Transient viscoelastic flows: detailed analysis of start-up and pulsating flows

A general numerical simulation code, based on the finite volume formulation is assessed in “simple” unsteady cases.

Test Problems: - Start-up planar Poiseuille flow;
- Pulsating flow.

Comparison between analytical and numerical results for Newtonian, UCM and Oldroyd-B fluids.

Actual flows of interest often occur in variable regimes (e.g. injection and blow moulding, blood flow, ...).
Governing Equations

- **Conservation Equations**
  
  **Mass**
  \[ \nabla \cdot \mathbf{u} = 0 \Leftrightarrow \n \]
  \[ \frac{\partial \mathbf{u}}{\partial x} = 0; \quad v = 0 \]

- **Momentum**
  
  **Newtonian**
  \[ \rho \frac{\partial \mathbf{u}}{\partial t} = \nu \frac{\partial^2 \mathbf{u}}{\partial y^2} - \frac{dp}{dx} \]
  
  **Viscoelastic**
  \[ \rho \frac{\partial \mathbf{u}}{\partial t} = \frac{\partial \tau_{xy}}{\partial y} - \frac{dp}{dx} \]

- **Constitutive Models**
  
  **UCM**
  \[ \tau + \lambda \nabla \tau = 2\eta_0 \mathbf{D} \]

  **Oldroyd-B**
  \[ \tau + \lambda \nabla \tau = 2\eta_0 \left( \mathbf{D} + \lambda \nabla \cdot \mathbf{D} \right) \]
FINITE VOLUME METHOD (FVM)

- Spatial discretization (over control volumes, $\Delta y$)
  - Convective fluxes vanish, thus simplifying the problem

- Temporal discretization (over time step, $\Delta t$)
  - Three time level method (2nd order accuracy; important)

- Pressure/velocity coupling according to Rhie and Chow (actually, not needed);
- Velocity/tension coupling according to Oliveira et al. (1998)
Problem Description

- **Start-up Poiseuille Flow**: A transient flow resulting from sudden application of a spatially constant pressure gradient to a fluid initially at rest.
- **Pulsating Flow**: A periodic flow resulting from application of a pressure gradient which varies in a sinusoidal manner in time:

\[-\frac{1}{\rho} \frac{dp}{dx} = K_e + K_0 \cos(\omega t)\]

- The calculations were carried out for different values of the elastic number $E$.

\[E = \frac{We}{Re} = \frac{\lambda \eta_0}{\rho h^2}\]
During the transient period the two fluids have very different behaviour;
Both flows become identical at steady state;
The UCM model takes much longer to reach steady-state than the Newtonian fluid;
The global plot shows an accurate result for the oscillatory response of velocity to instantaneously applied pressure gradient; the analytical velocity profile at the centreline has discontinuities in derivative that result in some numerical oscillations seen in the local plots.
Results:
UCM fluid – $\beta=\lambda_r/\lambda=0$ (start-up Poiseuille flow)

- Increasing $E$ leads to an increase in oscillatory frequency and amplitude.
- The error in frequency is amplified as time proceeds eventually leading to an erroneous result.
Results:
Oldroyd-B fluid — $\beta = \lambda_r / \lambda = 1/9$ (start-up Poiseuille flow)

- Good agreement between analytic and numerical results, thus demonstrating improved accuracy when some solvent viscosity is present.
- Smooth development of the transient evolution and spatial variation of the flow field (free of physical “shocks” and numerically induced oscillations).
Results:
PTT fluid (start-up Poiseuille flow)

- Similar behaviour to the UCM fluid (in both $\eta_s=0$);
- Steady state is more rapidly achieved (a consequence of shear-thinning effect);
- The evolution of velocity with time changes with the extensibility parameter ($\varepsilon$) $\varepsilon \rightarrow 0 \Rightarrow \text{PTT} \rightarrow \text{UCM}$;
Results: Newtonian Fluid (Pulsating flow)

- Very good agreement between theoretical and numerical results under steady conditions.
- During the cycle there is also a good agreement between the theoretical and numerical solution.

Note: $k_{osc}/K_{steady} = 2.6; \quad \alpha = 4.9$
Results:
Newtonian vs UCM (Pulsating flow)

- During the oscillation period the two fluids have very different behaviour;
- Both the amplitude and phase of the oscillation differ when going from a Newtonian fluid to a viscoelastic one;
Results:
UCM Fluid (Pulsating flow)

- A much finer mesh was required (NY=1000);
- Hard to find accurate numerical solutions when comparing with the Oldroyd-B case.
Results:
Oldroyd-B fluid – $\beta=\lambda_r/\lambda=0.1, 0.01$ (Pulsating flow)
Results:
Oldroyd-B fluid – $\beta = \lambda_r/\lambda = 0.005, 0.001$ (Pulsating flow)

- As the $\beta$ value goes to 0 (i.e., tends towards the UCM model) the numerical solution is less accurate when compared with the analytical.
Conclusions

- The finite volume code used (fully 3-D), allows a relatively accurate description of Newtonian and viscoelastic fluids (UCM and Oldroyd-B models) for the transient cases tested;
- The discretization errors can be minimized by choosing optimal steps in time and mesh refinement.

**Start-up Poiseuille Flow**
- A comparison between Newtonian and UCM fluids shows different behaviour during the transient before reaching steady-state. It was also verified that the Maxwell fluid takes longer to reach steady state;
- **UCM Fluid**: Observation of small numerical oscillations when the time derivative is discontinuous;
- **Oldroyd-B Fluid**: Smooth development of the transient evolution and of the spatial variation of the flow field.

**Pulsating Flow**
- Good agreement between the theoretical and numerical solution of the Newtonian fluid for the steady state and during the cycle;
- Newtonian and viscoelastic fluids show very different behaviour;
- **UCM Fluid**: Difficulty in obtaining numerical solutions – needs extremely refined mesh;
- **Oldroyd-B Fluid**: No trouble in obtaining good results with this model and improved accuracy as $\beta$ becomes larger.
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