Unsteady Viscoelastic Flow in a Planar “T” Junction

Helder M. Matos, Paulo J. Oliveira

Universidade da Beira Interior
1 - T-JUNCTION FLOWS (test case)

- The human circulatory system


2 - Rheology of Blood

Small arteries

Non-Newtonian behaviour

Shear-thinning viscoelasticity

Relevant in unsteady flows

Small arteries

Non-Newtonian behaviour

Shear-thinning viscoelasticity

Relevant in unsteady flows

www.vilastic.com/
3 - OBJECTIVES

- Investigate unsteady flow through a 2D bifurcation using a viscoelastic fluid model (FENE CR: Constant viscosity).
- First approach to blood flow simulations (elasticity usually neglected).
- Evaluate influence of flow rate ratio ($\beta$) and elasticity (Deborah number (De), at constant extensibility ($L^2$) and polymer concentration ($c$)).

\[
De = \frac{\lambda u_1}{H}
\]

\[c = \frac{\eta_p}{\eta_s} = 0.11\]

\[\beta = \frac{Q_3}{Q_1}\]

\[L^2 = 100\]

\[0 \leq De \leq 10\]

\[\eta_0 = \eta_s + \eta_p\]

\[0.1 \leq \beta \leq 0.9\]
4 - EQUATIONS

- Conservation of mass
  \[ \nabla \cdot \mathbf{u} = 0 \]

- Conservation of linear momentum
  \[ \rho \frac{Du}{Dt} = -\nabla p + \nabla \cdot \mathbf{\tau} + \nabla \cdot (\eta_s \mathbf{D}) \]

- Constitutive equation
  - FENE CR Model
    \[ f(\mathbf{\tau}) = \frac{L^2 + \left(\frac{\lambda}{\eta_p}\right)tr(\mathbf{\tau})}{L^2 - 3} \]
    \[ \mathbf{\tau} + \lambda \left( \begin{array}{c} \nabla \\ \mathbf{\tau} \\ \frac{\nabla \cdot \mathbf{\tau}}{f(\mathbf{\tau})} \end{array} \right) = 2\eta_p \mathbf{D} \]
5 - NUMERICAL PROCEDURE

- Finite-volume method for discretization of equations.
- Nonstaggered mesh arrangement.
- Pressure-correction: SIMPLEC algorithm with time-marching.
### Flow Geometry

**Generated by an sinusoidal pressure gradient**

\[-\frac{dp}{dx} = \rho K_s + \rho K_0 \cos(\omega t)\]

\[\rho K_s = 75.1 \text{ Pa/m} \quad \rho K_0 = 190 \text{ Pa/m} \quad \omega = 2\pi f = 2.2\pi \text{ s}^{-1}\]

\[K_0 / K_s = 2.53 \quad T = 0.91 \text{ s}\]

**Womersley number**

\[\alpha = \left[ \frac{H}{2 \left( \frac{\omega}{\nu} \right)^{1/2}} \right] = 4.864\]
Vanishing axial variation for all quantities
\[ \frac{\partial}{\partial x} = 0 \text{ or } \frac{\partial}{\partial y} = 0 \]
except pressure (linear extrapolation)

Outlet

Y_{tot} = 21 \, H

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Y_{tot} = 21 \, H

Inlet

Orthogonal but non uniform meshes

12,800 VC

\[ \Delta x_{\text{min}} = \Delta y_{\text{min}} = 2.5 \times 10^{-2} \]
8 - RESULTS

Streamwise results (β = Q_3 / Q_1 = 0.7)

- H for length scale
- τ_{W1} for stresses (τ_{W1} = 6η_0 (u_1 / H))
- 2π/ω for time scale

De=5.0
Δt = 0.000
T=000/200
ωt = 0.0
8 - RESULTS (variation of elasticity)

- Separation and reattachment points ($\beta = 0.7$)

![Graph showing separation and reattachment points with De=0.0]
Recirculation lengths ($\beta = 0.7$)

**Vertical**  \[(E) \approx 0.01 - 0.1\]

- $t = 0.125$, $\Delta Y_L = 13.9\%$
- $t = 0.61$, $\Delta X_L = 18.7\%$
- $t = 0.64$, $\Delta t = 20.7\%$
- $\Delta t = 3.8\%$

**Horizontal**
8 - RESULTS (variation of elasticity)

- Vortex strength ($\beta = 0.7$)

Vertical

Horizontal
Shear stress field ($\beta = 0.7$)

- RESULTS (variation of elasticity)
8 - RESULTS (variation of elasticity)

- Shear stress field (polymeric components only)
8 - RESULTS (variation of elasticity)

- Shear stress field

De=0.0
Δt =0.000
T=000/200
ωt = 0.0

De=5.0
Δt =0.000
T=000/200
ωt = 0.0
9 - RESULTS (variation of extraction ratio)

- Separation and reattachment points

De=0

De=5

β=0.9

β=0.1
Streamlines for a cycle ($\beta = 0.9$)

- RESULTS (variation of extraction ratio)
9 - RESULTS (variation of extraction ratio)

Recirculation lengths (Vertical)

\( De=0 \)

\( De=5 \)

\( De=0.0 \)

\( De=5.0 \)
9 - RESULTS (variation of extraction ratio)

- Recirculation lengths (Horizontal)

For $De=0$

$\beta=0.1$
$\beta=0.2$
$\beta=0.3$
$\beta=0.4$
$\beta=0.5$
$\beta=0.6$
$\beta=0.7$
$\beta=0.8$
$\beta=0.9$

For $De=5$

$\beta=0.1$
$\beta=0.2$
$\beta=0.3$
$\beta=0.4$
$\beta=0.5$
$\beta=0.6$
$\beta=0.7$
$\beta=0.8$
$\beta=0.9$
9 - RESULTS (variation of extraction ratio)

- Vortex strength (Vertical)

De=0

De=5
9 - RESULTS (variation of extraction ratio)

- Vortex strength (Horizontal)

De=0

\[ \psi_H \]

\( t \) 0.0 0.2 0.4 0.6 0.8 1

\( \beta \) 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

De=5

\[ \psi_H \]

\( t \) 0.0 0.2 0.4 0.6 0.8 1

\( \beta \) 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Shear stress field (polymeric components only)

De=5.0
\( \beta=0.2 \)
\( \Delta t =0.000 \)
\( T=000/200 \)
\( \omega t= 0.0 \)

De=5.0
\( \beta=0.4 \)
\( \Delta t =0.000 \)
\( T=000/200 \)
\( \omega t= 0.0 \)

De=5.0
\( \beta=0.6 \)
\( \Delta t =0.000 \)
\( T=000/200 \)
\( \omega t= 0.0 \)

De=5.0
\( \beta=0.8 \)
\( \Delta t =0.000 \)
\( T=000/200 \)
\( \omega t= 0.0 \)
10 - CONCLUSIONS

- Size and intensity of recirculations decrease with De.
- \( Y_L \) increase with \( \beta \); \( X_L \) increase with \( \beta \) for \( \beta \leq 0.6 \) and decrease for \( \beta > 0.6 \).
- Intensity \( \psi_H \) increase with \( \beta \).
- Horizontal recirculation not always present: residence interval decrease with De and increase with \( \beta \).
- Maximum size and intensity occur after middle of the cycle.
During the cycle the shear stress field follows the velocity variation, with maximum for $\omega t \approx 90^\circ$ and minimum values for $\omega t \approx 270^\circ$.

Low stresses inside recirculating zones and high stresses in the re-entrant corners of the bifurcation.

Polymeric shear stress component increase with $D_e$ and $\beta$. 
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