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Unsteady Viscoelastic Flow in a Planar "T" Junction

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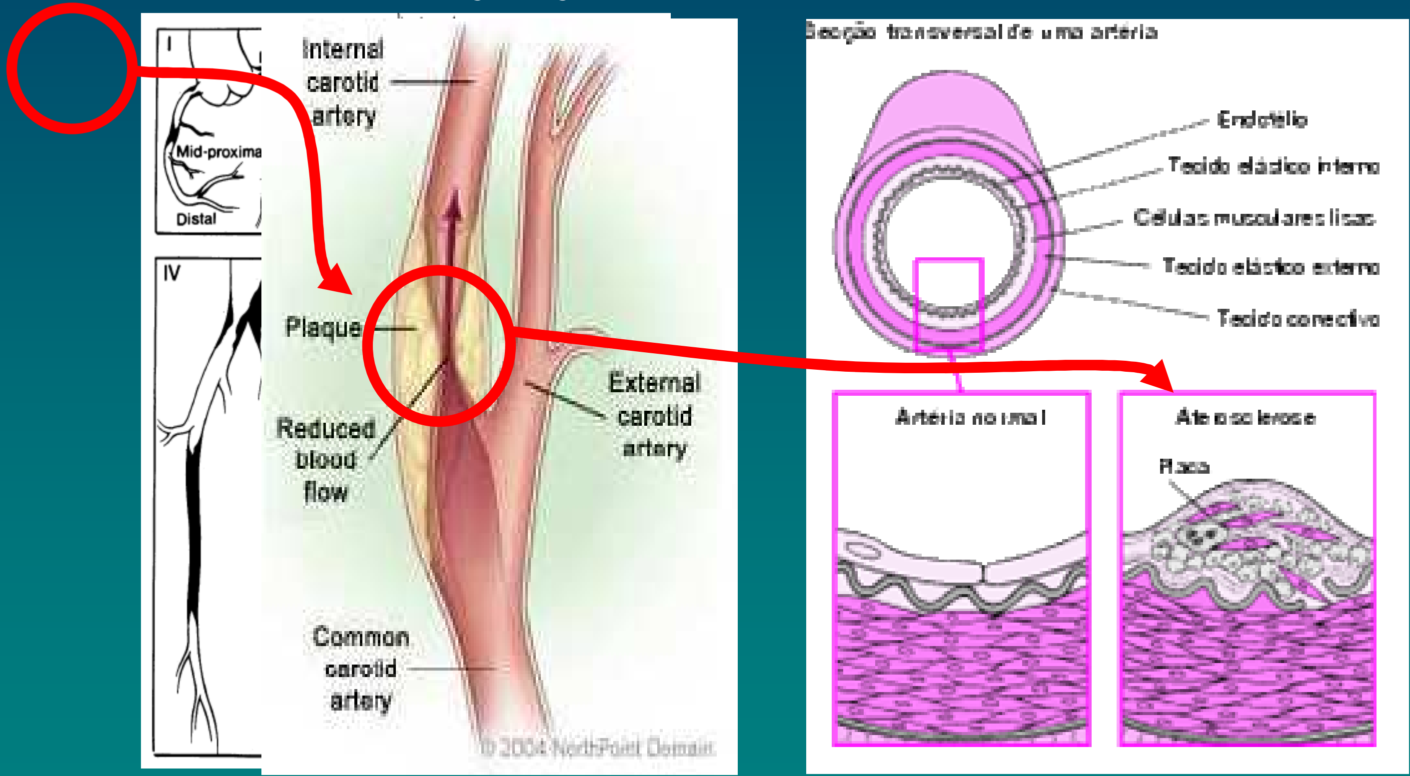
1 - T-JUNCTION FLOWS (test case)

- The human circulatory system



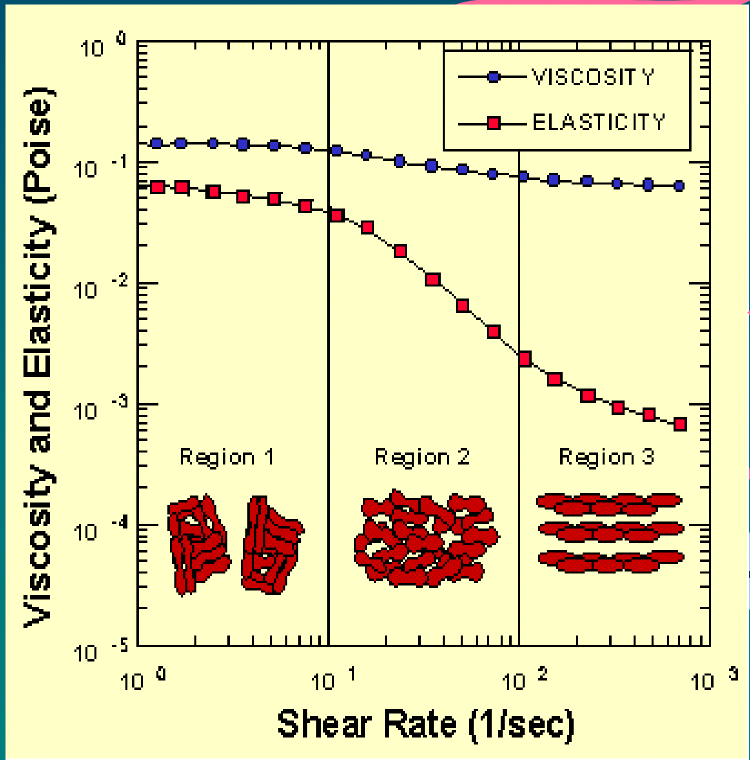
Del www.tcg.cardiologydomain.com/
Ann Surg. 201(2), 115–131.

www.manualmerck.net/

2 - RHEOLOGY OF BLOOD

BLOOD

Shear-thinning viscoelasticity



www.vilastic.com/

Small arteries

Non-Newtonian behaviour

-thinning

Elasticity

Relevant in unsteady flows

3 - OBJECTIVES

- Investigate unsteady flow trough a 2D bifurcation using a viscoelastic fluid model (FENE CR: Constant viscosity).
- First approach to blood flow simulations (elasticity usually neglected).
- Evaluate influence of flow rate ratio (β) and elasticity (Deborah number (De), at constant extensibility (L^2) and polymer concentration (c)).

$$De = \frac{\lambda u_1}{H}$$

$$0 \le De \le 10$$

$$c = \frac{\eta_P}{\eta_S} = 0.11$$

$$\eta_0 = \eta_S + \eta_P$$

$$\beta = \frac{Q_3}{Q_1}$$

$$0.1 \le \beta \le 0.9$$

$$L^2 = 100$$

4 - EQUATIONS

- Conservation of mass

$$\nabla \cdot \boldsymbol{u} = 0$$

- Conservation of linear momentum

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \nabla \cdot \boxed{\boldsymbol{\tau}} + \nabla \cdot (\eta_s \boldsymbol{D})$$

- Constitutive equation
  - FENE CR Model
    (Chilcott e Rallison, 1988)

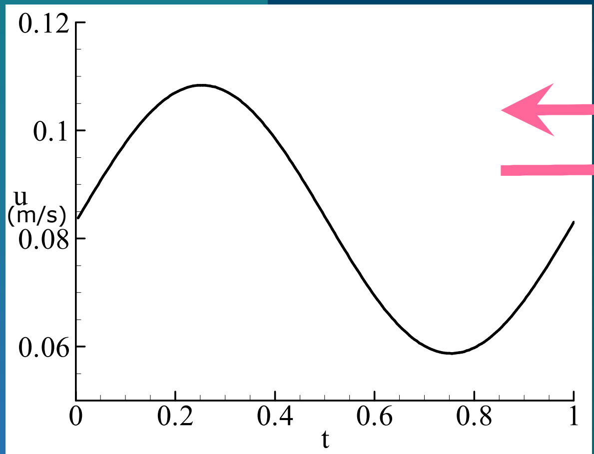
$$\boldsymbol{\tau} + \lambda \left(\frac{\overset{\nabla}{\boldsymbol{\tau}}}{\boxed{f(\boldsymbol{\tau})}} \right) = 2\eta_p \boldsymbol{D}$$

$$\boxed{f(\boldsymbol{\tau}) = \frac{L^2 + (\lambda / \eta_p) tr(\boldsymbol{\tau})}{L^2 - 3}}$$

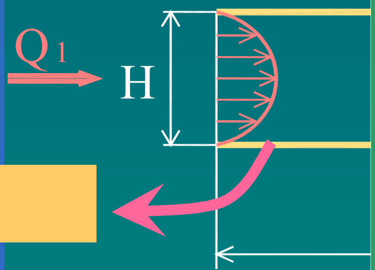
5 - NUMERICAL PROCEDURE

- Finite-volume method for discretization of equations.
- Nonstaggered mesh arrangement.
  - Pressure-velocity coupling: Rhie & Chow, 1983.
  - Stress-velocity coupling: Oliveira et al., 1999.
- Spatial discretisation (convective terms): CUBISTA scheme (Alves et al. 2003).
- Temporal discretisation (unsteady term): Three-time level scheme (Oliveira 2001).
- Pressure-correction: SIMPLEC algorithm with time-marching.

6 - FLOW GEOMETRY



Miranda, A.I.P., et al. (2008), Int. J. Numer. Meth. Fluids, 57, 295-328.



Inlet

Generated by an sinusoidal pressure gradient

$$-\frac{dp}{dx} = \rho K_S + \rho K_0 \cos(\omega t)$$

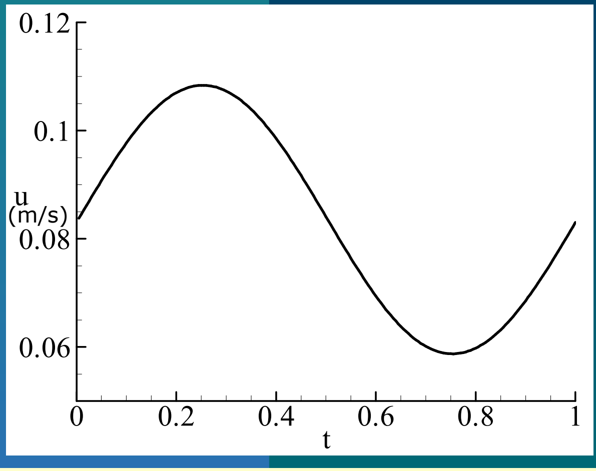
$\rho K_S = 75.1 \text{ Pa/m}$ | $\rho K_0 = 190 \text{ Pa/m}$ | $\omega = 2\pi f = 2.2\pi \text{ s}^{-1}$

$K_0 / K_S = 2.53$ | $T = 0.91 \text{ s}$

Womersley number

$$\alpha = \left[\frac{H}{2} \bullet \left(\frac{\omega}{\nu}\right)^{1/2}\right] = 4.864$$

6 - FLOW GEOMETRY



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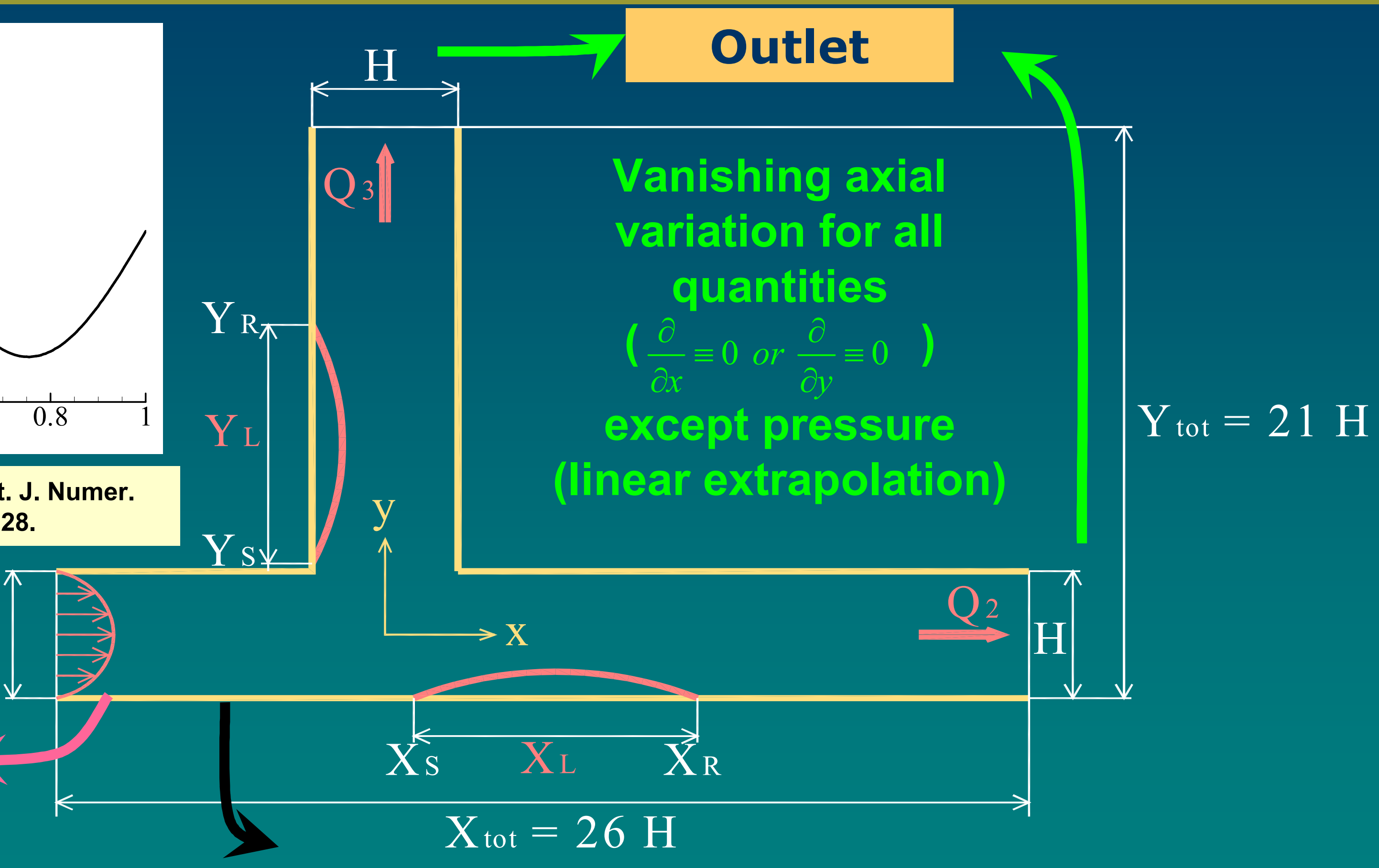
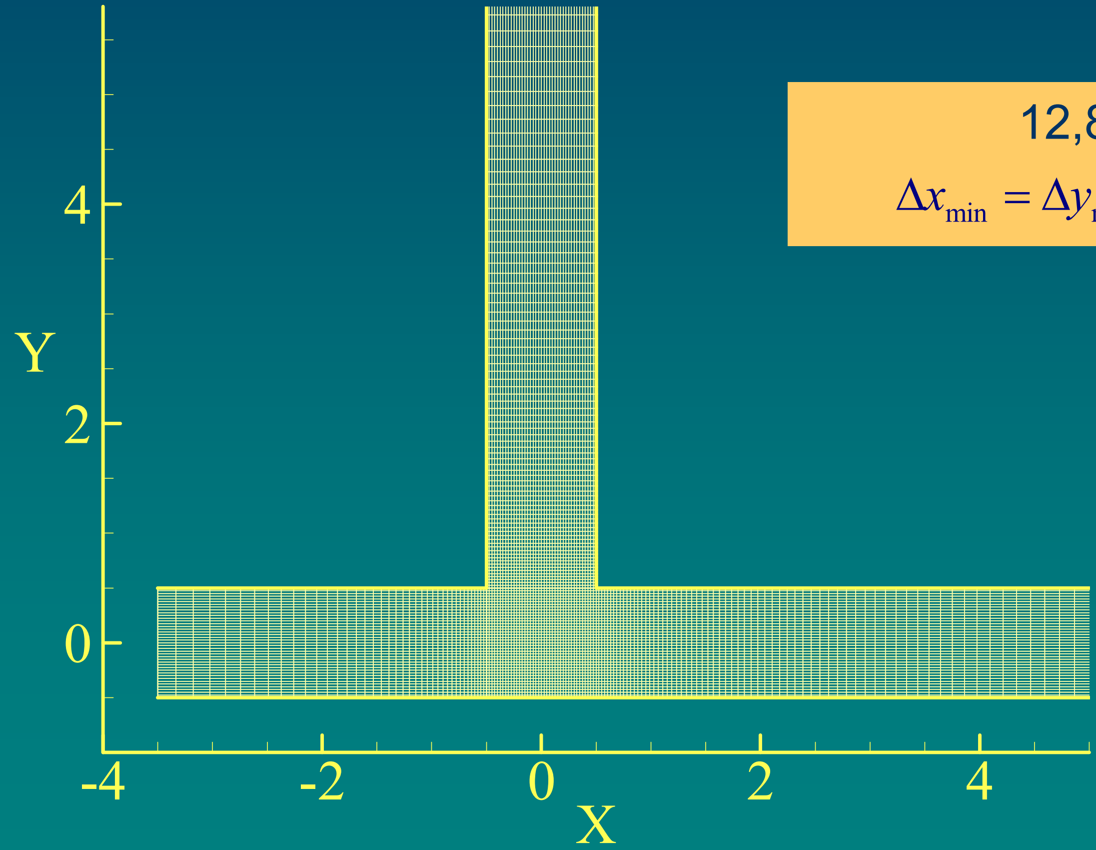


Walls — **No slip condition u = v = 0**

7 - MESH

- Orthogonal but non uniform meshes

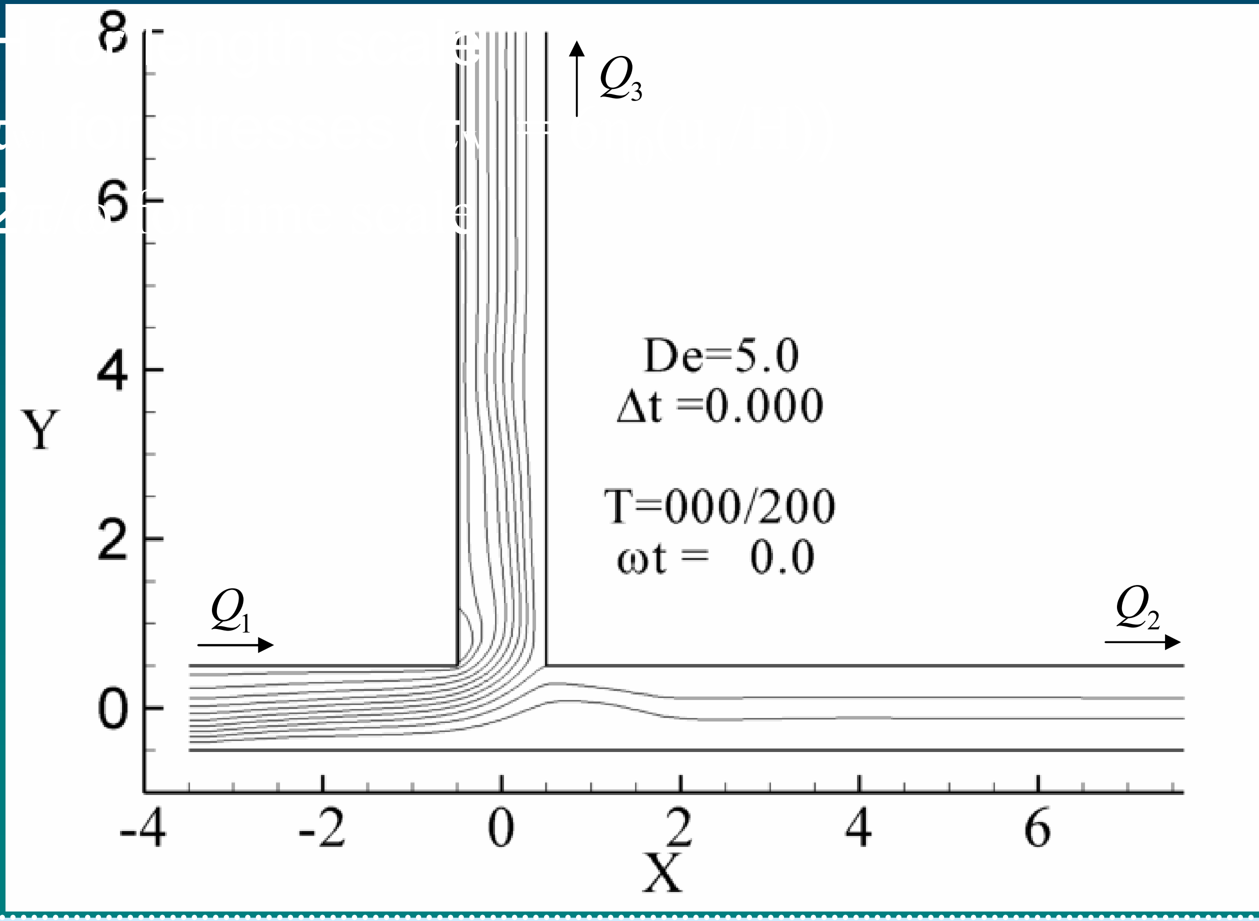


12,800 VC

$$\Delta x_{min} = \Delta y_{min} = 2.5\times10^{-2}$$

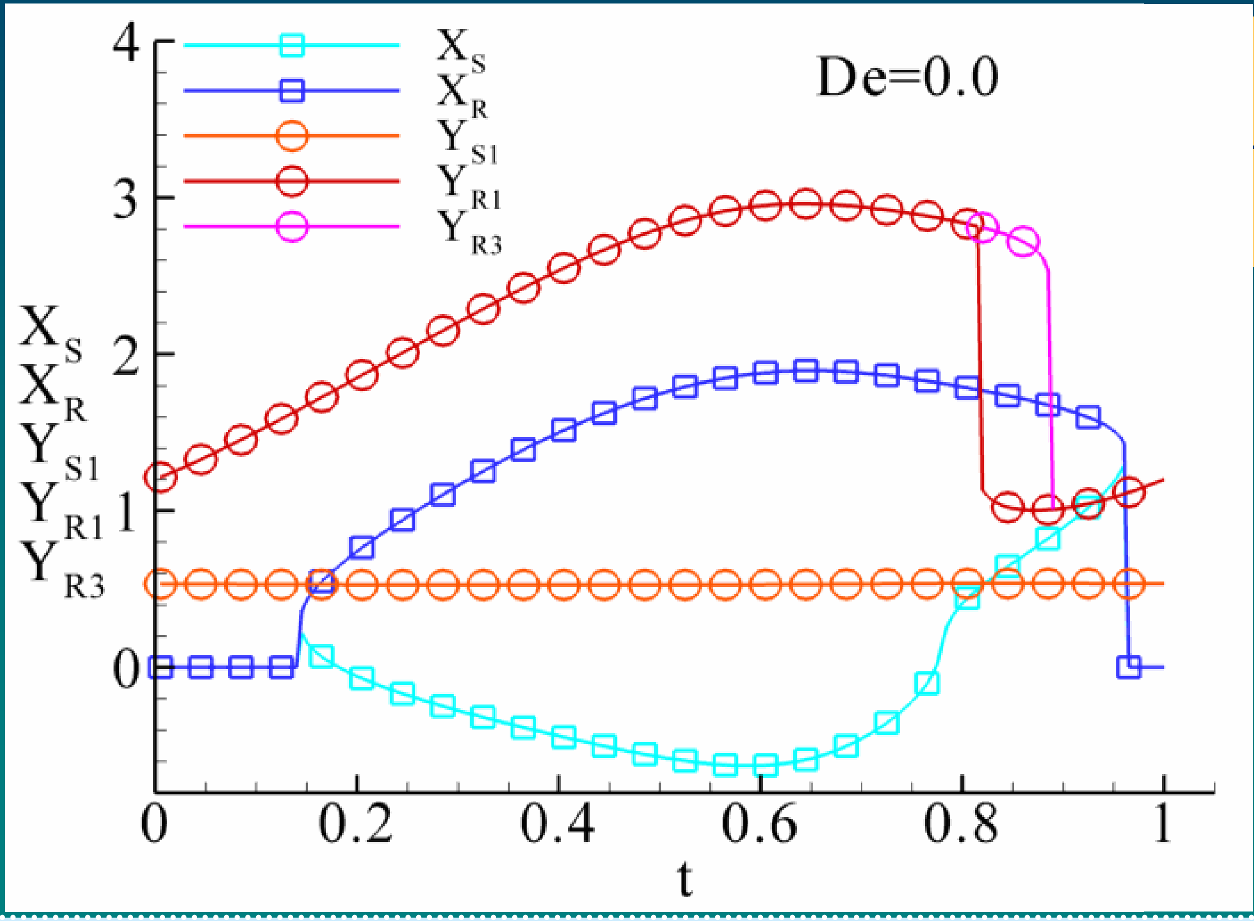
8 - RESULTS

- Streamlines for a cycle ($\beta = Q_3/ Q_1 = 0.7$)



8 - RESULTS (variation of elasticity)

- Separation and reattachment points ($\beta = 0.7$)



residence

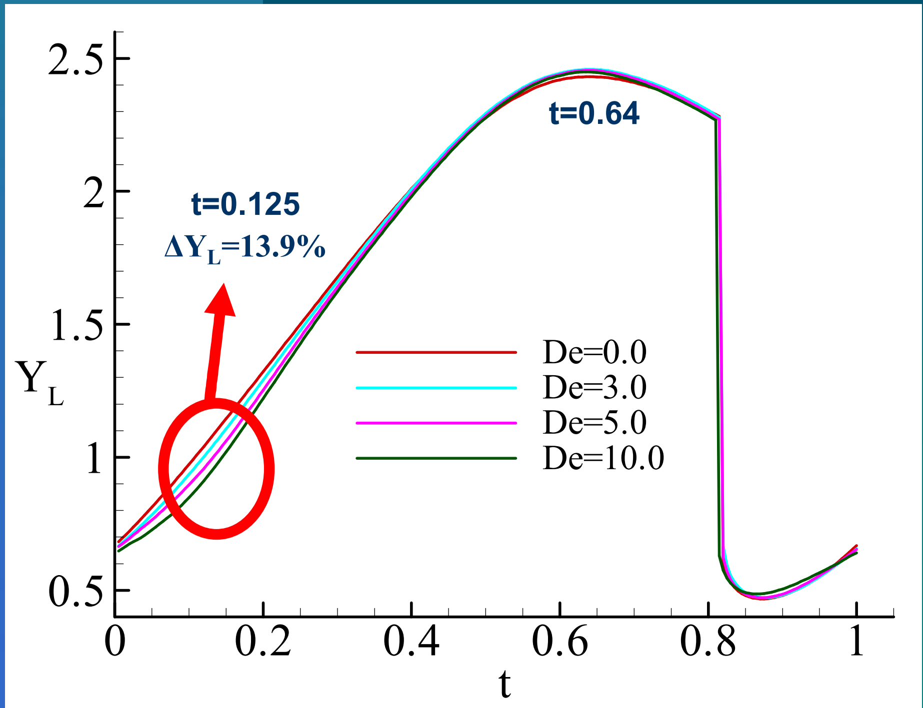
– 81.5%

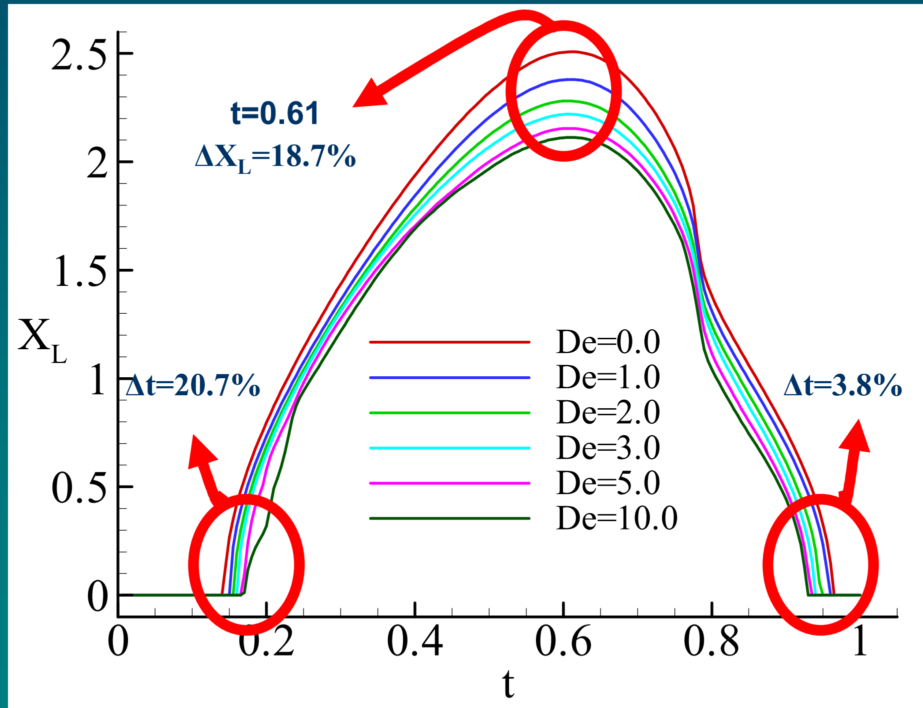
– 75.5%

8 - RESULTS (variation of elasticity)

- Recirculation lengths ($\beta = 0.7$)

Vertical *(E)≈0.01 - 0.1* ***Horizontal***

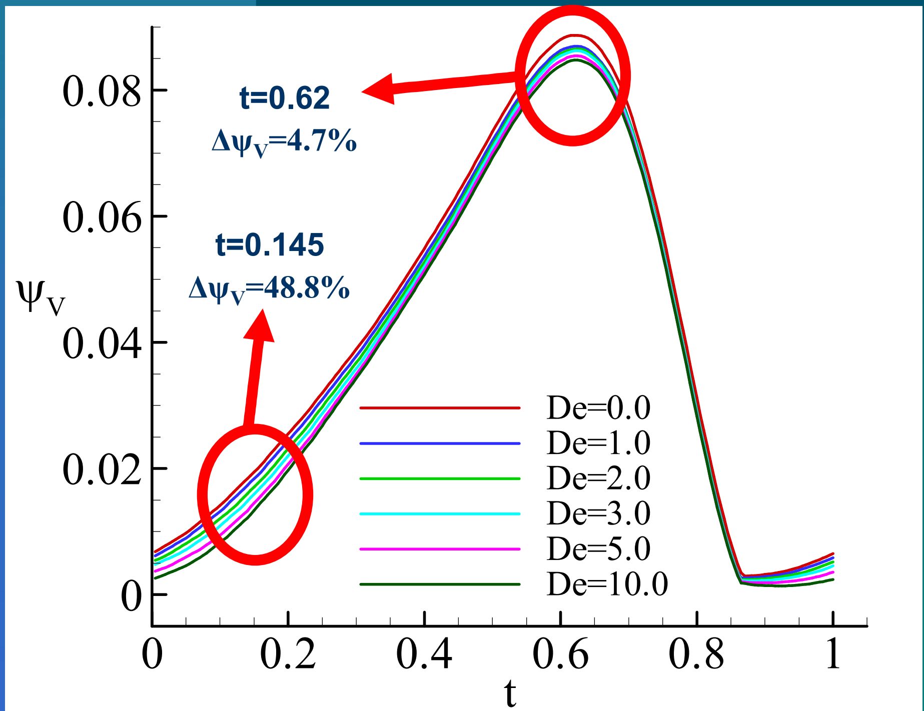




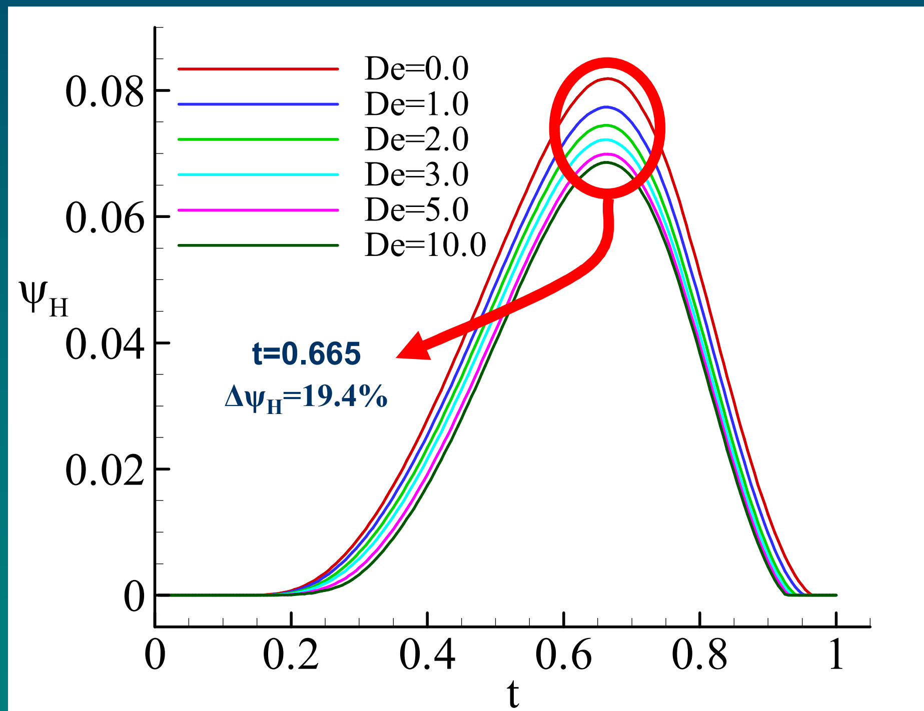
8 - RESULTS (variation of elasticity)

- Vortex strength ($\beta = 0.7$)

Vertical

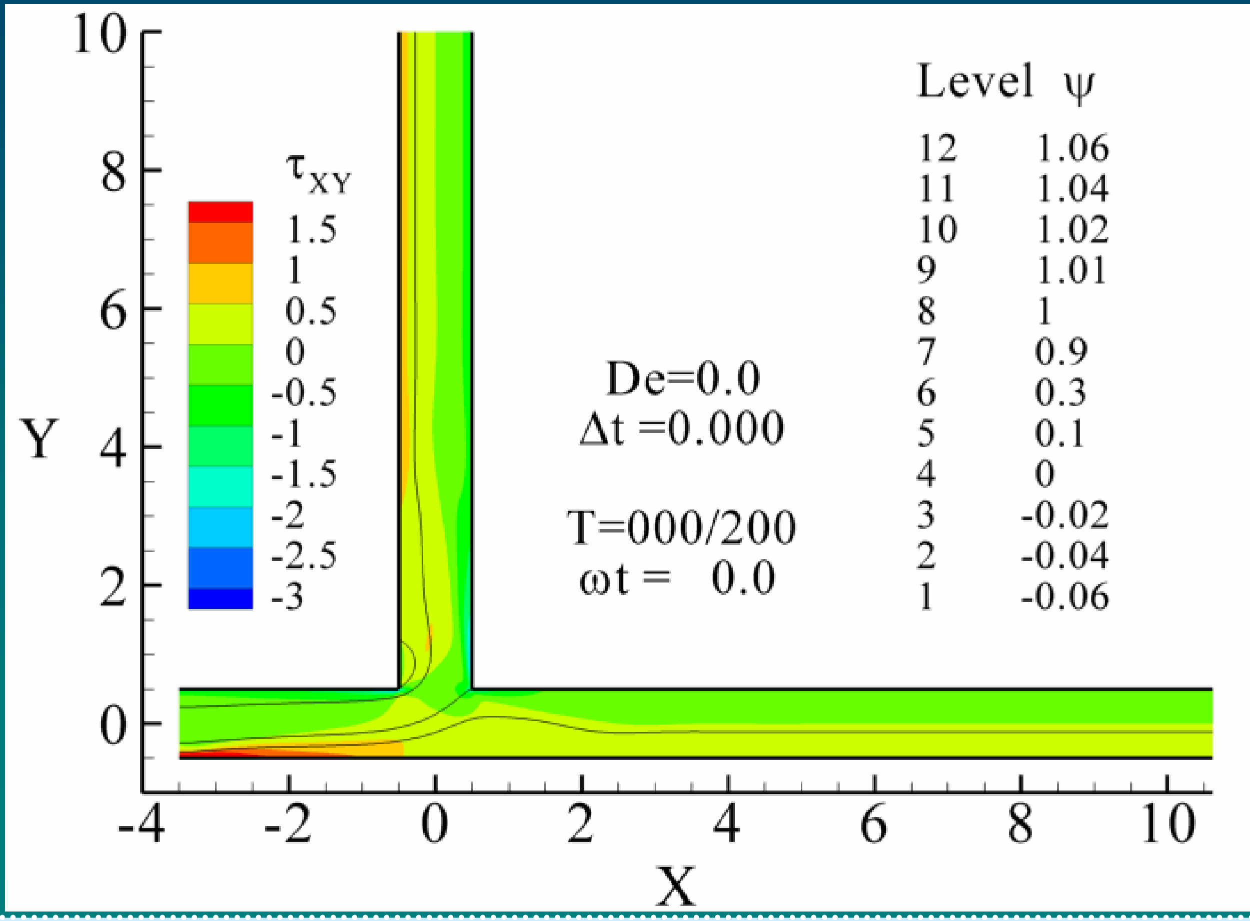


Horizontal



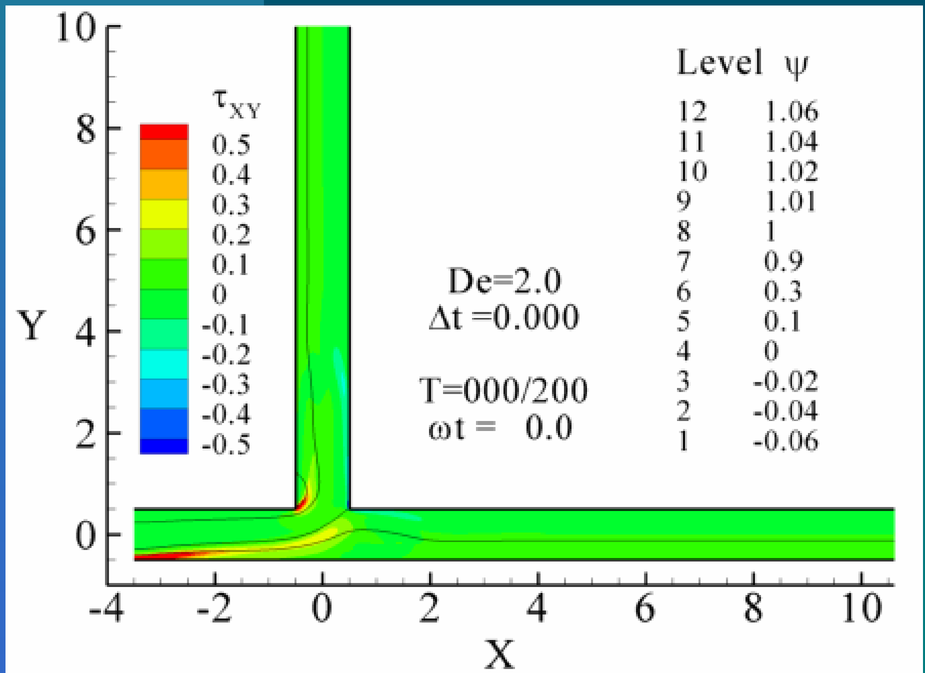
8 - RESULTS (variation of elasticity)

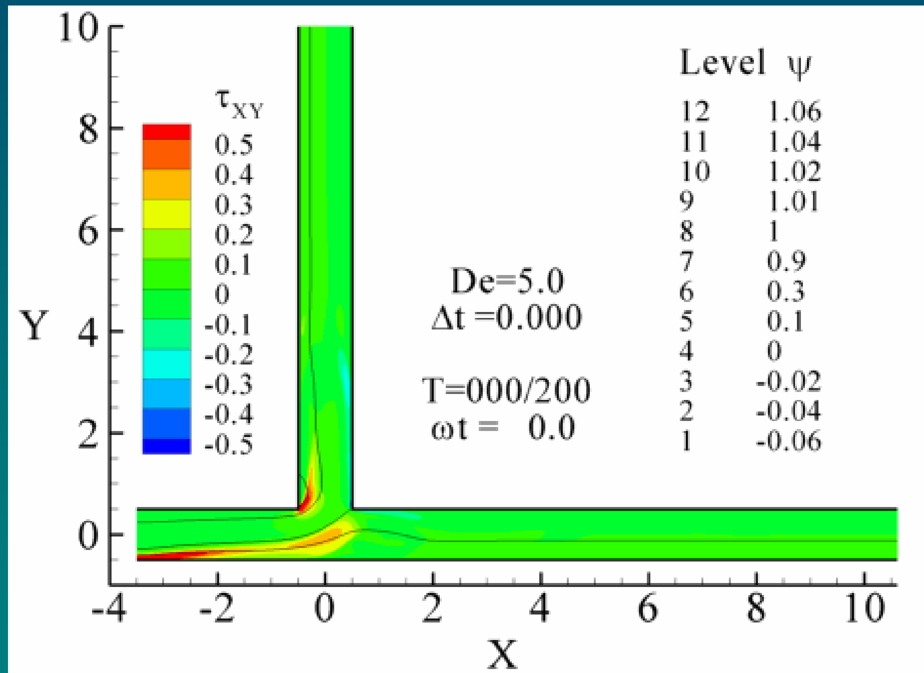
- Shear stress field ($\beta = 0.7$)



8 - RESULTS (variation of elasticity)

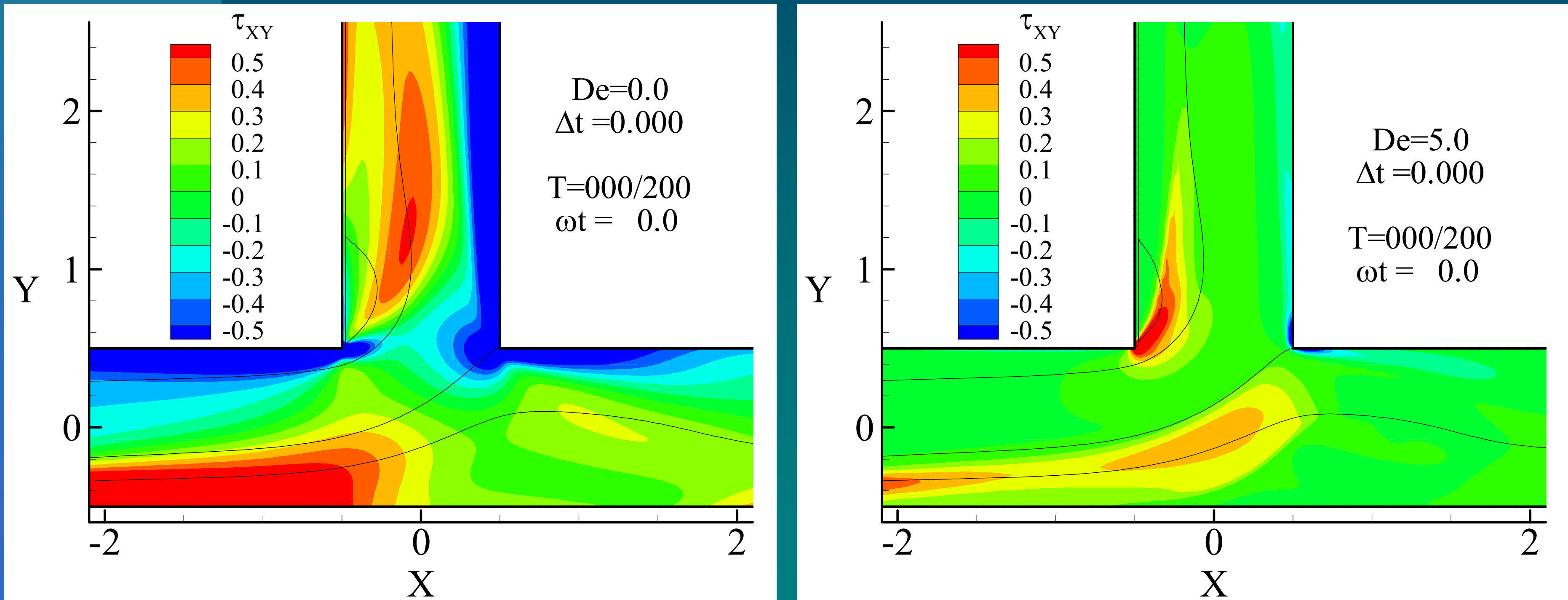
- Shear stress field (polymeric components only)





8 - RESULTS (variation of elasticity)

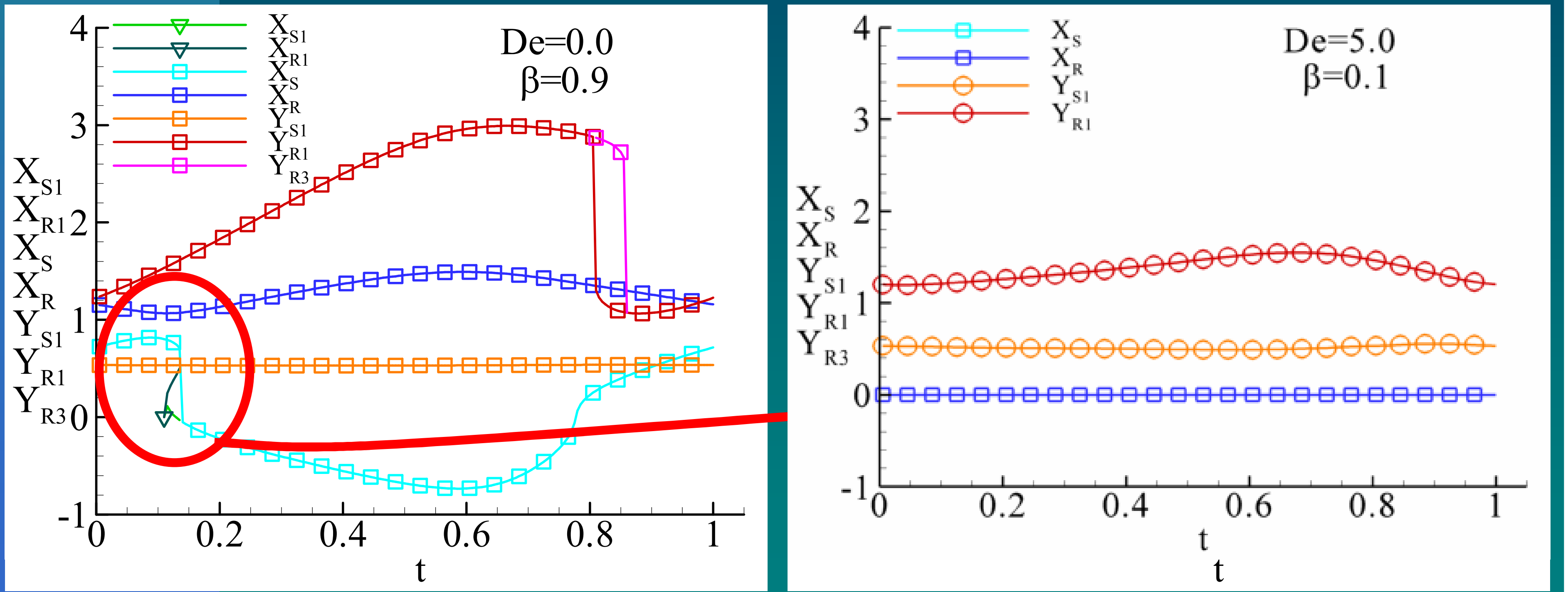
- Shear stress field



9 - RESULTS (variation of extraction ratio)

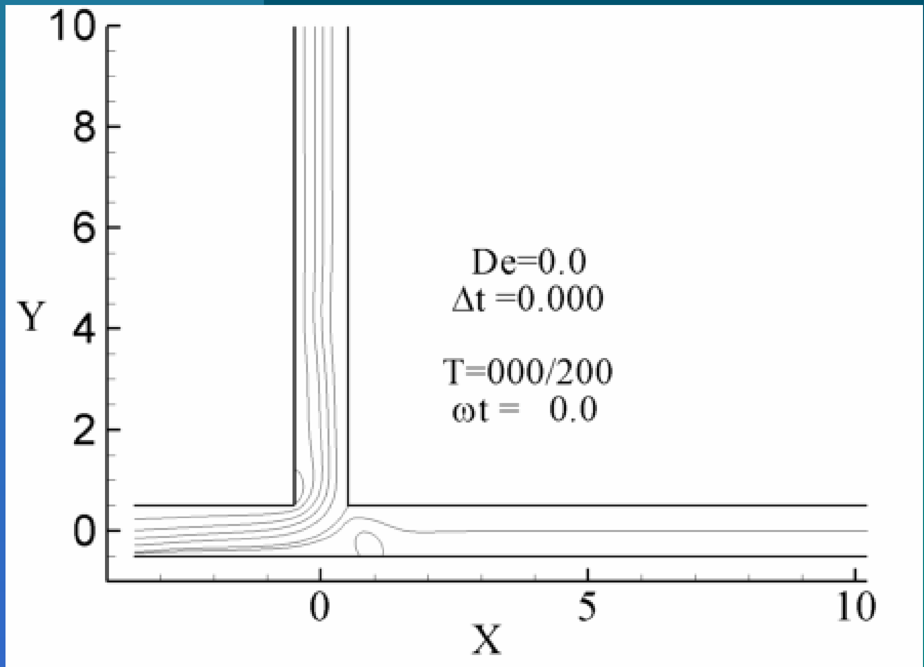
- Separation and reattachment points

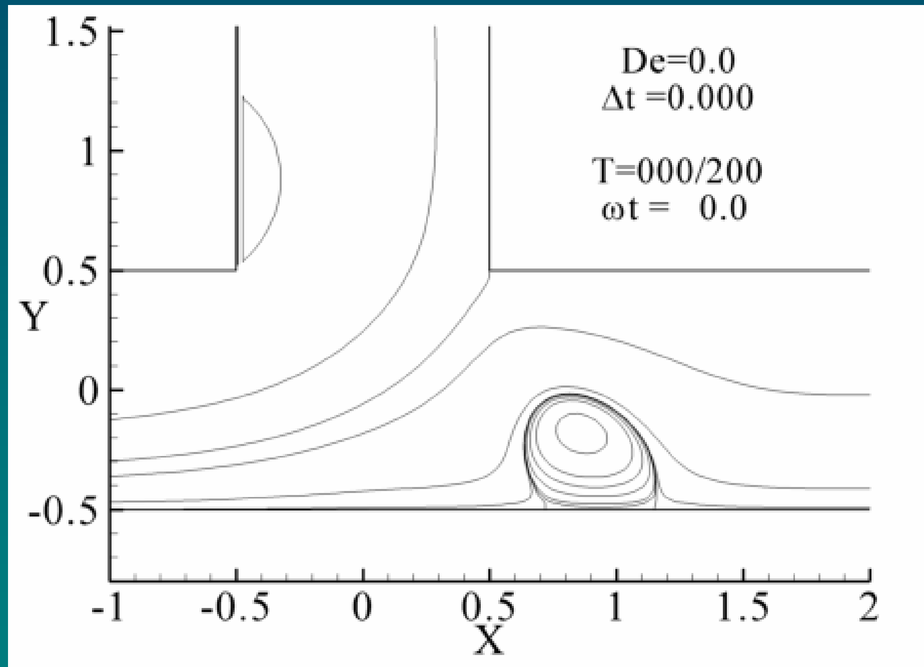
De=0 ***De=5***



9 - RESULTS (variation of extraction ratio)

- Streamlines for a cycle ($\beta = 0.9$)

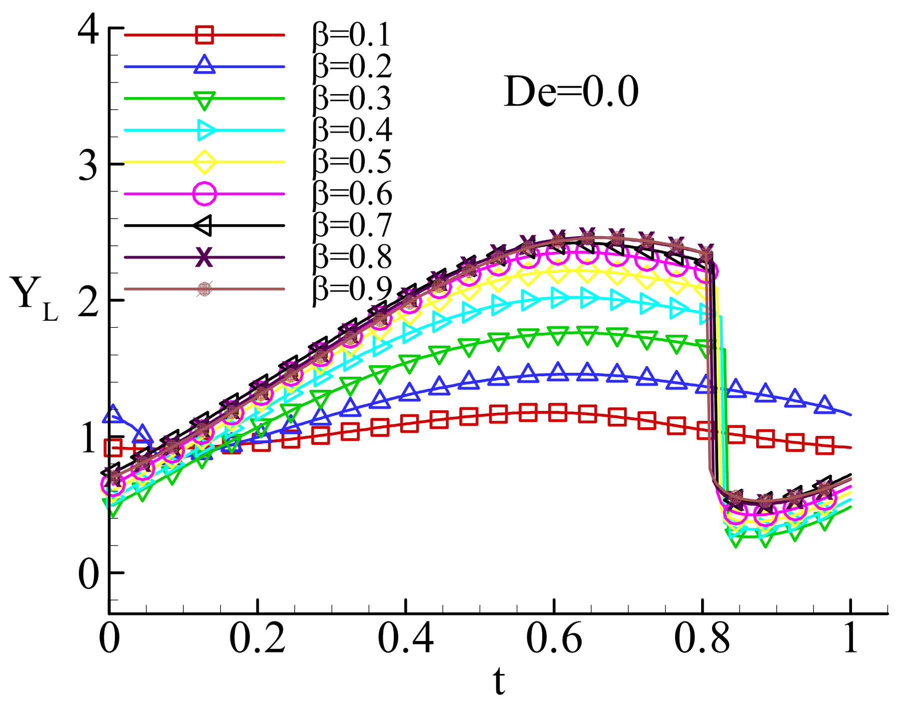


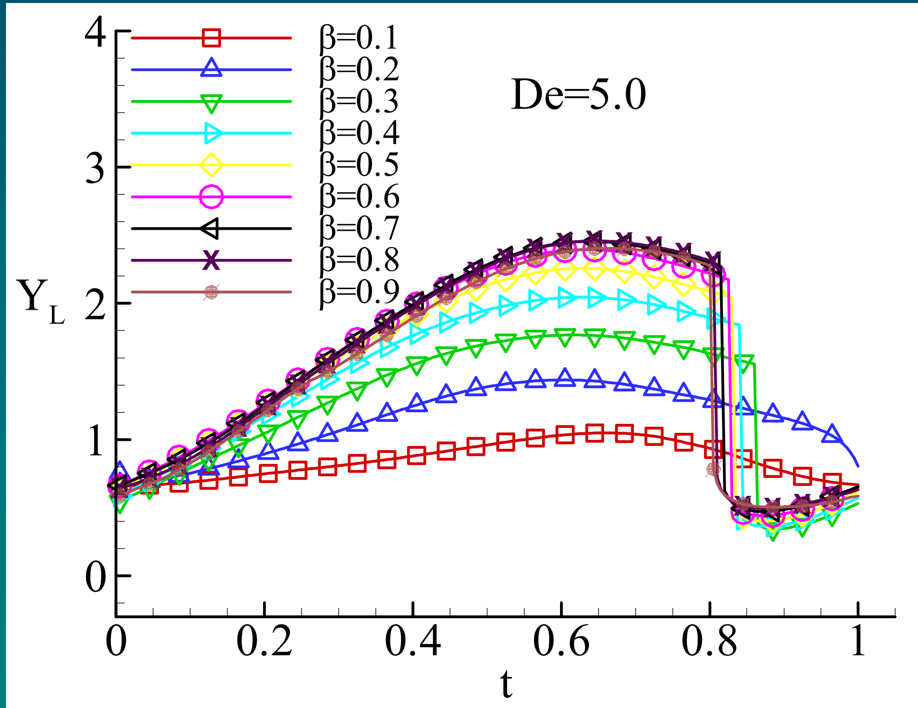


9 - RESULTS (variation of extraction ratio)

- Recirculation lengths (Vertical)

De=0 ***De=5***

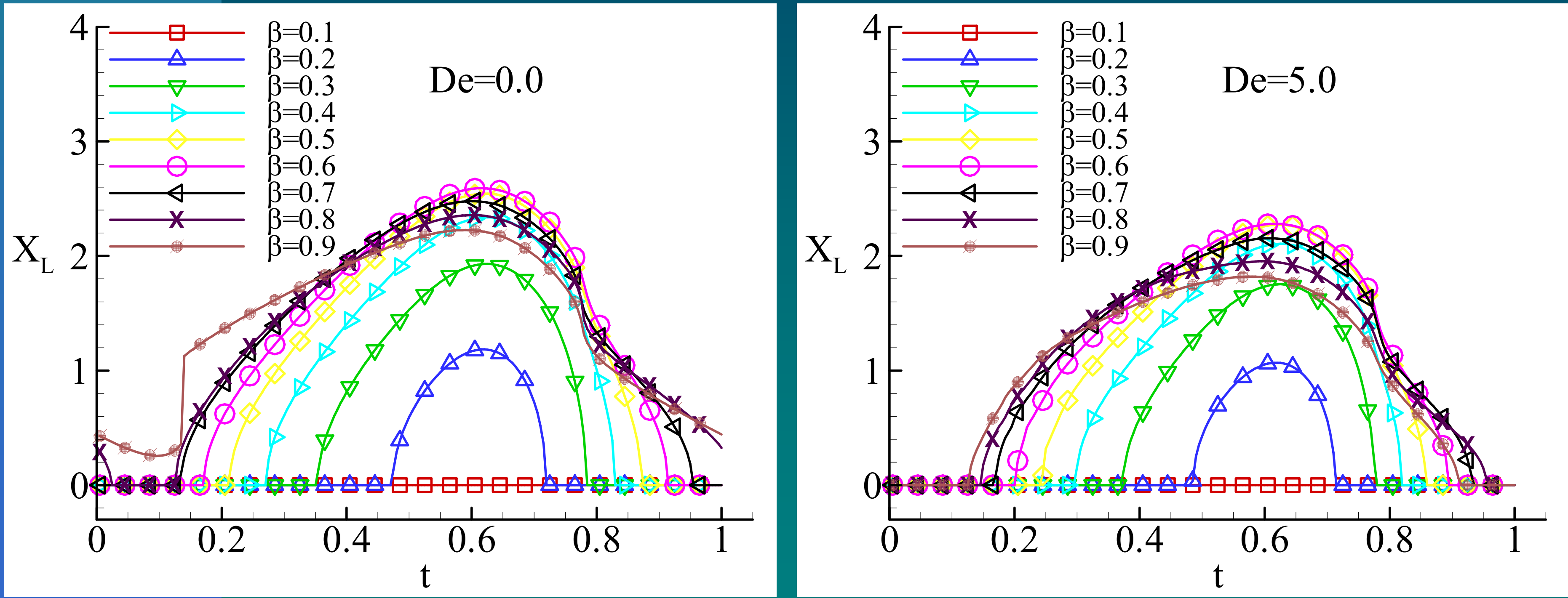




9 - RESULTS (variation of extraction ratio)

- Recirculation lengths (Horizontal)

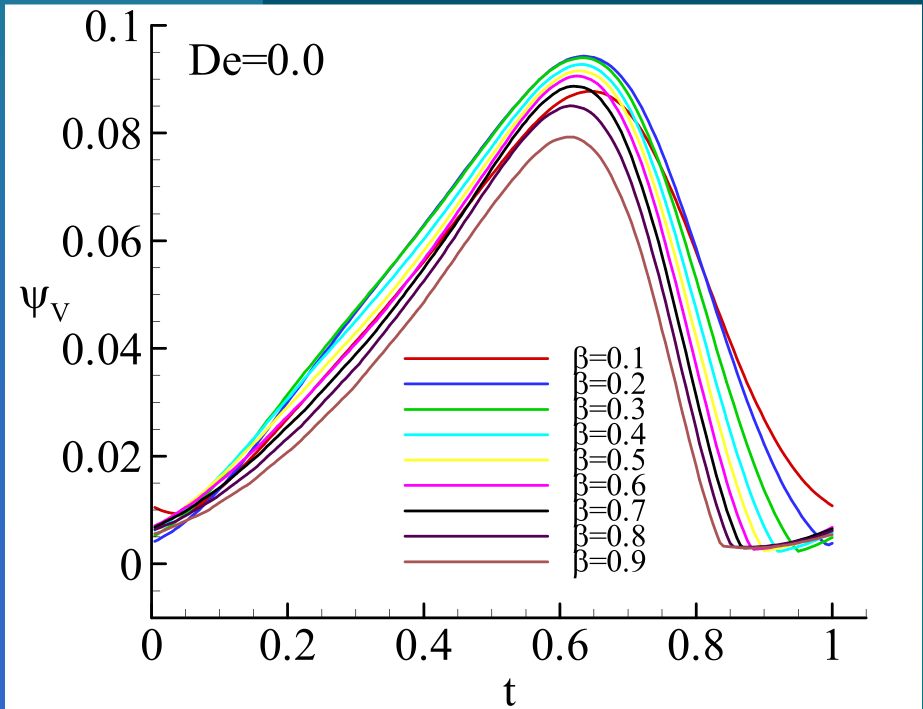
De=0 ***De=5***

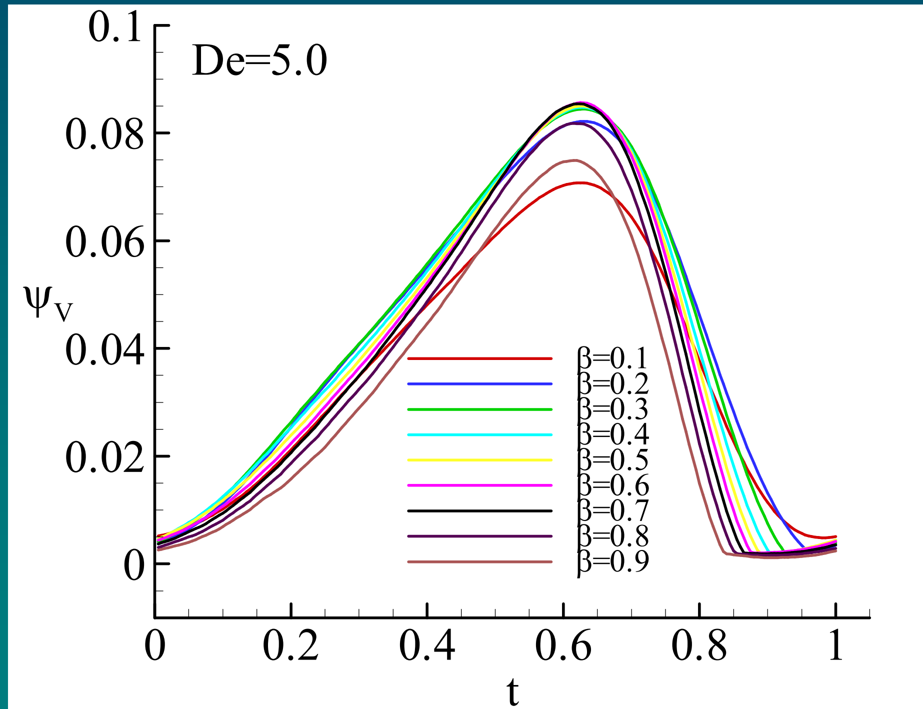


9 - RESULTS (variation of extraction ratio)

- Vortex strength (Vertical)

De=0 ***De=5***

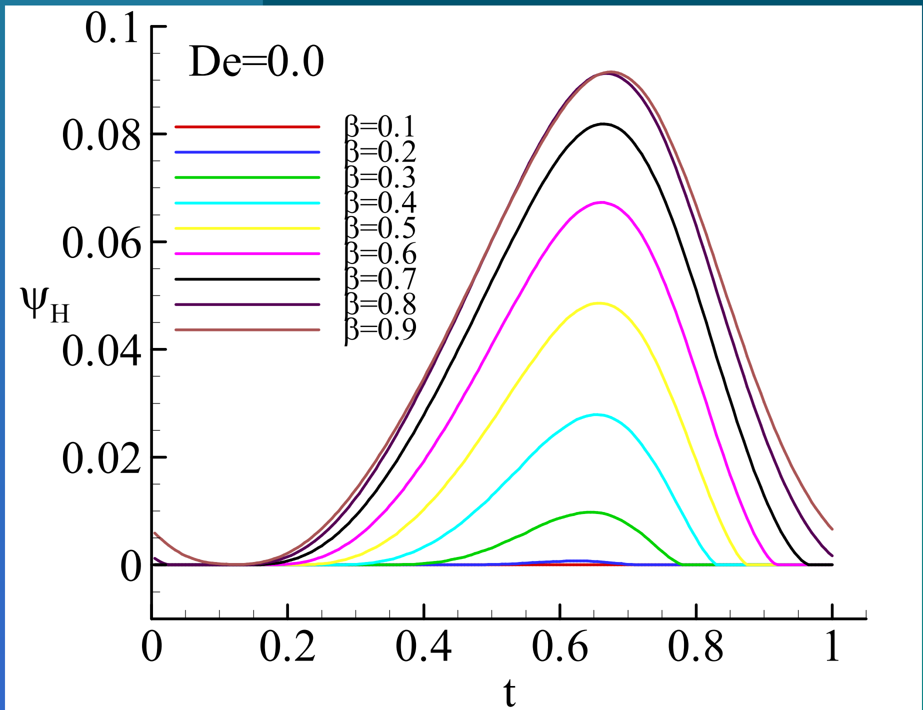


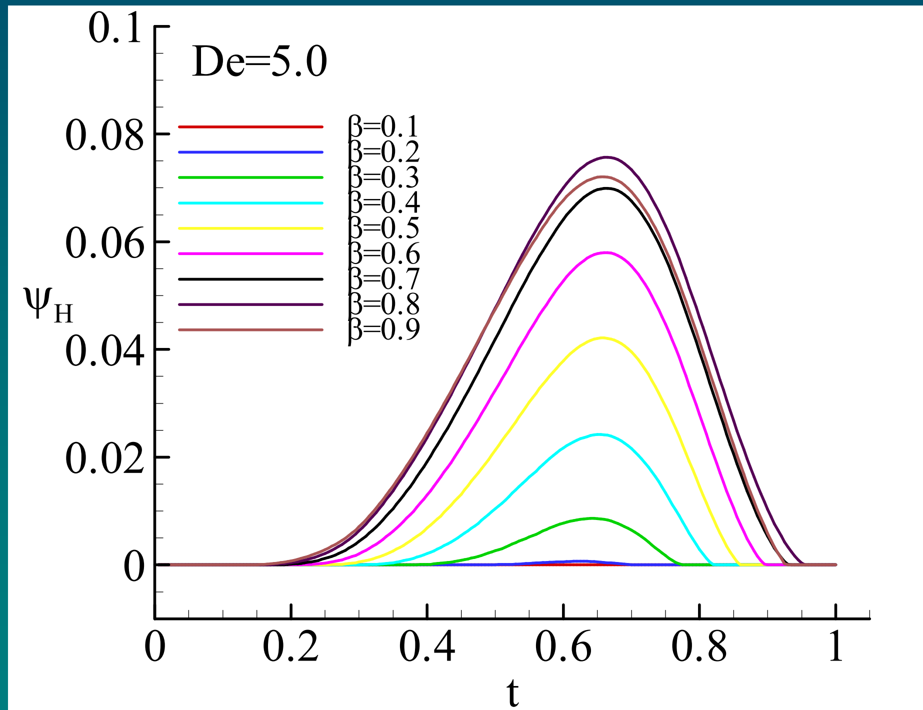


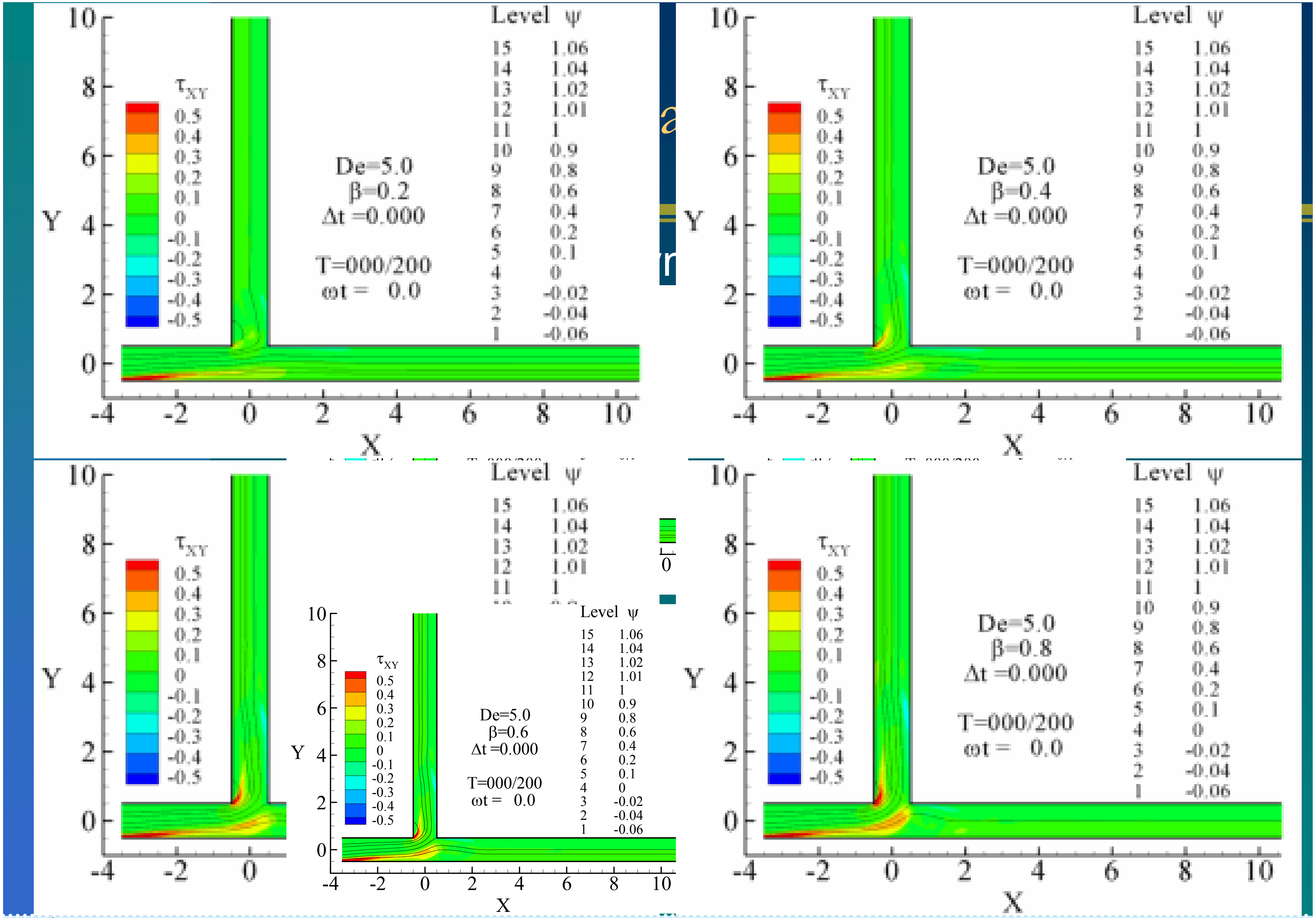
9 - RESULTS (variation of extraction ratio)

- Vortex strength (Horizontal)

De=0 ***De=5***






De=5.0
β=0.2
Δt =0.000
T=000/200
ωt = 0.0
De=5.0
β=0.4
Δt =0.000
T=000/200
ωt = 0.0
De=5.0
β=0.6
Δt =0.000
T=000/200
ωt = 0.0
De=5.0
β=0.8
Δt =0.000
T=000/200
ωt = 0.0
τ_{XY}
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Level ψ
15 1.06
14 1.04
13 1.02
12 1.01
11 1
10 0.9
9 0.8
8 0.6
7 0.4
6 0.2
5 0.1
4 0
3 -0.02
2 -0.04
1 -0.06
X
Y

10 - CONCLUSIONS

- Size and intensity of recirculations decrease with De.
- Y_L increase with β; X_L increase with β for $\beta \leq 0.6$ and decrease for $\beta > 0.6$.
- Intensity ψ_H increase with β.
- Horizontal recirculation not always present: residence interval decrease with De and increase with β.
- Maximum size and intensity occur after middle of the cycle.

10 - CONCLUSIONS

- Polymeric shear stress component increase with De and β.
- Low stresses inside recirculating zones and high stresses in the re-entrant corners of the bifurcation.
- During the cycle the shear stress field follows the velocity variation, with maximum for $\omega t \approx 90^o$ and minimum values for $\omega t \approx 270^o$.

11 - ACKNOWLEDGEMENTS

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- ***University of Beira Interior (Portugal)***