying the degree of asymmetry

$Re = 0$; $L^2 = 100$ and $\beta = 0.1$

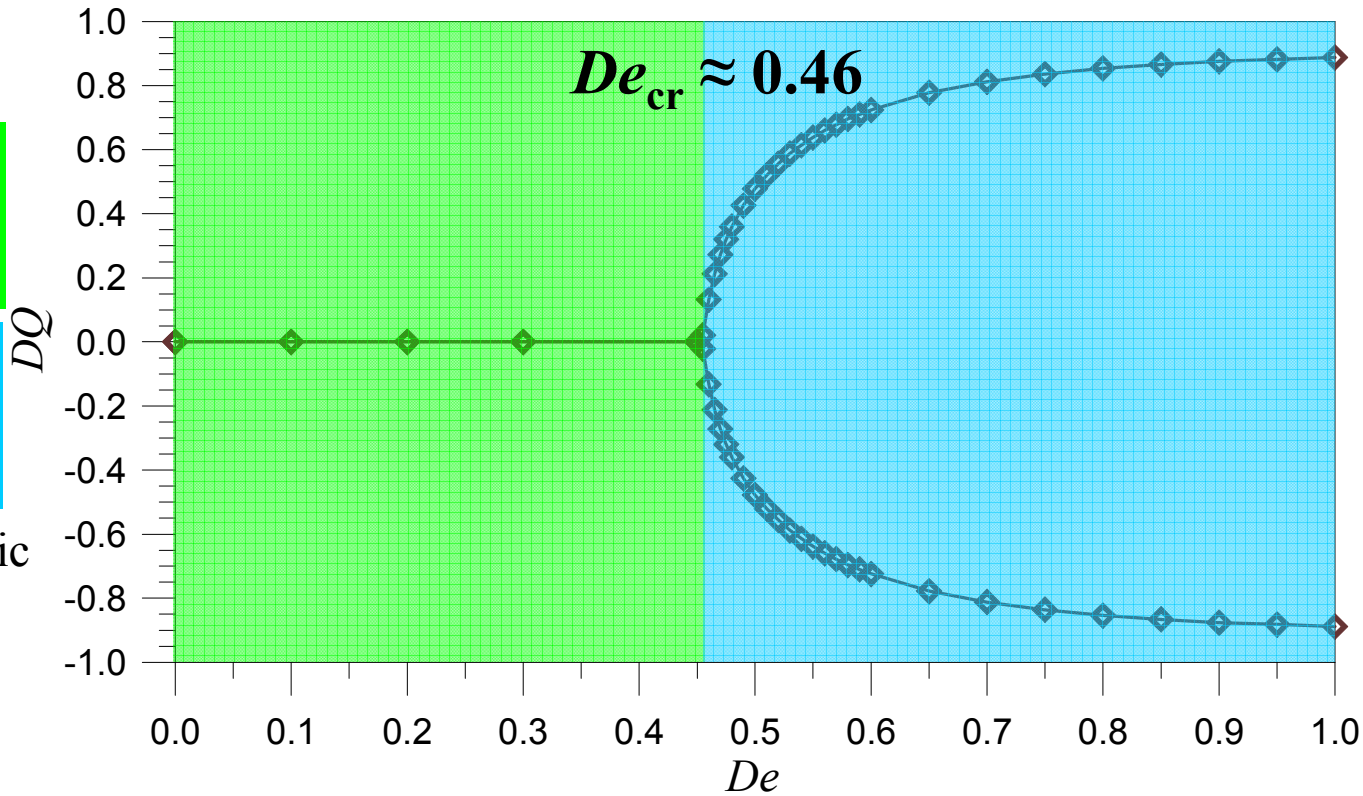
Symmetric flow:

$DQ = 0 \quad (Q_1 = Q_2)$

Asymmetric flow:

$DQ \neq 0 \quad (Q_1 \neq Q_2)$

(for a completely asymmetric flow $DQ = \pm 1$)



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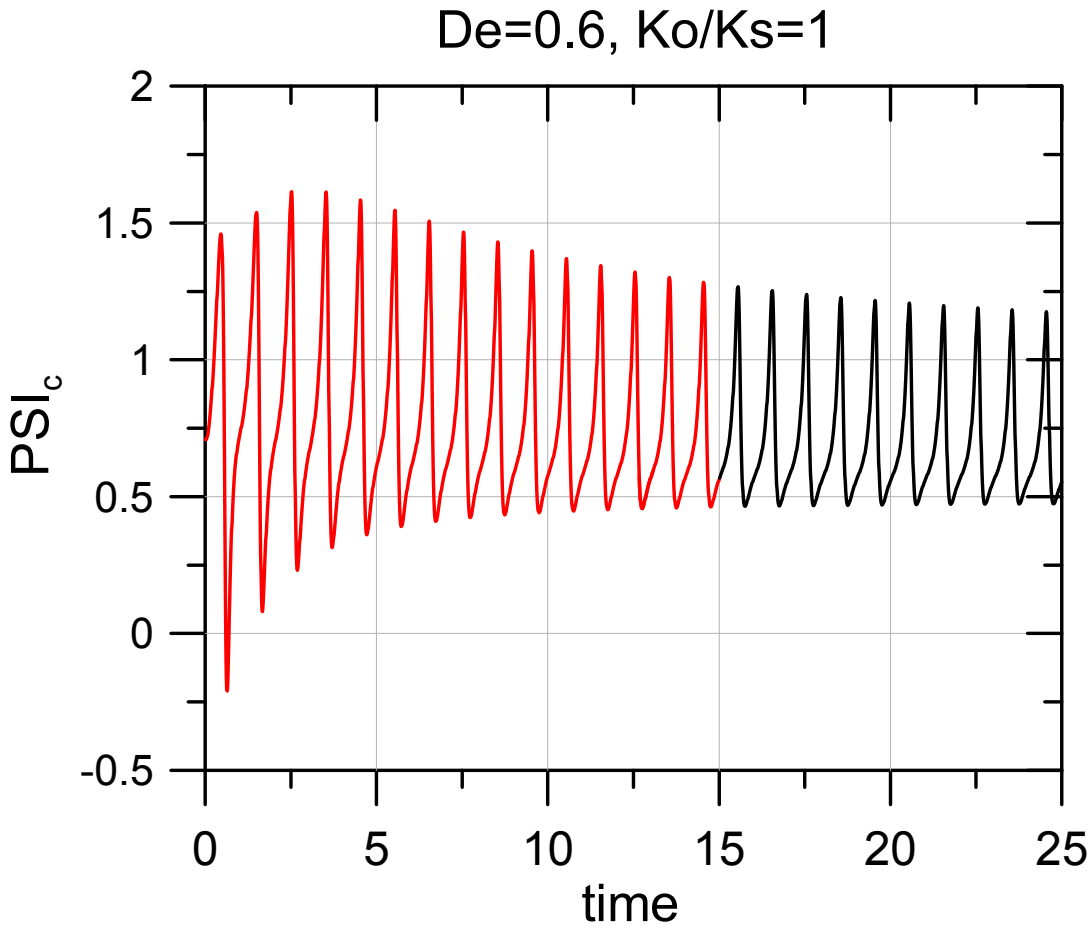
Pulsating flow:

Stream function at cross-slot center



Newt. $\psi_c = 0.5$

FENECR.

$\psi_c = 0.86$

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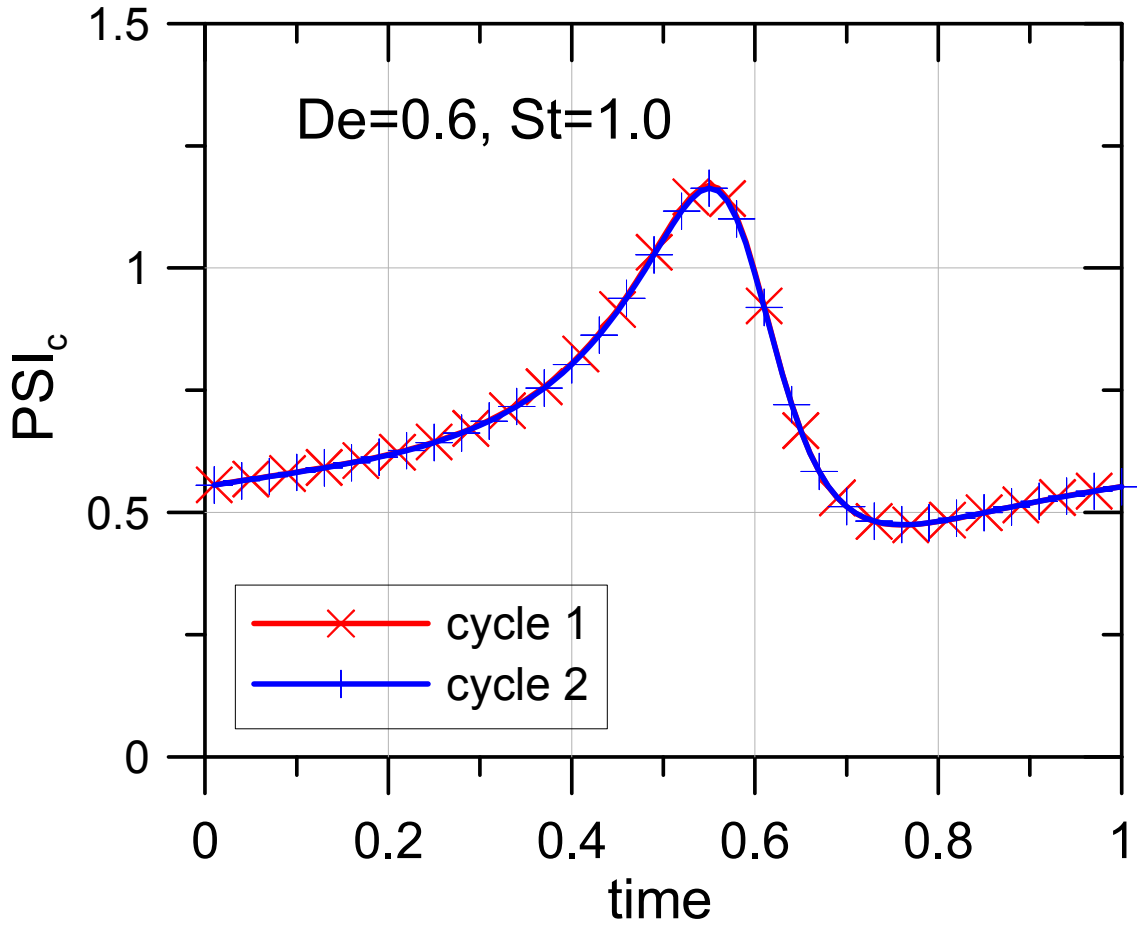
Pulsating flow (numerical aspects)

Repeatability between cycles



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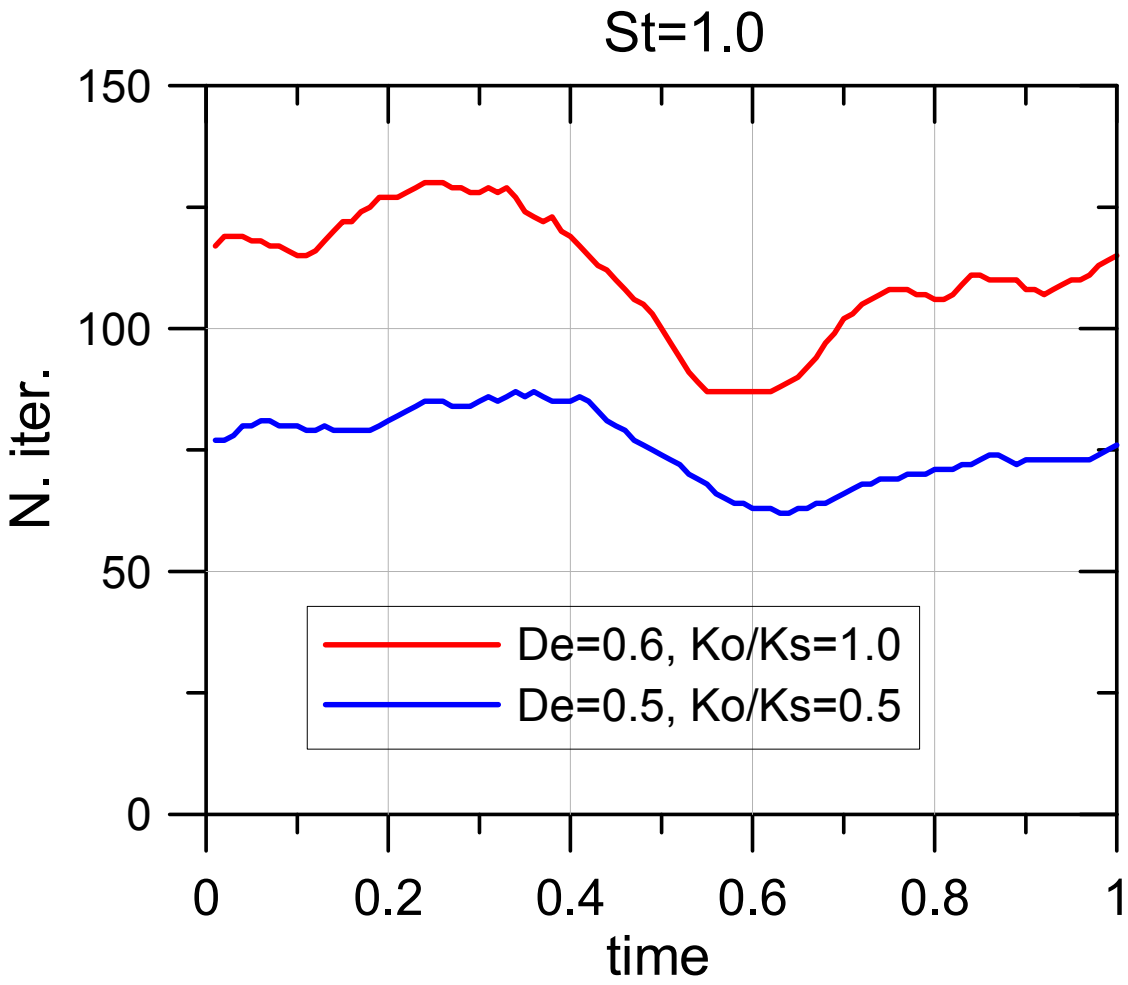
Pulsating flow (numerical aspects)

Number of inner iterations within each time step; dt=0.01



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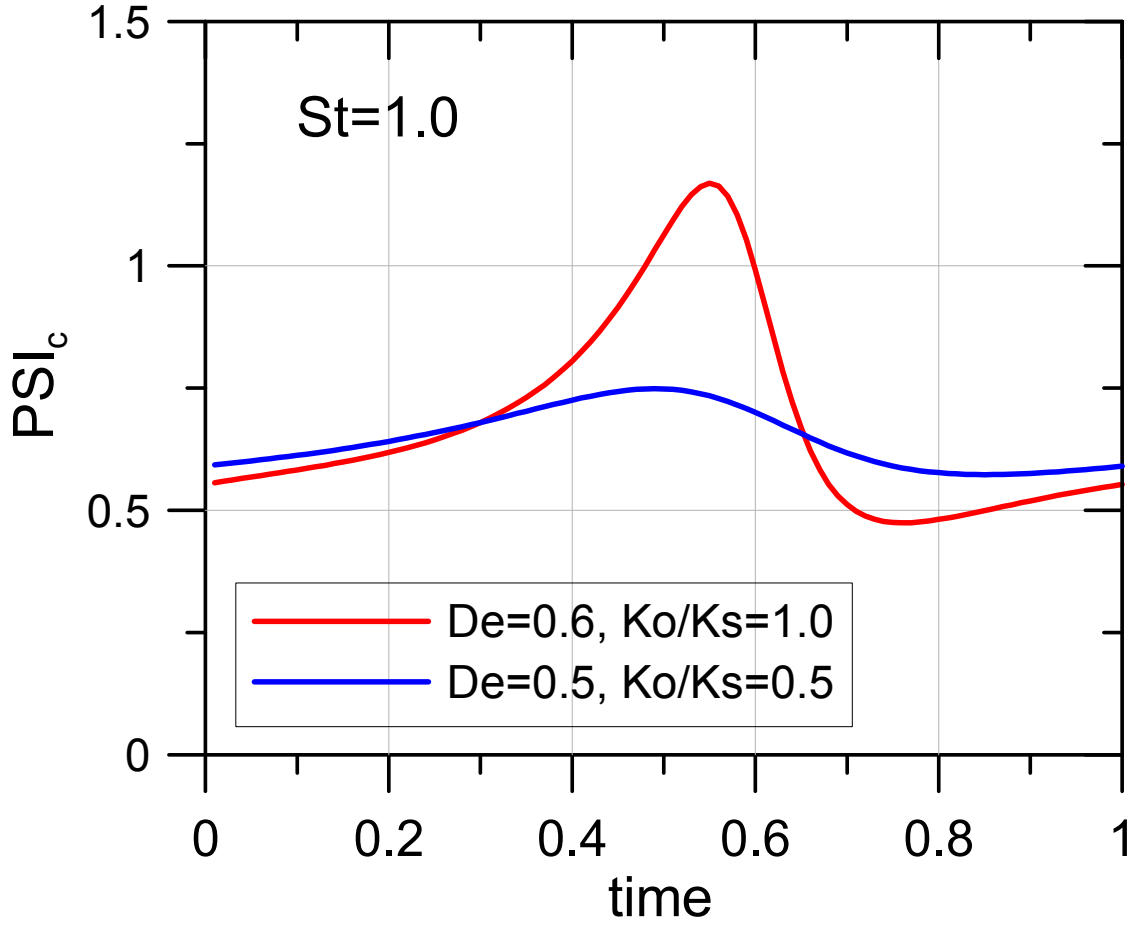
Pulsating flow:

Variation of centre streamfuntction in a period: 2 pressure gradient ratios



$$\psi_c = 0.86$$

$$DQ = 0.724$$

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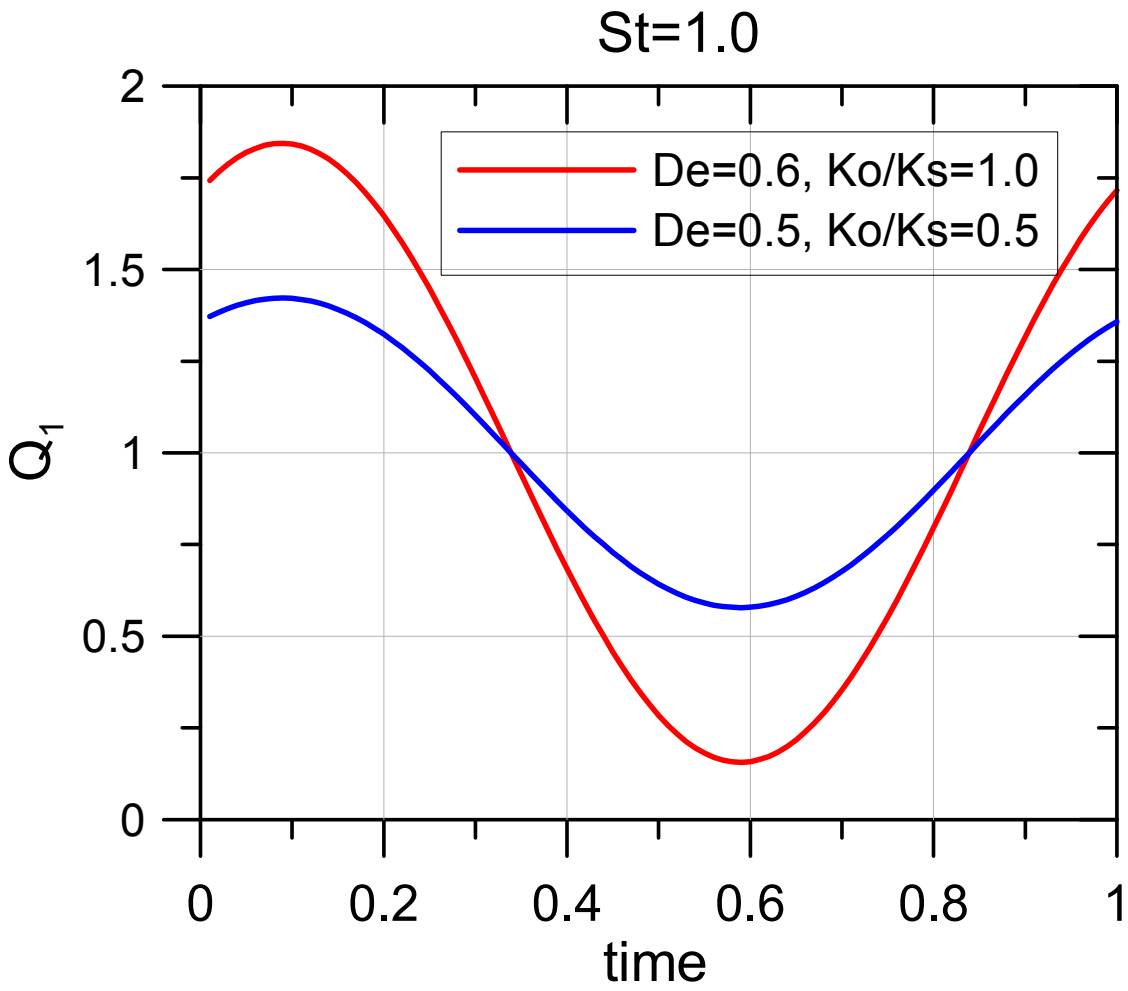
Pulsating flow:

Variation of inlet flow rate in a period



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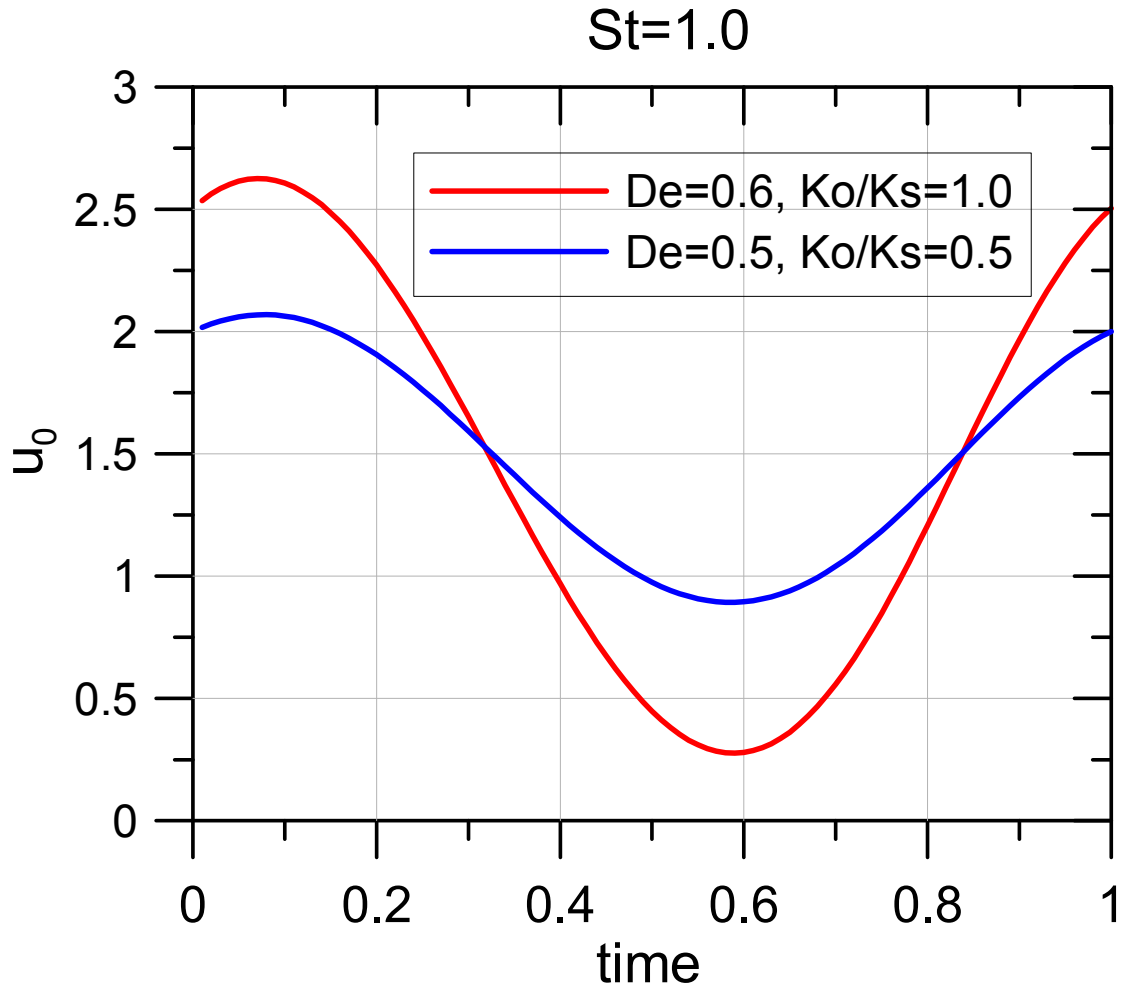
Pulsating flow:

Variation of maximum velocity at inlet during 1 period



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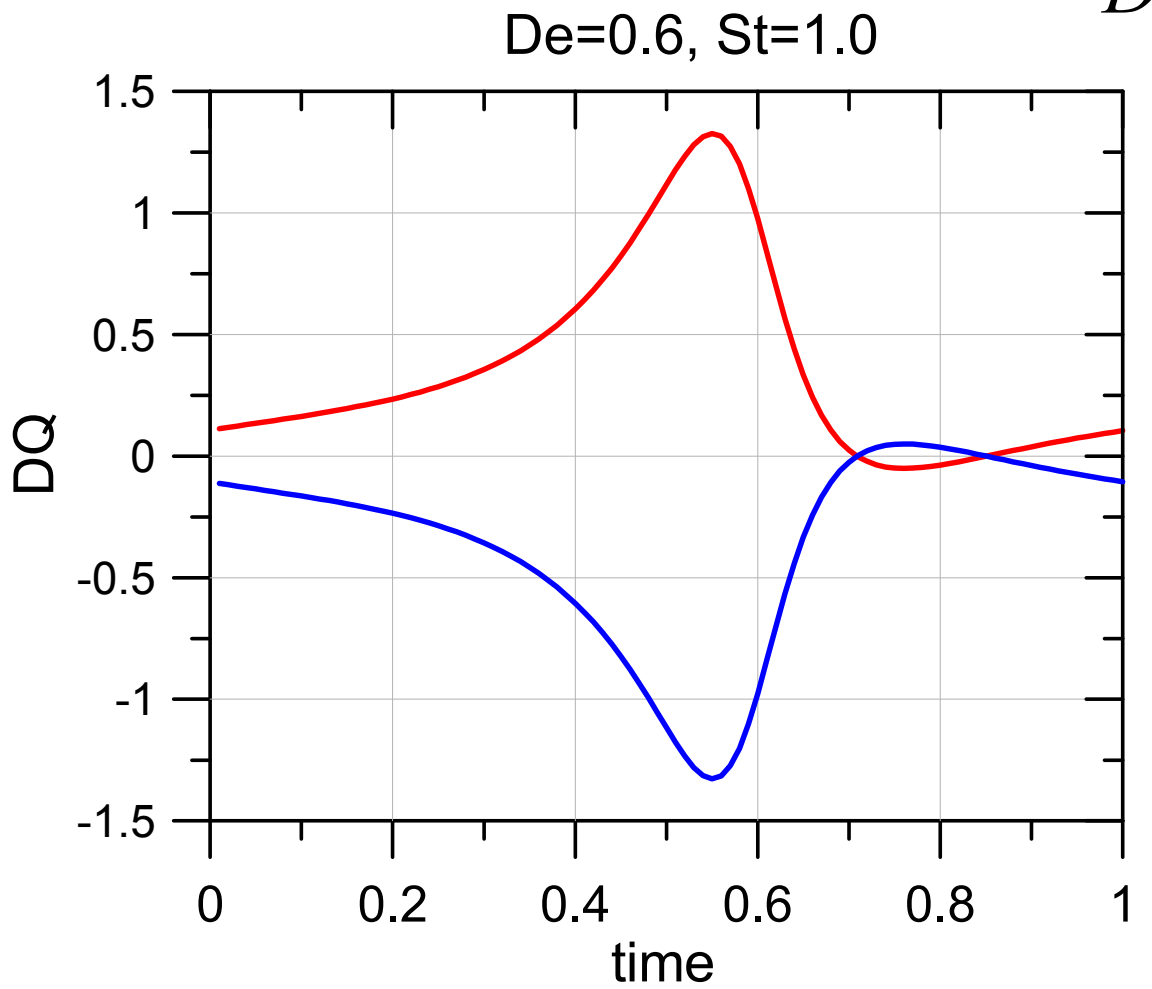
Pulsating flow:

Asymmetry parameter for 1 cycle

$$DQ = 0.724$$



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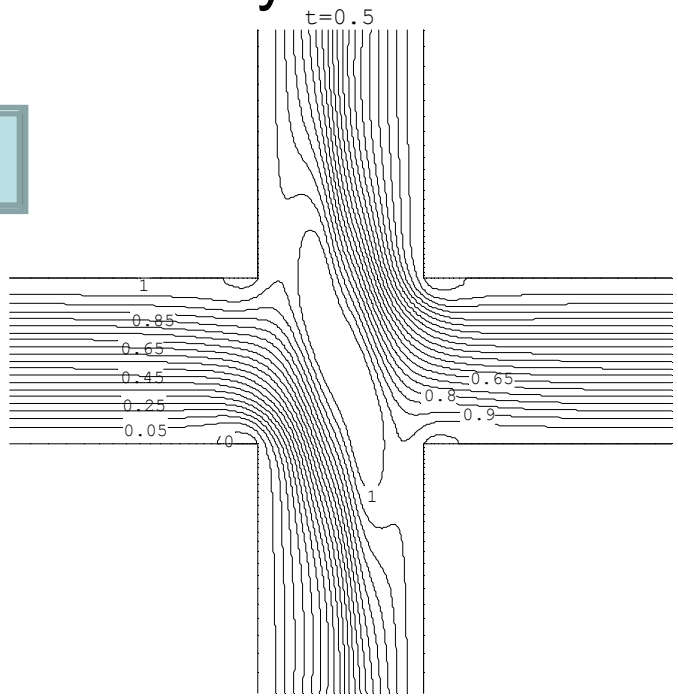
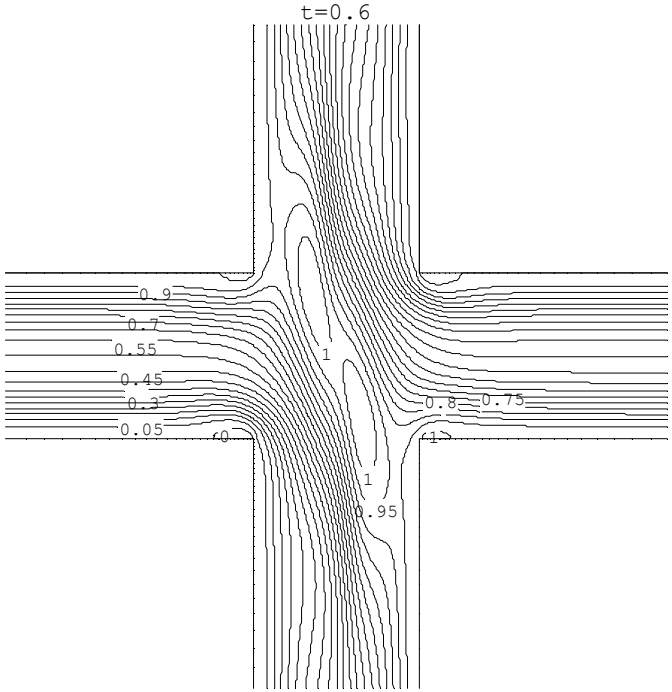
Pulsating flow:

Visualization by streamlines: *De* = 0.6, K_0/K_s = 1.0





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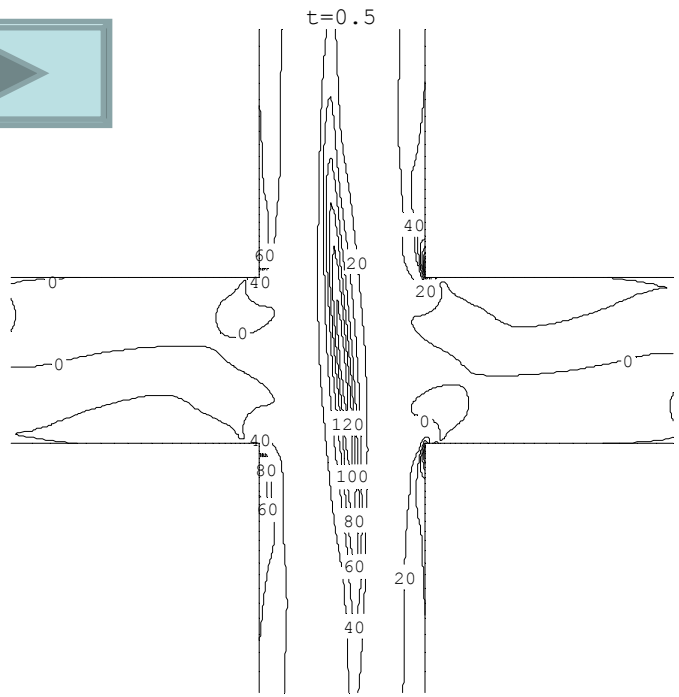
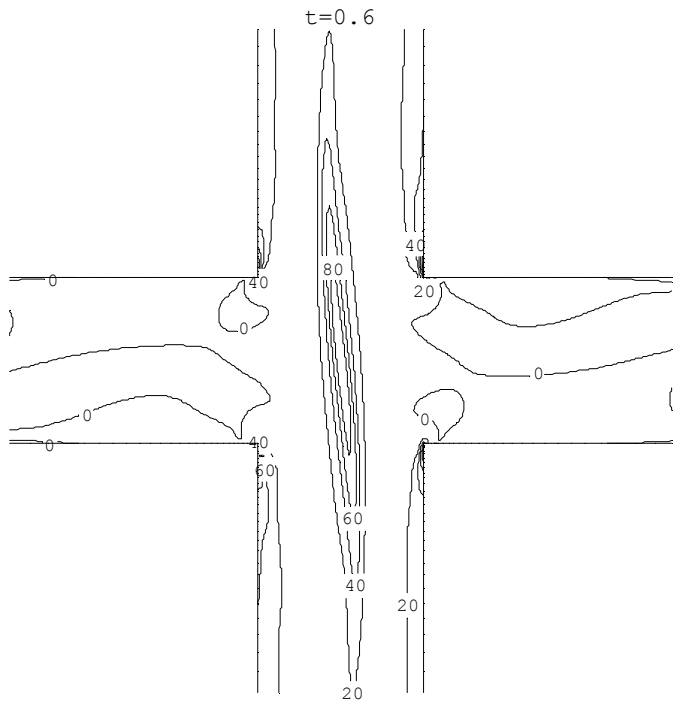
Pulsating flow:

Contours of vertical normal stress: *De* = 0.6, $K_0/K_s = 1$





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Pulsating flow:

Streaklines: De = 0.6, K_0/K_s = 1.0

Initially, there is a row of 15 tracer particles along centerline at cross-slot center: y=0

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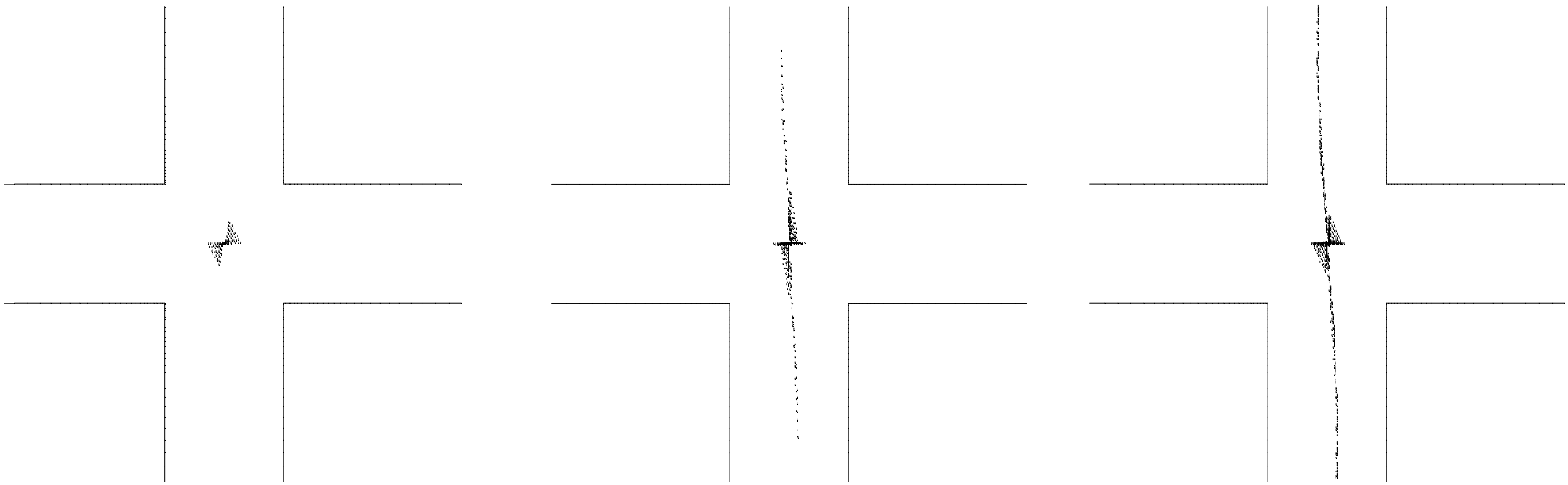
Pulsating flow:

Streaklines: *De* = 0.6, K_0/K_s = 1.0



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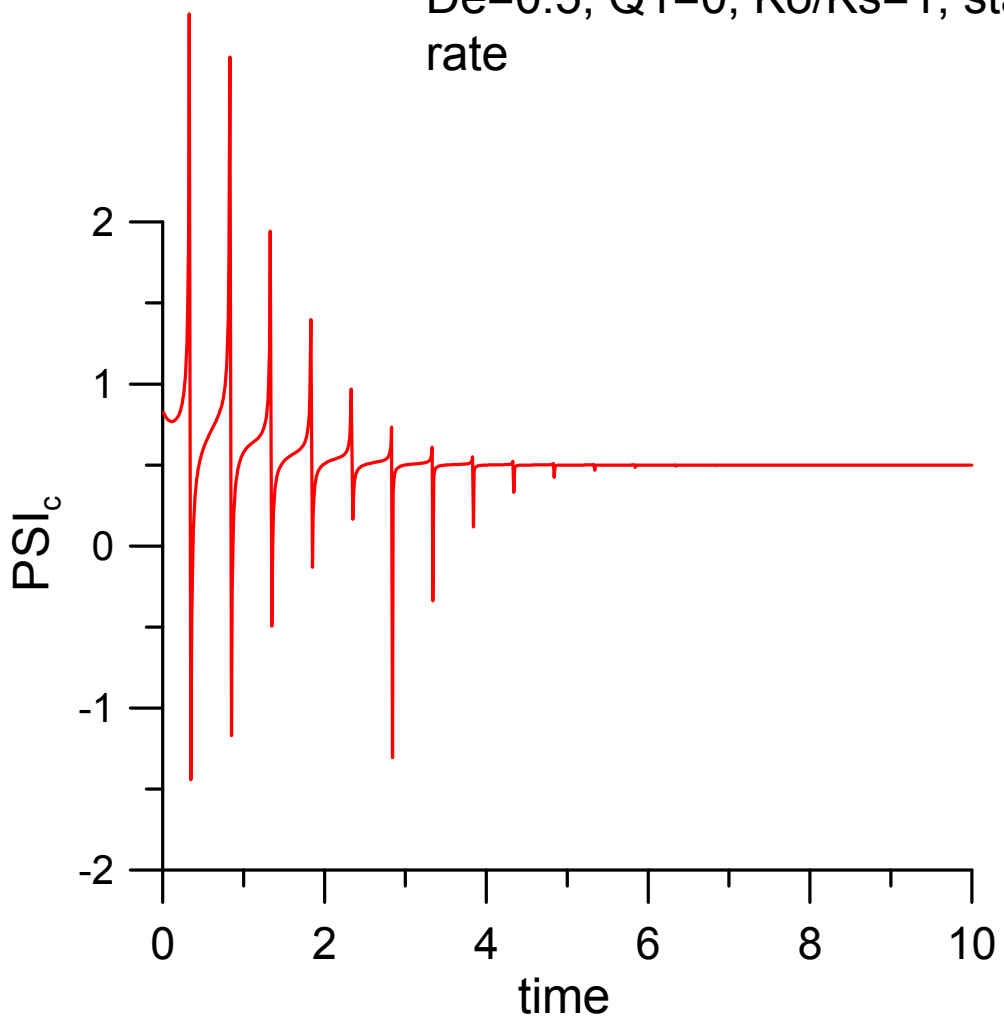
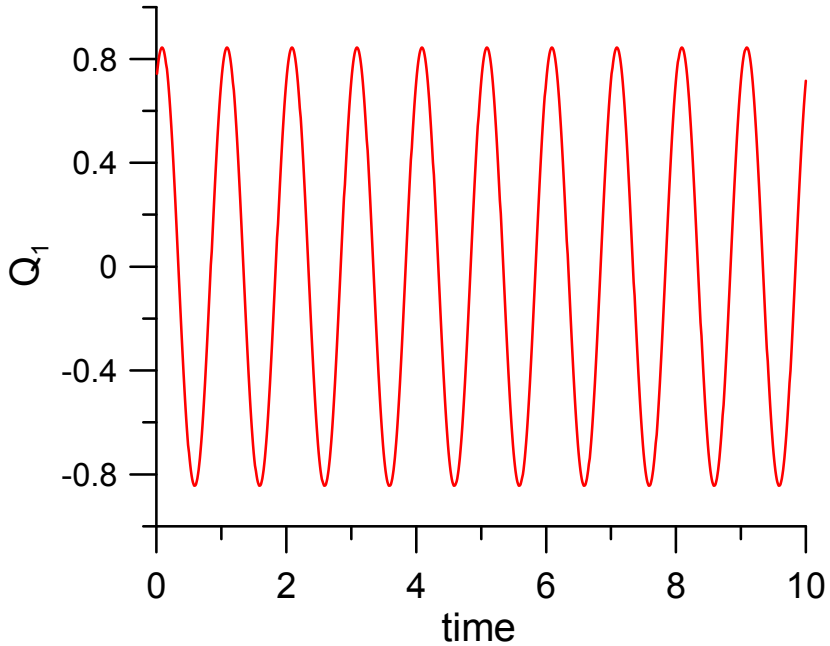
Purely pulsating case

De=0.5, Q1=0; Ko/Ks=1; starting from previous case with flow rate





Conclusions

- Flow becomes more complex when there is pulsation (this should enhance the mixing rate ??)
- Purely oscillating case remains symmetric (no bifurcation) in all cases tested
- Need to implement the actual inlet conditions used by Phelean et al. (with crossing of streamlines)

Acknowledgments:

FCT Fundação para a Ciência e a Tecnologia

MINISTÉRIO DA CIÊNCIA, INOVAÇÃO E DO ENSINO SUPERIOR Portugal

- **Under projects:**
  - **SFRH/BD/22644/2005 (G.N. Rocha)**
  - **PTDC/EQU-FTT/71800/2006**
  - **PTDC/EME-MFE/70186/2006**

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