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Transient viscoelastic flows: detailed analysis of start-up and pulsating flows

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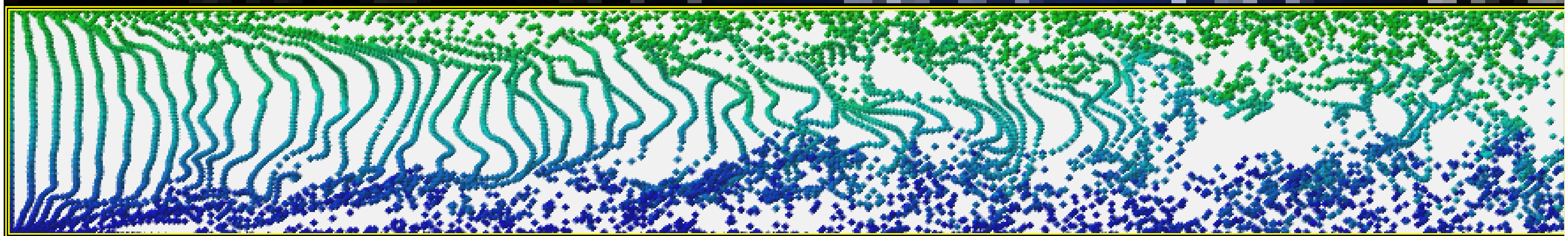

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Abstract

- Actual flows of interest often occur in variable regimes (e.g. injection and blow moulding, blood flow, …).

- A general numerical simulation code, based on the finite volume formulation is assessed in “simple” unsteady cases.

- Test Problems: - Start-up planar Poiseuille flow;

  - Pulsating flow.

Comparison between analytical and numerical results for Newtonian, UCM and Oldroyd-B fluids.

Governing Equations

Conservation Equations

Mass

$$\nabla \cdot \boldsymbol{u} = 0 \Leftrightarrow$$

$$\frac{\partial u}{\partial x} = 0; \; v = 0$$

Momentum

Newtonian $\rho \dfrac{\partial u}{\partial t} = \nu \dfrac{\partial^2 u}{\partial y^2} - \dfrac{dp}{dx}$

Viscoelastic $\rho \dfrac{\partial u}{\partial t} = \dfrac{\partial \tau_{xy}}{\partial y} - \dfrac{dp}{dx}$

Constitutive Models

UCM

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta_0 \boldsymbol{D}$$

Oldroyd-B

$$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\eta_0 \left(\boldsymbol{D} + \lambda_r \overset{\nabla}{\boldsymbol{D}} \right)$$

Numerical Method

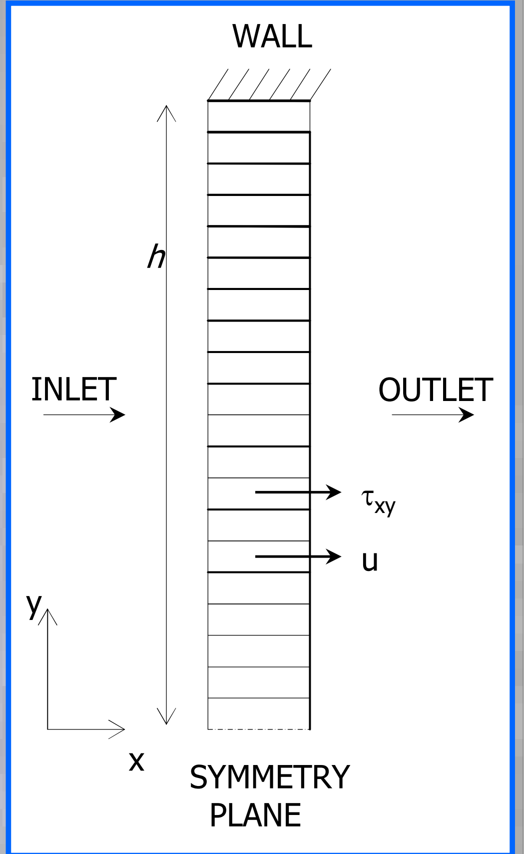
- FINITE VOLUME METHOD (FVM)

  - Spatial discretization (over control volumes, Δy)

    → Convective fluxes vanish, thus simplifying the problem

  - Temporal discretization (over time step, Δt)

    → Three time level method (2nd order accuracy; important)

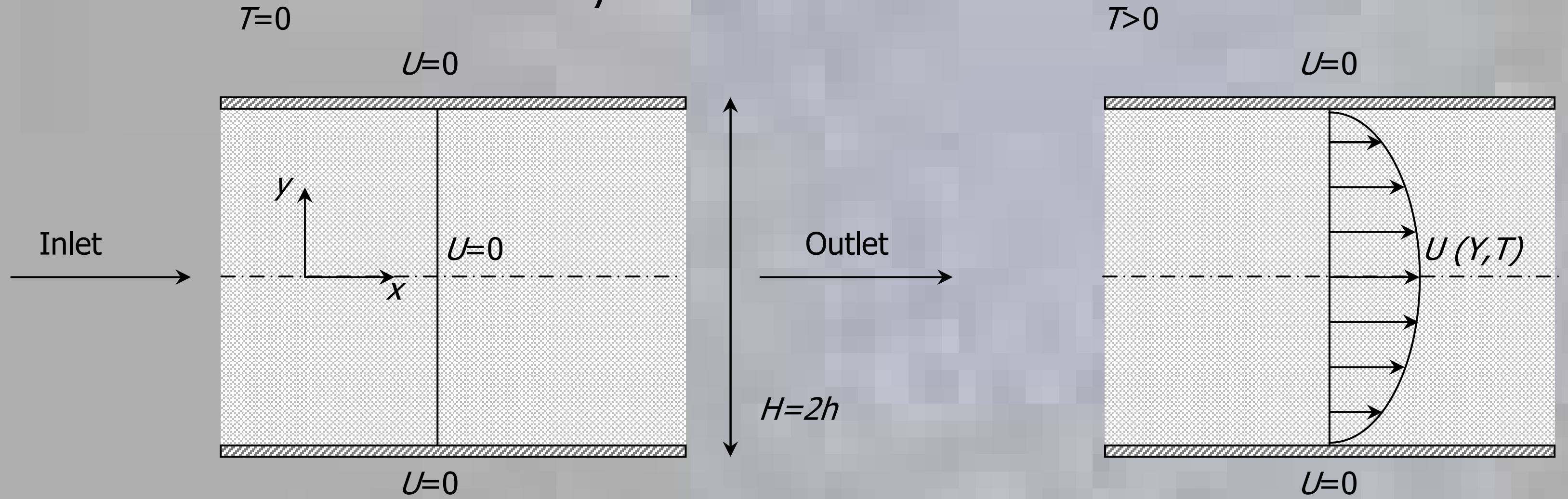


- Pressure/velocity coupling according to Rhie and Chow (actually, not needed);
- Velocity/tension coupling according to Oliveira *et al*. (1998)

Problem Description

- ***Start-up Poiseuille Flow***: A transient flow resulting from sudden application of a spatially constant pressure gradient to a fluid initially at rest.
- ***Pulsating Flow***: A periodic flow resulting from application of a pressure gradient which varies in a sinusoidal manner in time:

$$-\frac{1}{\rho}\frac{dp}{dx} = K_e + K_0\cos(\omega t)$$

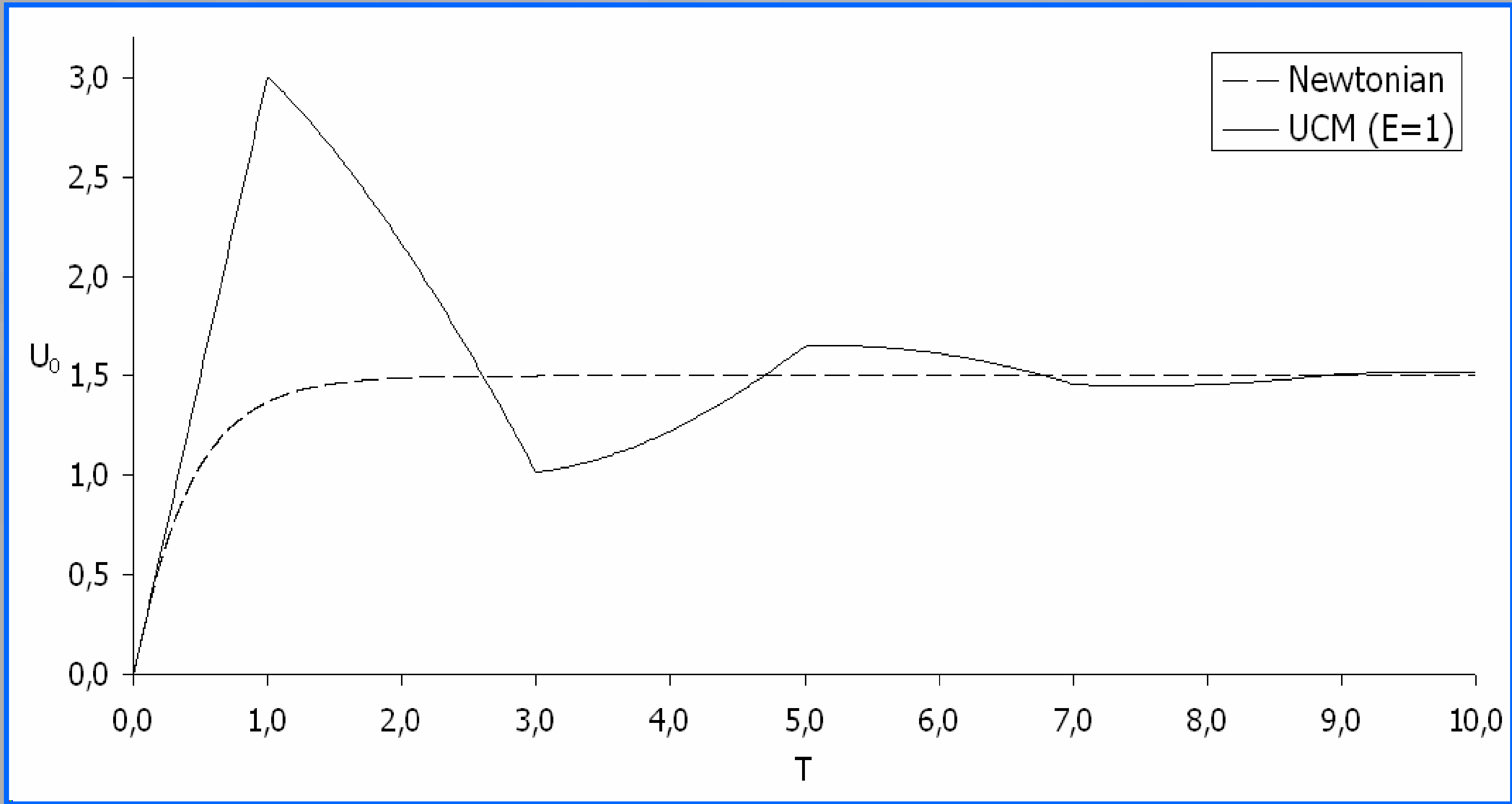


- The calculations were carried out for different values of the elastic number *E*.

$$E = \frac{We}{\mathrm{Re}} = \frac{\lambda\eta_0}{\rho h^2}$$

Results:

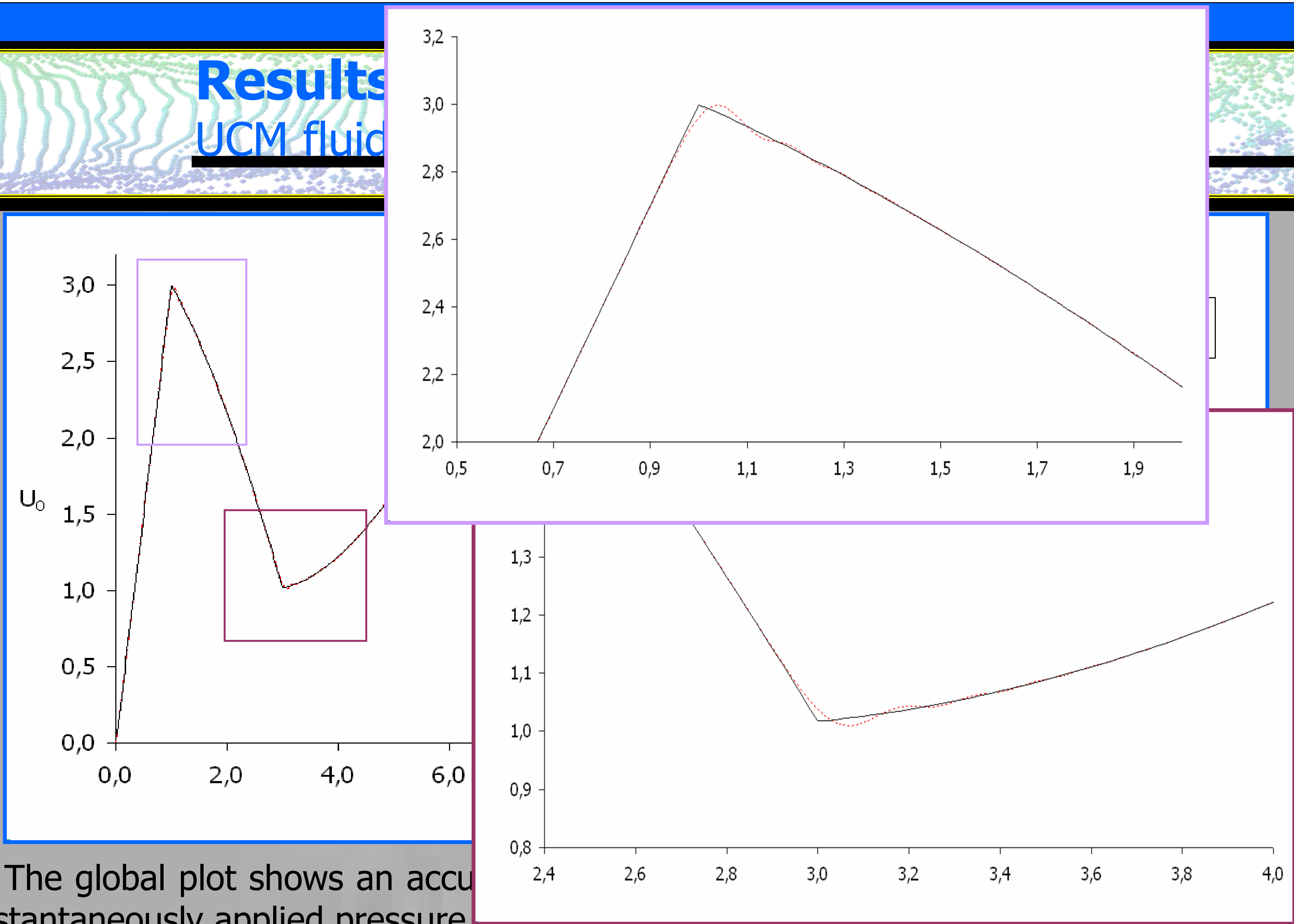
Newtonian *vs* UCM (start-up Poiseuille flow)



- During the transient period the two fluids have very different behaviour;
- Both flows become identical at steady state;
- The UCM model takes much longer to reach steady-state than the Newtonian fluid;

Results

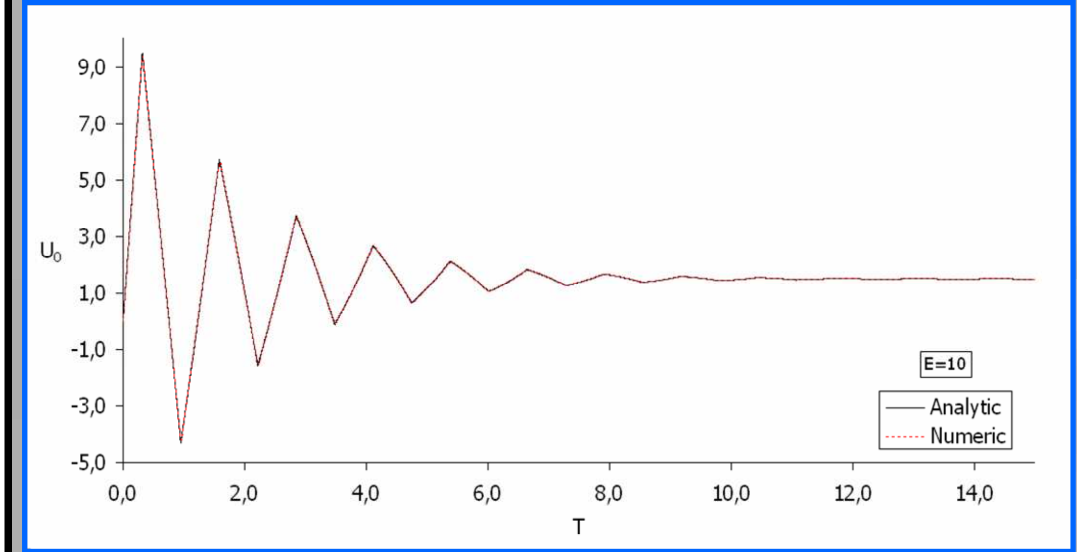
UCM fluid



- The global plot shows an accu... instantaneously applied pressure gradient;
- The analytical velocity profile at the centreline has discontinuities in derivative that result in some numerical oscillations seen in the local plots.

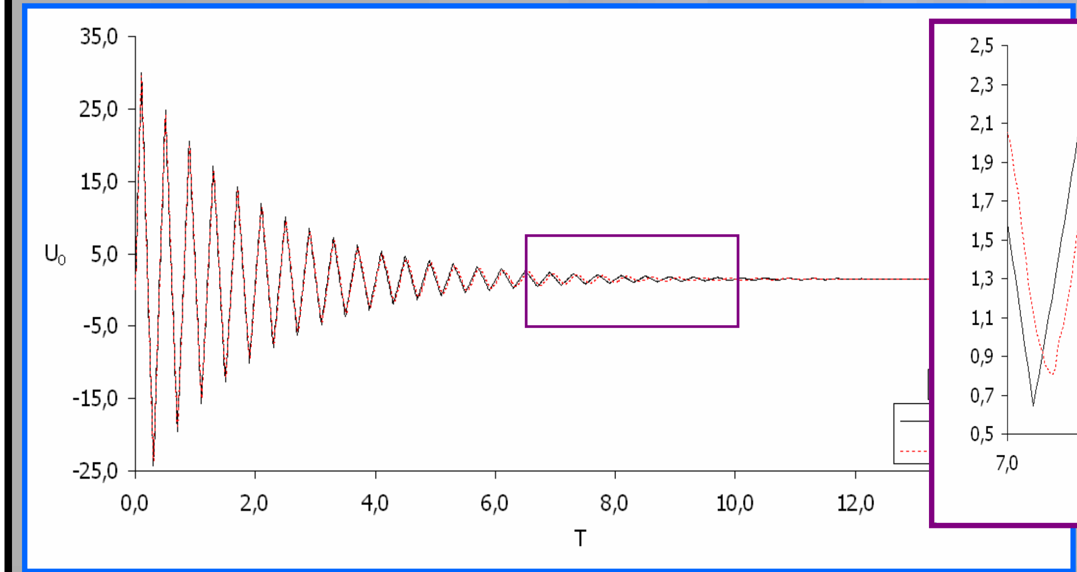
Results:

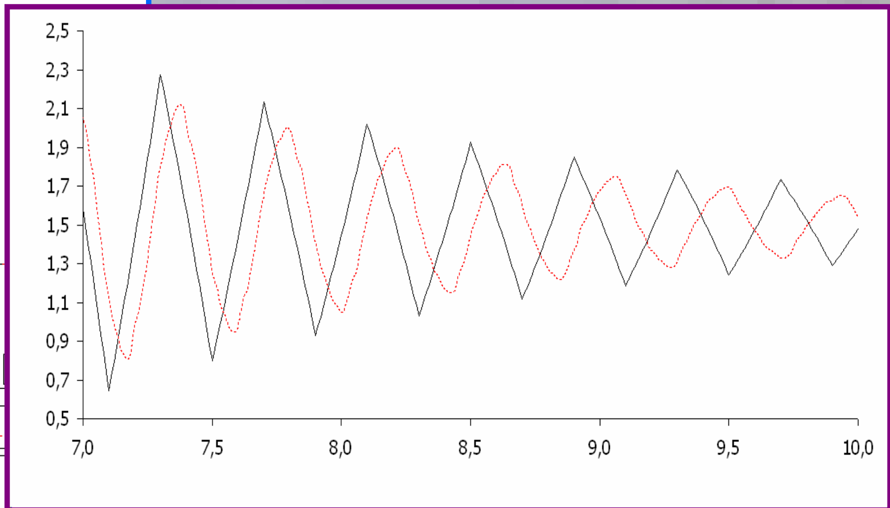
UCM fluid – $\beta=\lambda_r/\lambda=0$ (start-up Poiseuille flow)



- Increasing E leads to an increase in oscillatory frequency and amplitude.

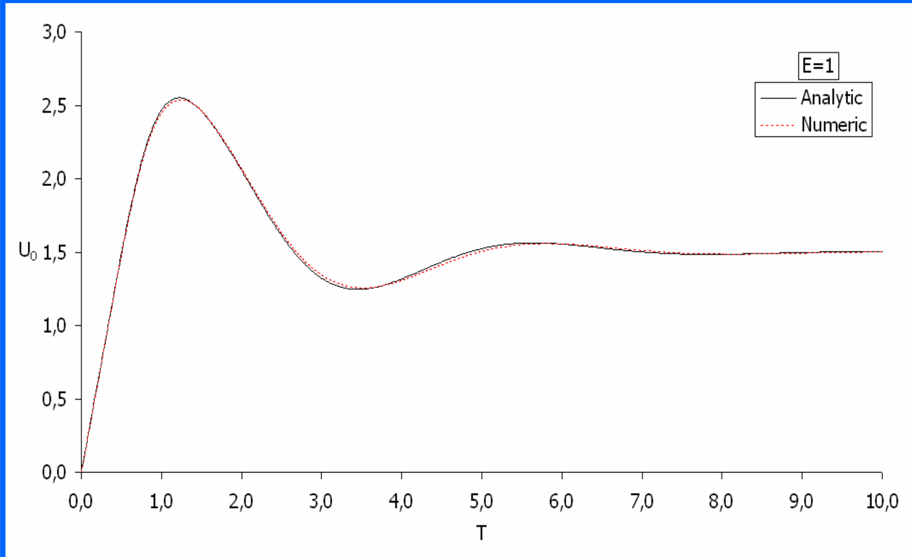
- The error in frequency is amplified as time proceeds eventually leading to an erroneous result.

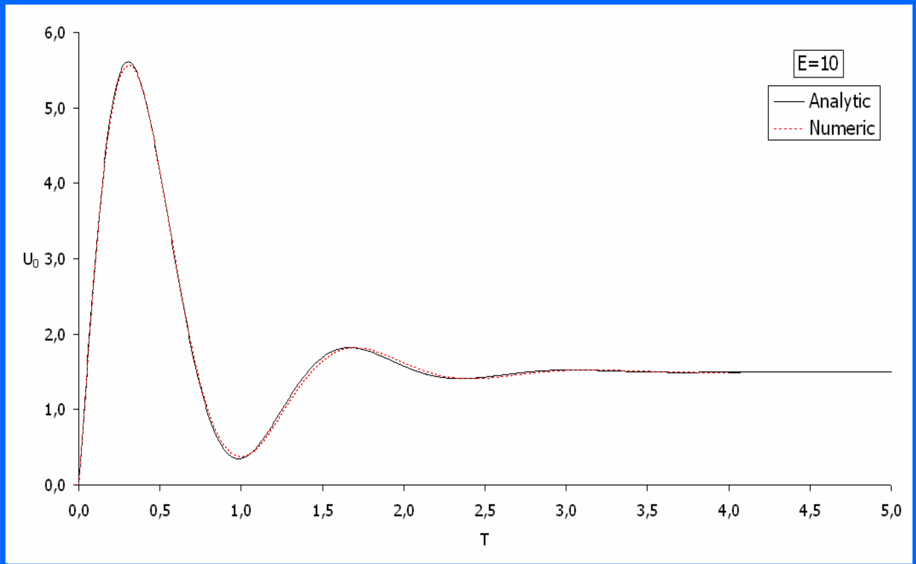


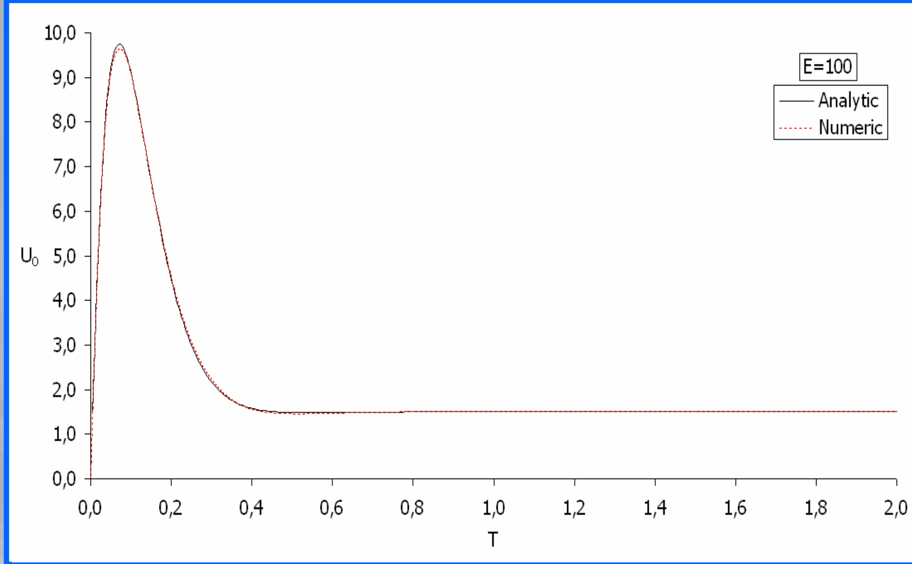


Results:

Oldroyd-B fluid – $\beta=\lambda_r/\lambda=1/9$ (start-up Poiseuille flow)

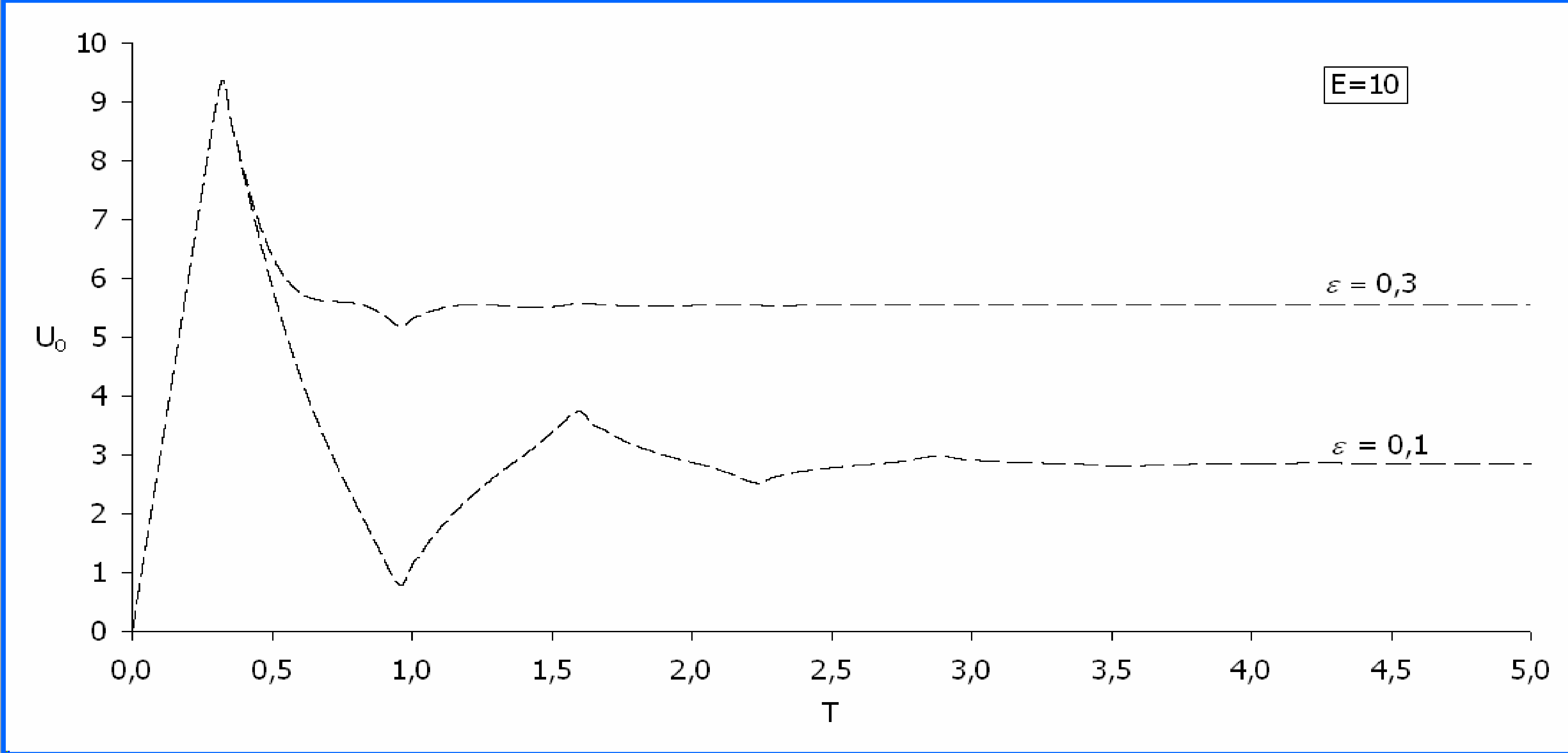






- Good agreement between analytic and numerical results, thus demonstrating improved accuracy when some solvent viscosity is present.
- Smooth development of the transient evolution and spatial variation of the flow field (free of physical "shocks" and numerically induced oscillations).

Results:
PTT fluid (start-up Poiseuille flow)

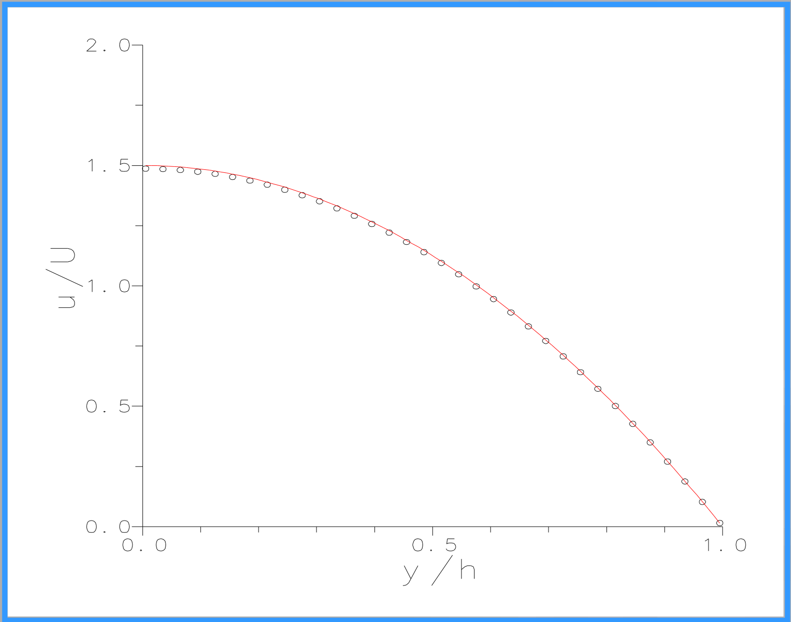


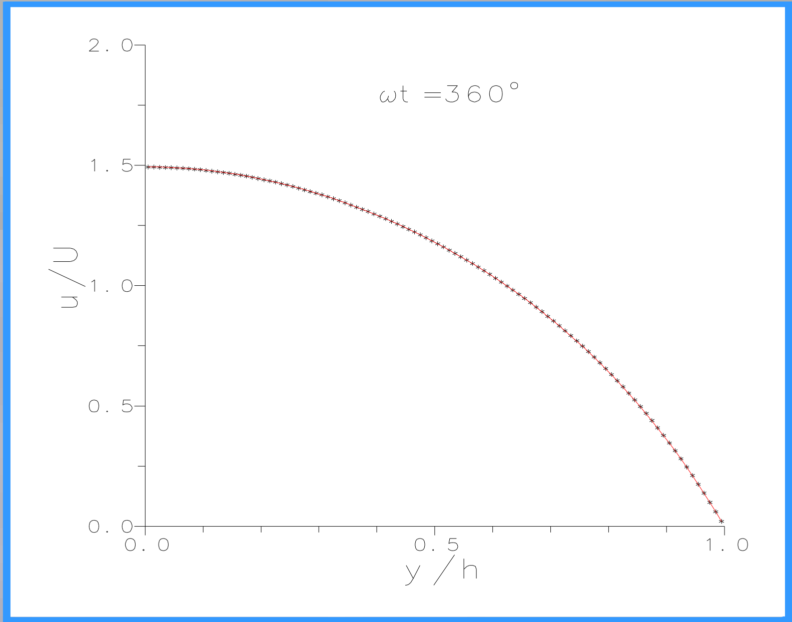
- Similar behaviour to the UCM fluid (in both η_s=0);
- Steady state is more rapidly achieved (a consequence of shear-thinning effect) ;
- The evolution of velocity with time changes with the extensibility parameter (ε)

$\varepsilon \rightarrow 0$ => PTT $\rightarrow$ UCM;

Results:

Newtonian Fluid (Pulsating flow)



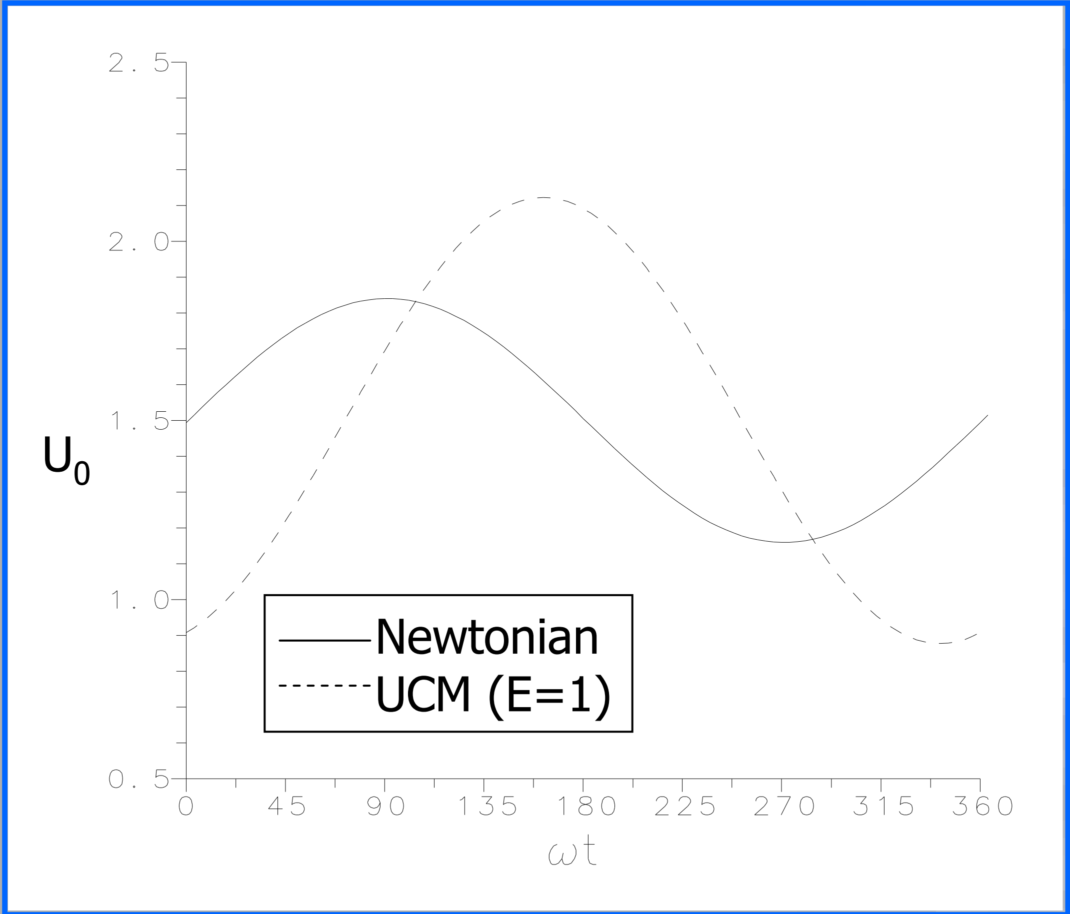


- Very good agreement between theoretical and numerical results under steady conditions.

- During the cycle there is also a good agreement between the theoretical and numerical solution.

Note: k_{osc} /K_{steady}= 2.6; α = 4.9

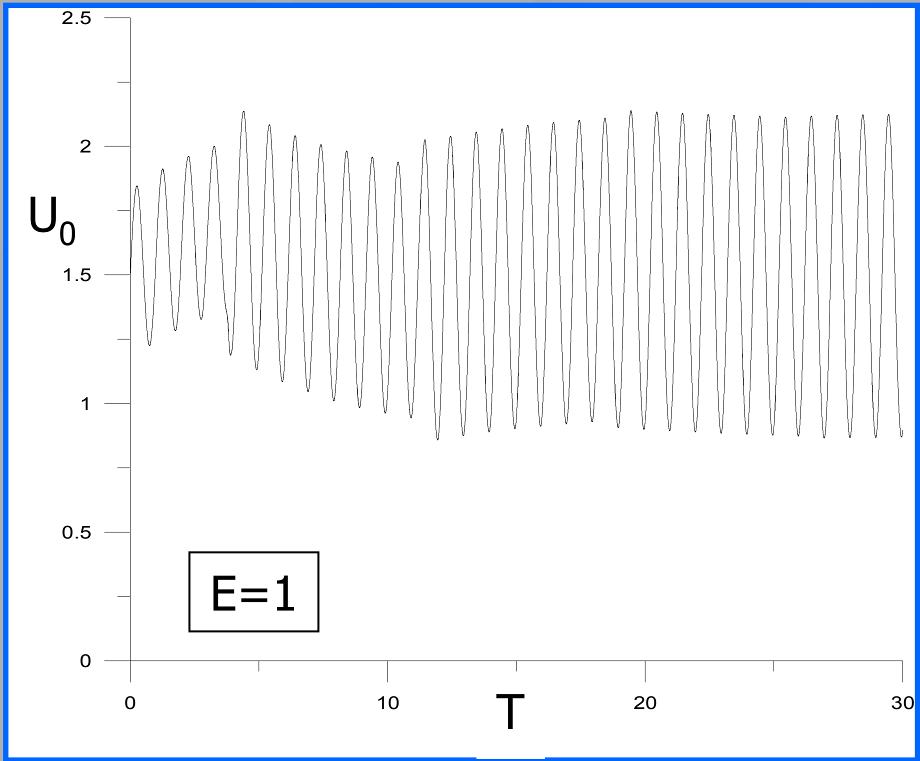
Results:
Newtonian *vs* UCM (Pulsating flow)

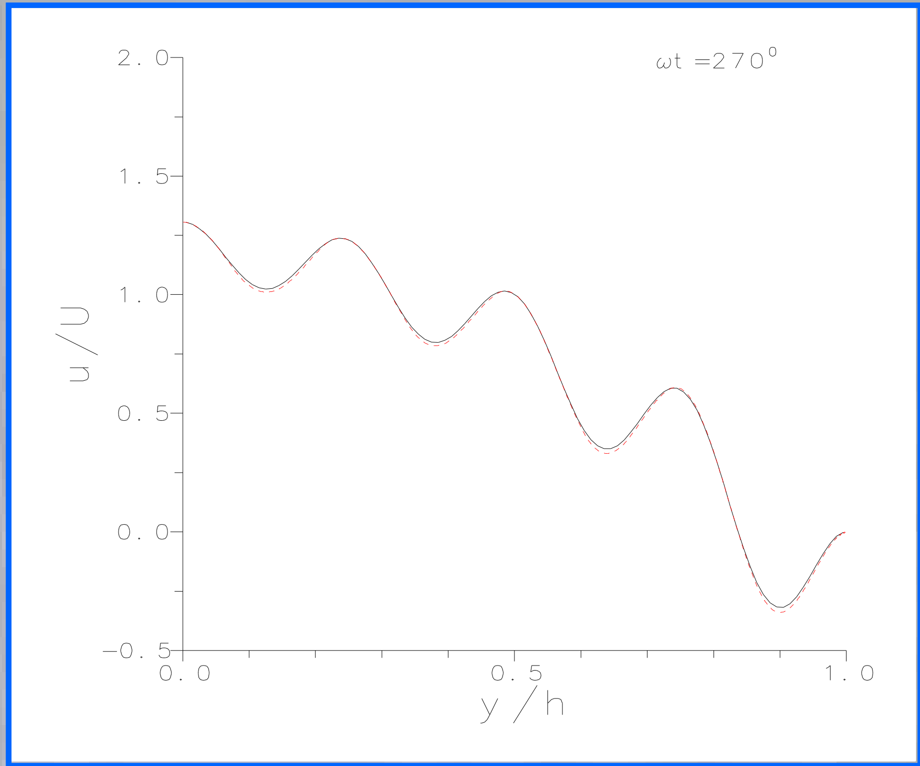


- During the oscillation period the two fluids have very different behaviour;

- Both the amplitude and phase of the oscillation differ when going from a Newtonian fluid to a viscoelastic one;

Results:
UCM Fluid (Pulsating flow)

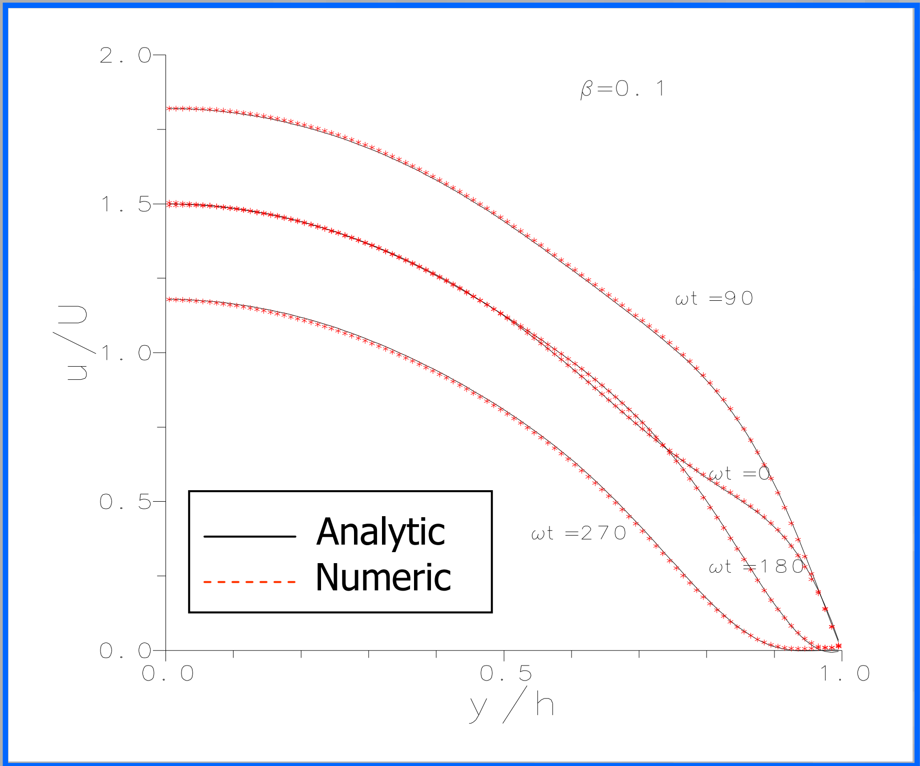


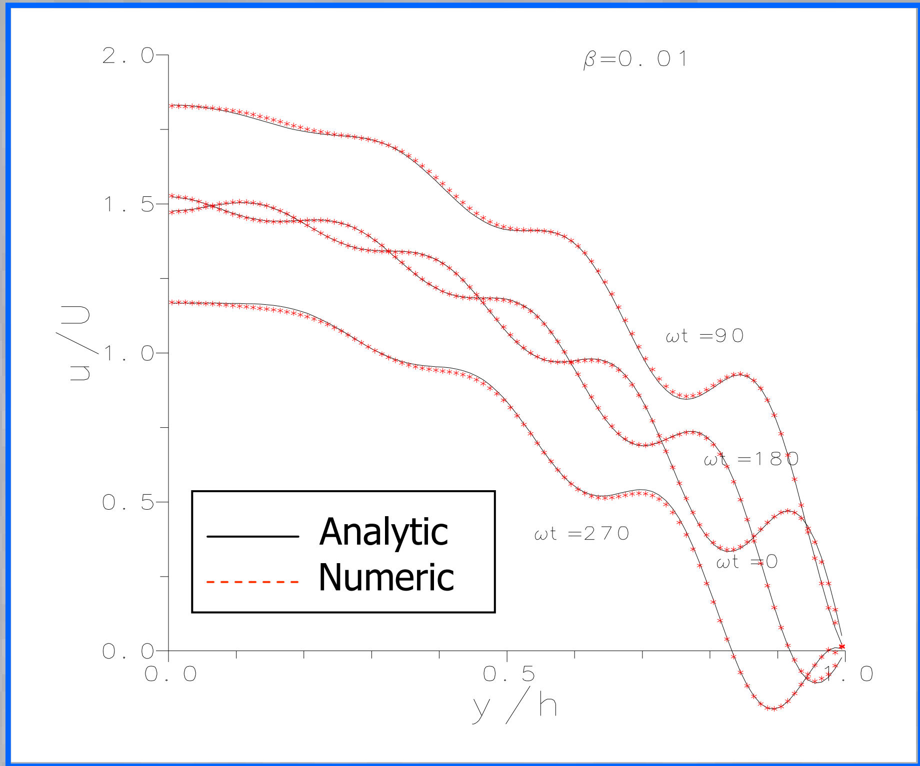


- A much finer mesh was required (NY=1000);
- Hard to find accurate numerical solutions when comparing with the Oldroyd-B case.

Results:

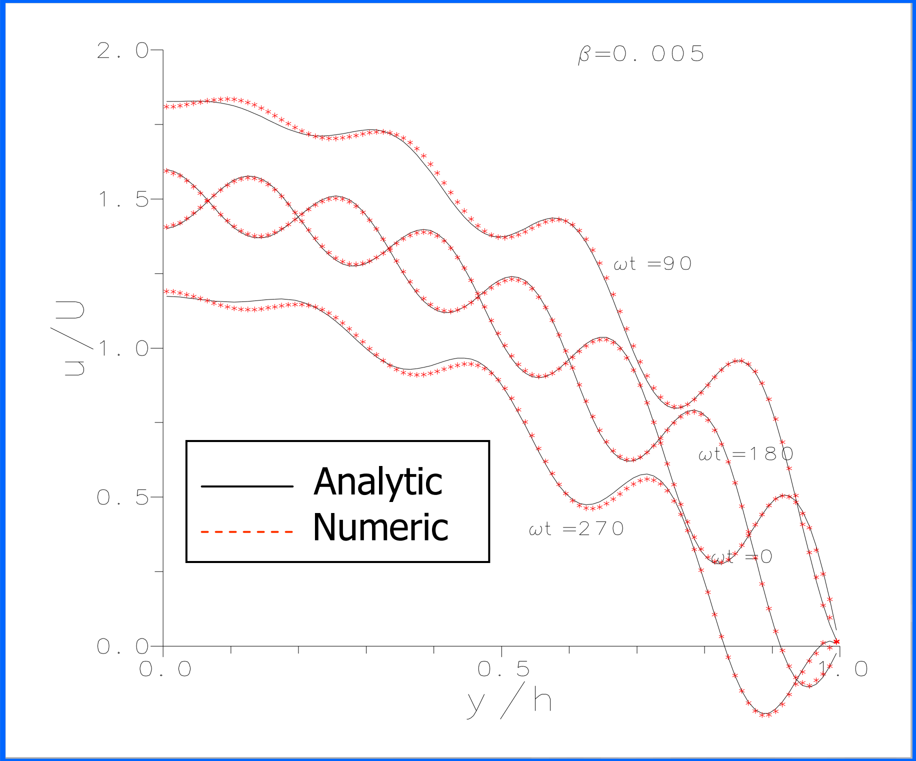
Oldroyd-B fluid – $\beta=\lambda_r/\lambda=0.1, 0.01$ (Pulsating flow)

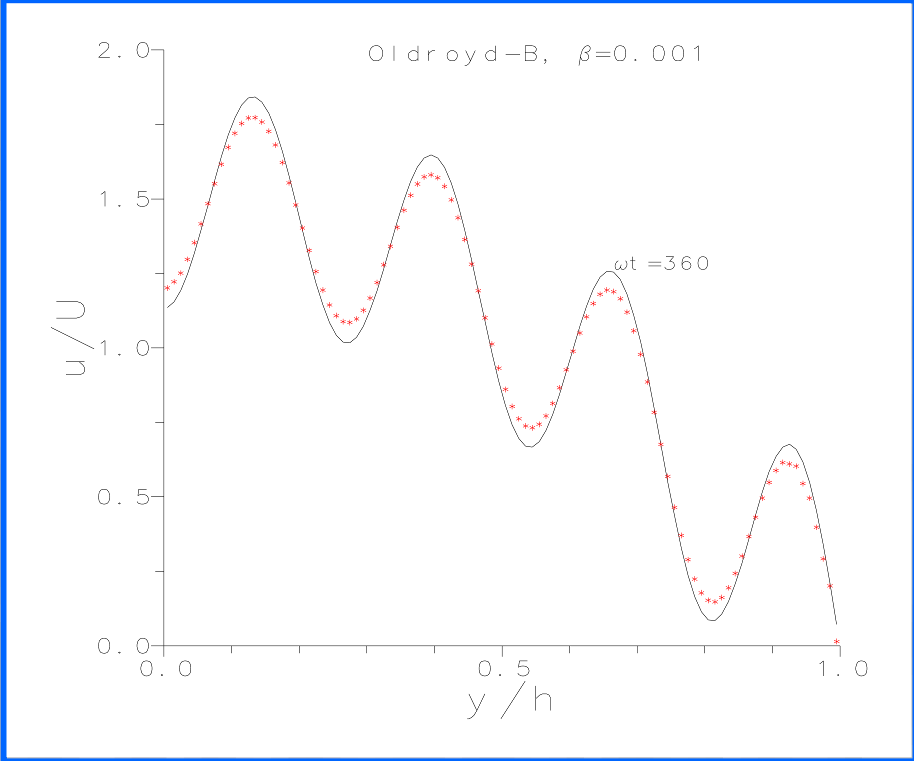




Results:

Oldroyd-B fluid – β=λr/λ=0.005, 0.001 (Pulsating flow)





- As the β value goes to 0 (i.e., tends towards the UCM model) the numerical solution is less accurate when compared with the analytical.

Conclusions

- The finite volume code used (fully 3-D), allows a relatively accurate description of Newtonian and viscoelastic fluids (UCM and Oldroyd-B models) for the transient cases tested;
- The discretization errors can be minimized by choosing optimal steps in time and mesh refinement.

Start-up Poiseuille Flow

- A comparison between Newtonian and UCM fluids shows different behaviour during the transient before reaching steady-state. It was also verified that the Maxwell fluid takes longer to reach steady state;
- UCM Fluid: Observation of small numerical oscillations when the time derivative is discontinuous;
- Oldroyd-B Fluid: Smooth development of the transient evolution and of the spatial variation of the flow field.

Pulsating Flow

- Good agreement between the theoretical and numerical solution of the Newtonian fluid for the steady state and during the cycle;
- Newtonian and viscoelastic fluids show very different behaviour;
- UCM Fluid: Difficulty in obtaining numerical solutions – needs extremely refined mesh;
- Oldroyd-B Fluid: No trouble in obtaining good results with this model and improved accuracy as β becomes larger.

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