

# **SIMULATION STUDY OF FENE-CR FLOWS IN TWO-DIMENSIONAL SUDDEN EXPANSIONS AT LOW-TO-MODERATE REYNOLDS NUMBERS**

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# OBJECTIVES

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- 📖 Numerical analysis of viscoelastic fluid flow in a planar expansion ( $E = D/d = 4$ ).
- 📖 Comparison between Newtonian and Non-Newtonian (viscoelastic) fluids.
  - ▶ Effect of elasticity;
  - ▶ Effect of inertia;
  - ▶ Effect of polymer concentration.
- 📖 Application the **FENE-CR\* model** in the analysis of the viscoelastic flow (no shear-thinning).
- 📖 Utilization the **Finite Volume Method (FVM)** for the discretization equations, using the High Resolution Schemes (*HRS*).

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\* **FENE-CR – Finitely Extensible Nonlinear Elastic – Chilcott and Rallison (1988)**

# EQUATIONS

 **Mass:**

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

 **Momentum:**

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \cdot \boldsymbol{\tau}_{\text{tot}} \quad (2)$$

Newtonian solvent

Polimeric solution

$$\boldsymbol{\tau}_{\text{tot}} = \underbrace{\boldsymbol{\tau}_s}_{\text{Newtonian solvent}} + \underbrace{\boldsymbol{\tau}}_{\text{Polimeric solution}} \quad (3)$$

 **Constitutive: Modified FENE-CR Model (FENE-MCR)**

$$\boldsymbol{\tau} + \frac{\lambda}{f(\boldsymbol{\tau})} \overset{\nabla}{\boldsymbol{\tau}} = 2\eta_P \mathbf{D} \quad (4)$$

$$f(\boldsymbol{\tau}) = \frac{L^2 + (\lambda/\eta_P) \text{tr}(\boldsymbol{\tau})}{L^2 - 3} \quad (5)$$

# DIMENSIONLESS GROUPS

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The relevant dimensionless groups to be varied in a parametric way are:

 The extensibility parameter of the FENE-CR model:

$$L^2 = 100 \quad (6)$$

 The Reynolds number:

$$Re = \frac{\rho U d}{\eta_0} \quad (7)$$

 The Weissenberg number:

$$We = \frac{\lambda U}{d} \quad (8)$$

 and the solvent viscosity ratio:

$$\beta = \frac{\eta_s}{\eta_0} \quad (9)$$

# GEOMETRY

## DIMENSIONS

$$d = 1$$

$$D = 4$$

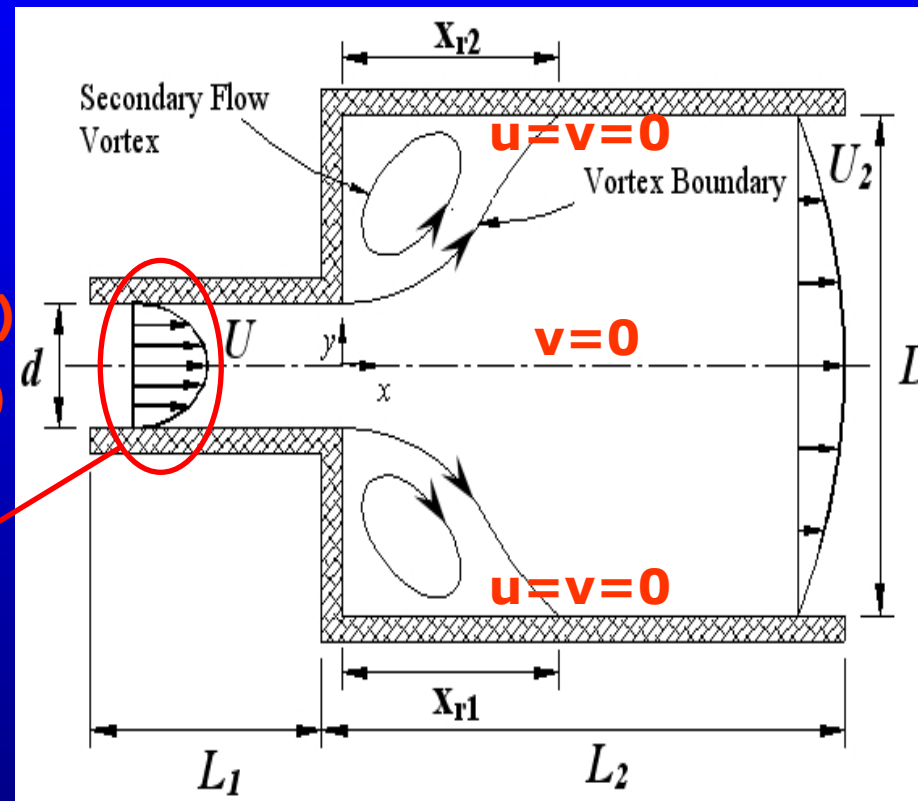
$$L_1 = 20d$$

$$L_2 = 50d$$

Inlet Conditions:

Fully developed  
Profile

$$u(y)$$
$$v=0$$



Wall Conditions:

No slip conditions:  $u=v=0$

Outlet Conditions:

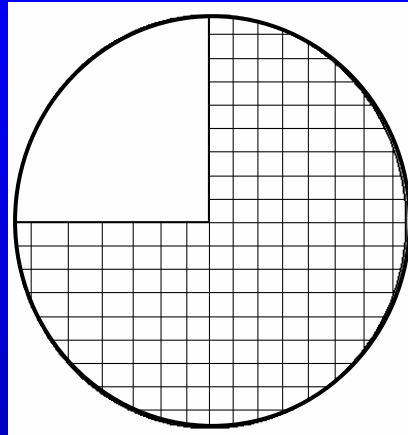
Vanishing  
axial variation  
for all  
quantities

$$\frac{\partial}{\partial x} \equiv 0$$

Except  
pressure  
which was  
linearly  
extrapolated  
from the  
inside

# NUMERICAL METHOD

## FINITE VOLUME METHOD (FVM).

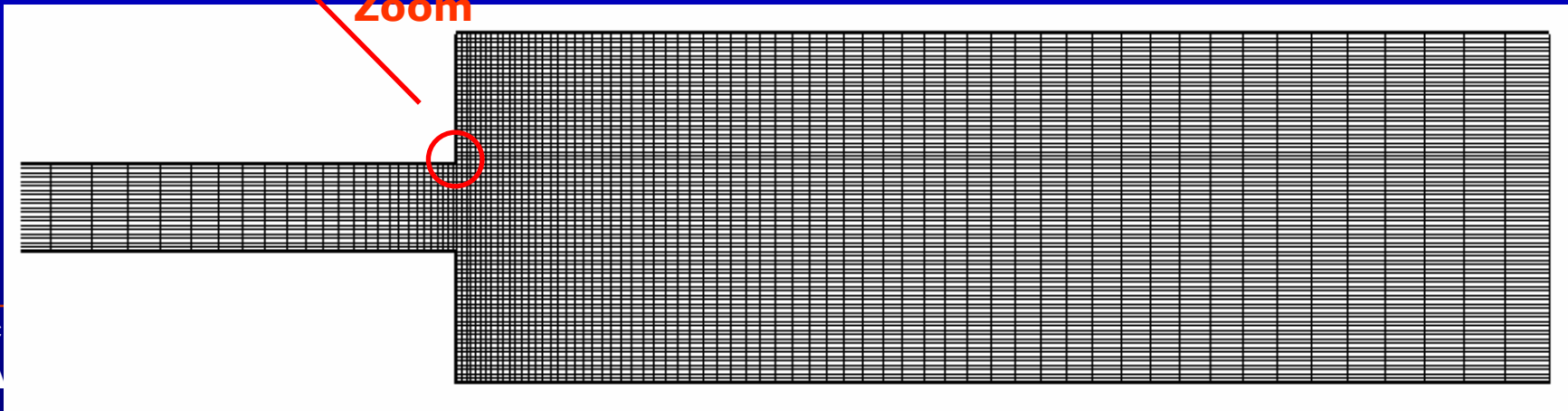


- For discretization equations.
- Non uniform orthogonal mesh.

**Pressure/velocity coupling** is followed by Rhie method (1983)

**Velocity/stress coupling** is followed by Oliveira et al. method (1998).

Zoom



\*  
A

of

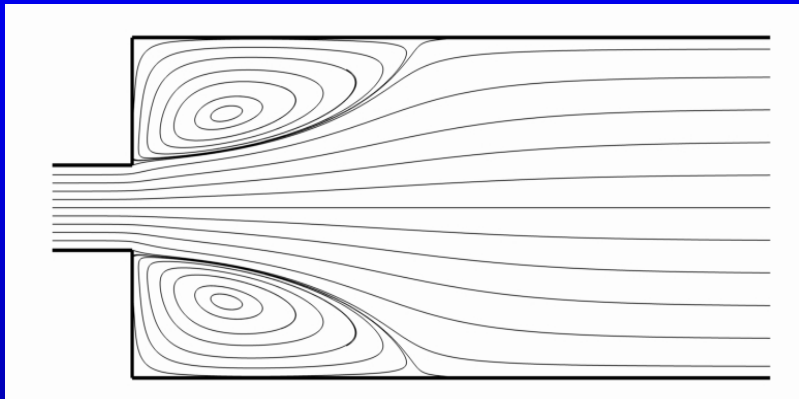
\*\* Semi-Implicit Method for Pressure-Linked Equations - Consistent (Van Doormaal and Raithby (1984)).

$$L_1 = 20d \text{ e } L_2 = 50d$$

# RESULTS *Effect of elasticity (We)*

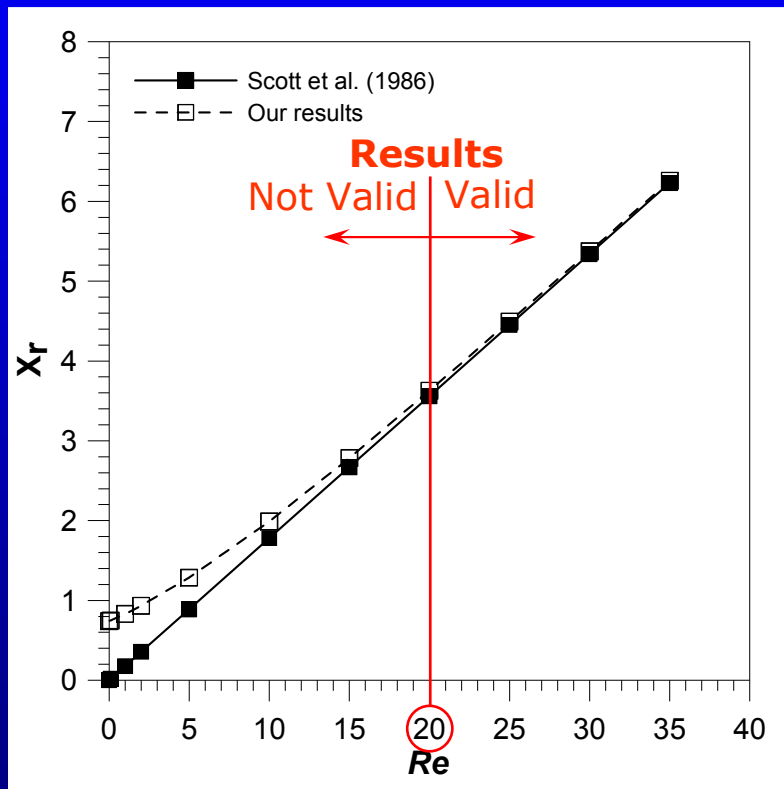
**$Re = 20; L^2 = 100; \beta = 0.5$**

**Newtonian:  $We = 0$**

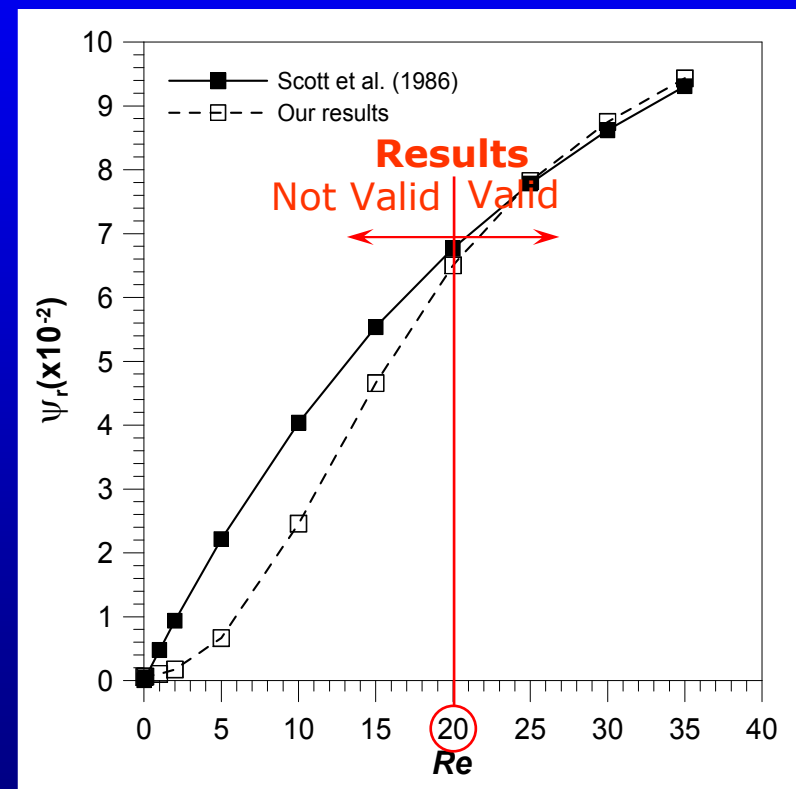


# Comparison our numerical data with the correlations proposed by Scott et al. (1986).

## VALIDATION



Vortex length ( $X_r$ )  
versus  $Re$  number.



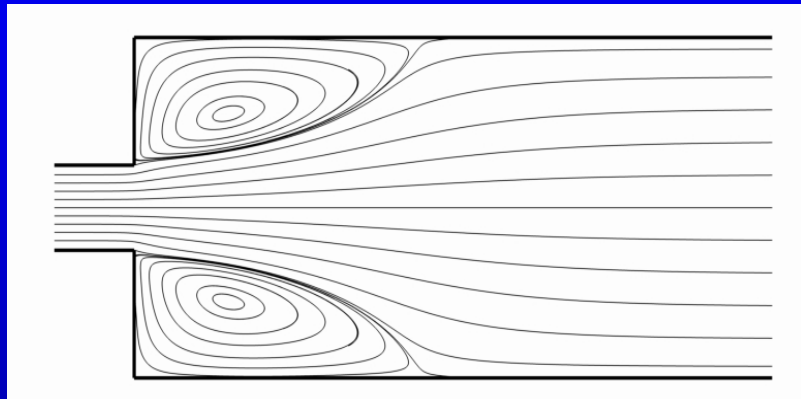
Vortex Intensity ( $\psi_r$ )  
versus  $Re$  number.



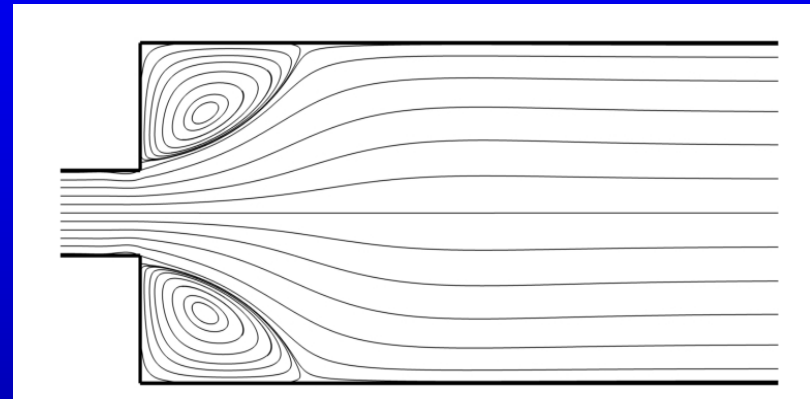
# RESULTS *Effect of elasticity (We)*

$$Re = 20; L^2 = 100; \beta = 0.5$$

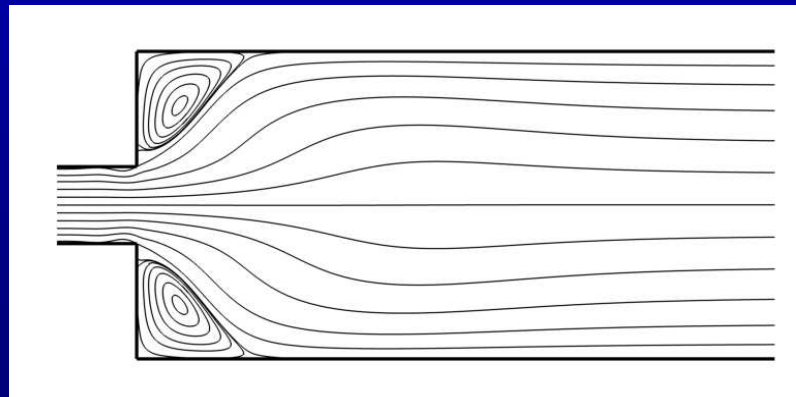
**Newtonian:  $We = 0$**



**Viscoelastic:  $We = 2$**



**Viscoelastic:  $We = 5$**

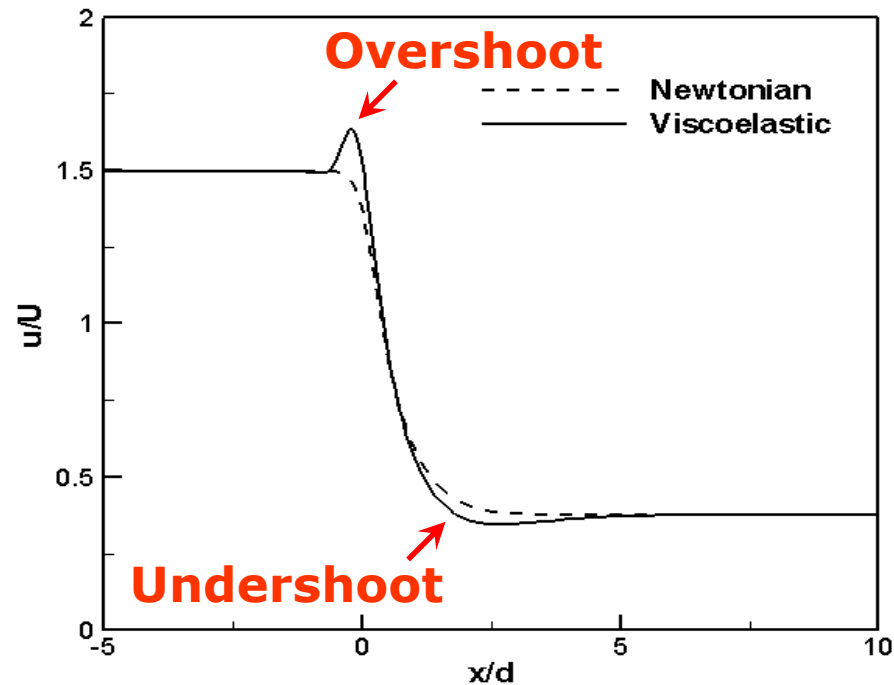


**Bulging  
of  
Streamlines**

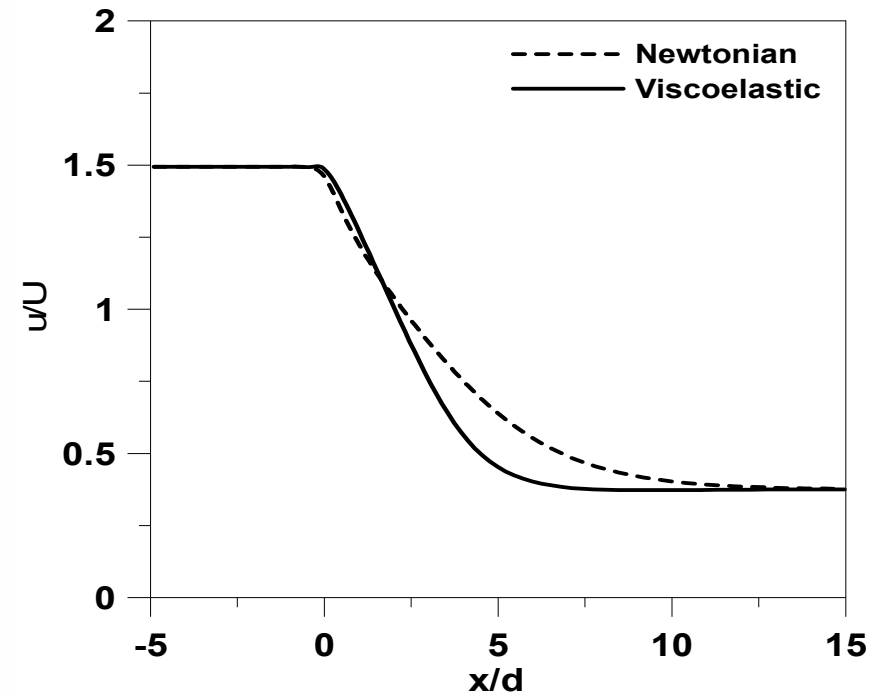
## Variation of streamwise velocity component along the central plane ( $y=0$ ).

$$L^2 = 100, We = 2, \beta = 0.5$$

$$Re = 0.01$$



$$Re = 35$$



# RESULTS *Effect of inertia (Re)*

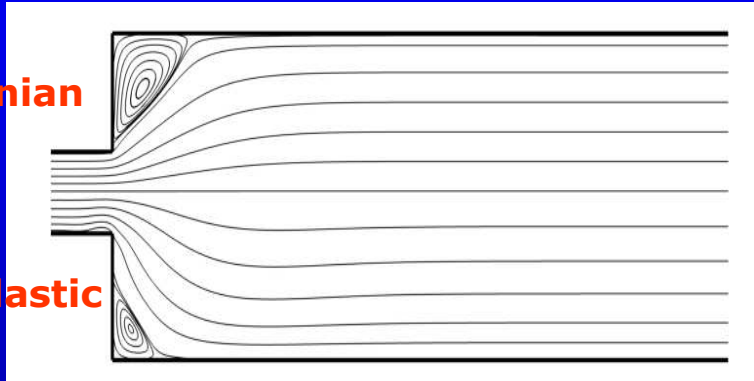
**Newtonian/Viscoelastic case**

$We = 2; L^2 = 100; \beta = 0.5$

**Newtonian  $Re = 5$**

Newtonian

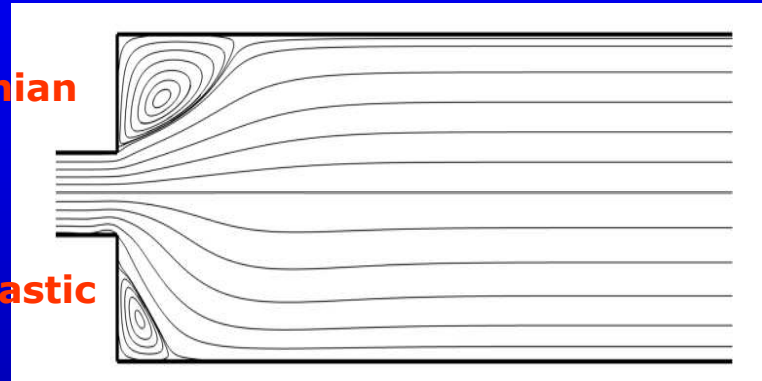
Viscoelastic



**Newtonian  $Re = 10$**

Newtonian

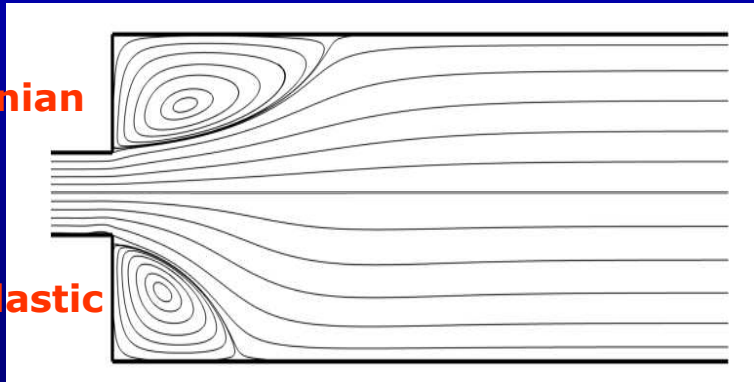
Viscoelastic



**Newtonian  $Re = 20$**

Newtonian

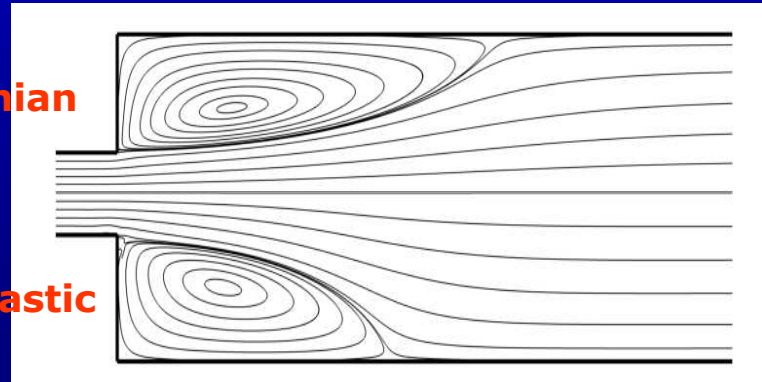
Viscoelastic



**Newtonian  $Re = 35$**

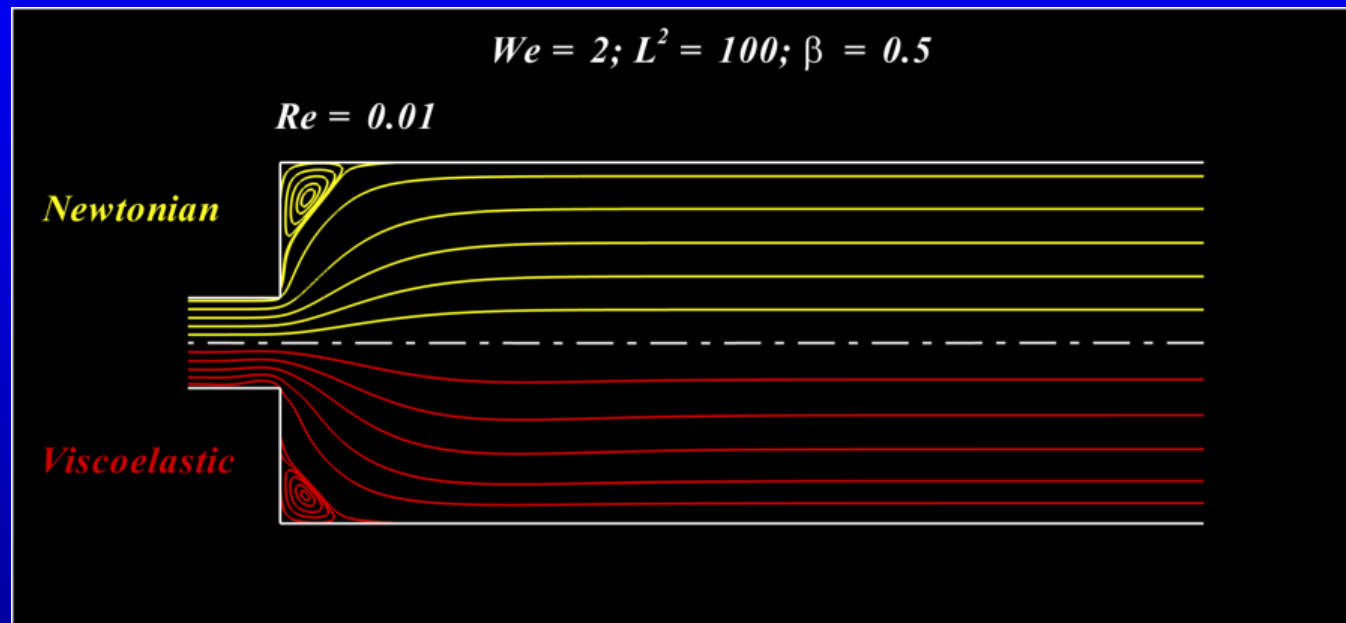
Newtonian

Viscoelastic

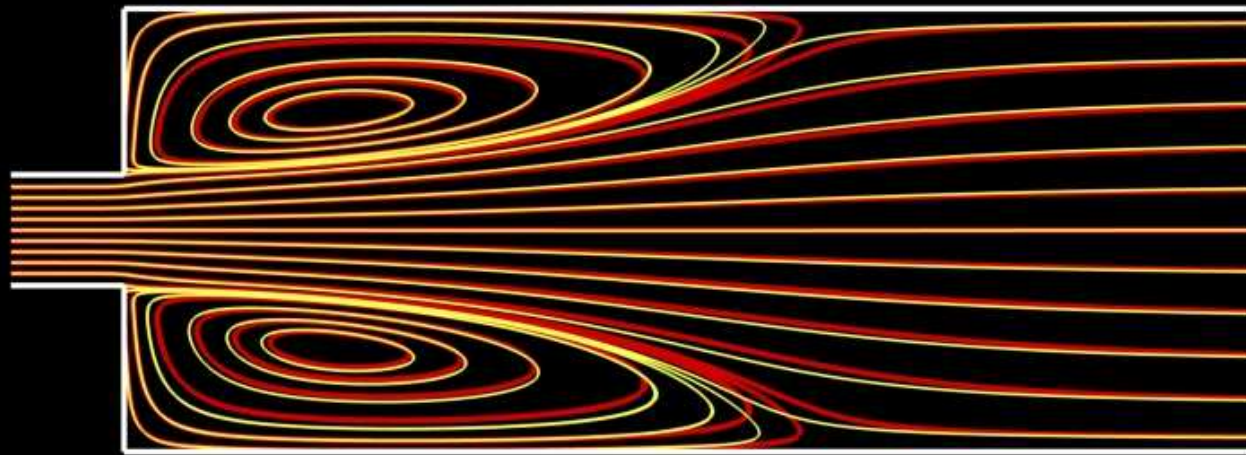


# RESULTS *Effect of inertia (Re)*

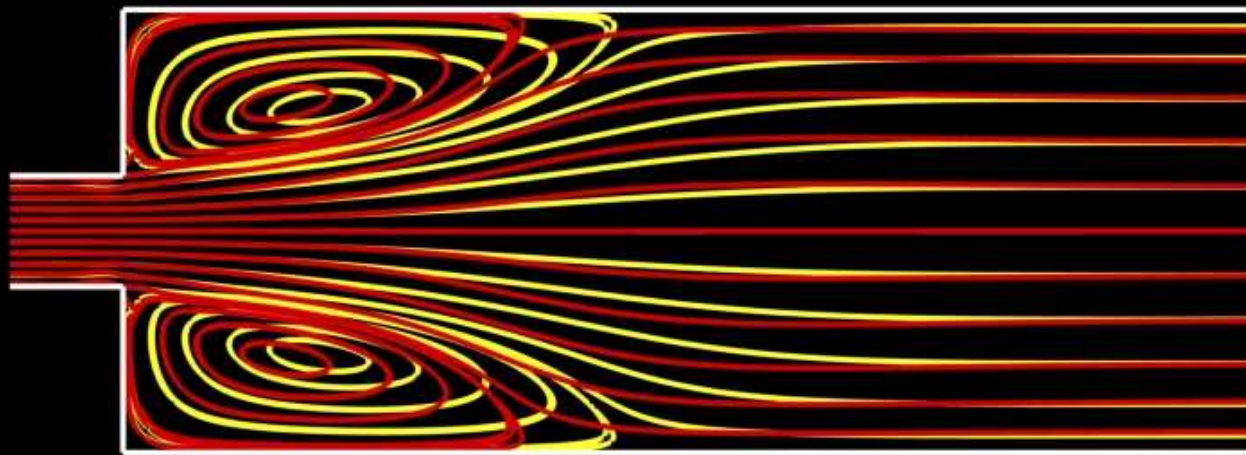
## Newtonian/Viscoelastic case



# RESULTS *Effect of concentration ( $\beta$ )*



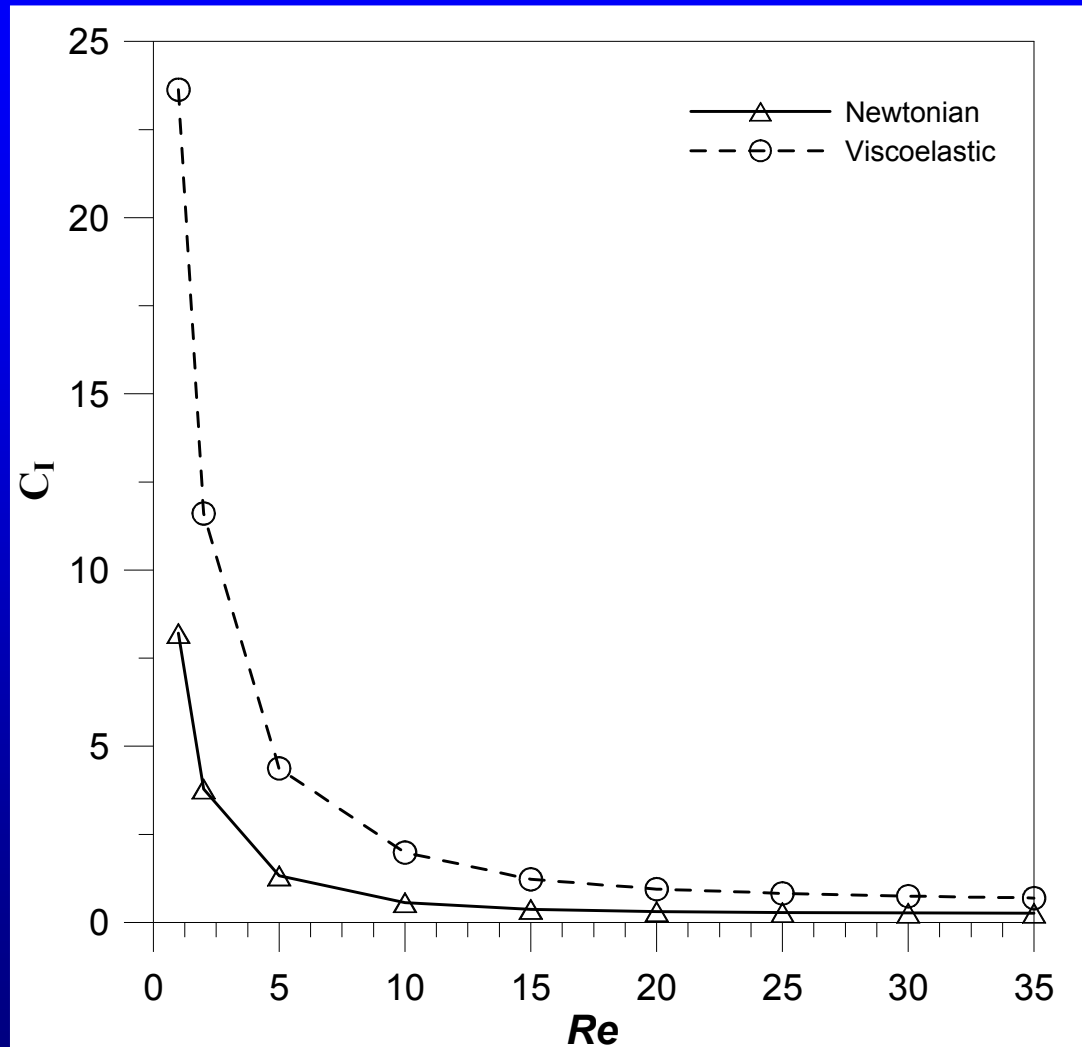
— Newtonian Case  
 $Re = 35$   
— Viscoelastic Case  
 $Re = 35$   
 $We = 2; L^2 = 100$   
 $\beta = 0.9$



$Re = 35$   
— Viscoelastic Case  
 $We = 2; L^2 = 100$   
 $\beta = 0.5$   
— Viscoelastic Case  
 $We = 2; L^2 = 100$   
 $\beta = 0.3$

## Variation of the localized loss coefficients with Reynolds number.

$$C_I = \frac{\Delta p - \Delta p_F - \Delta p_R}{\frac{1}{2} \rho U^2}$$

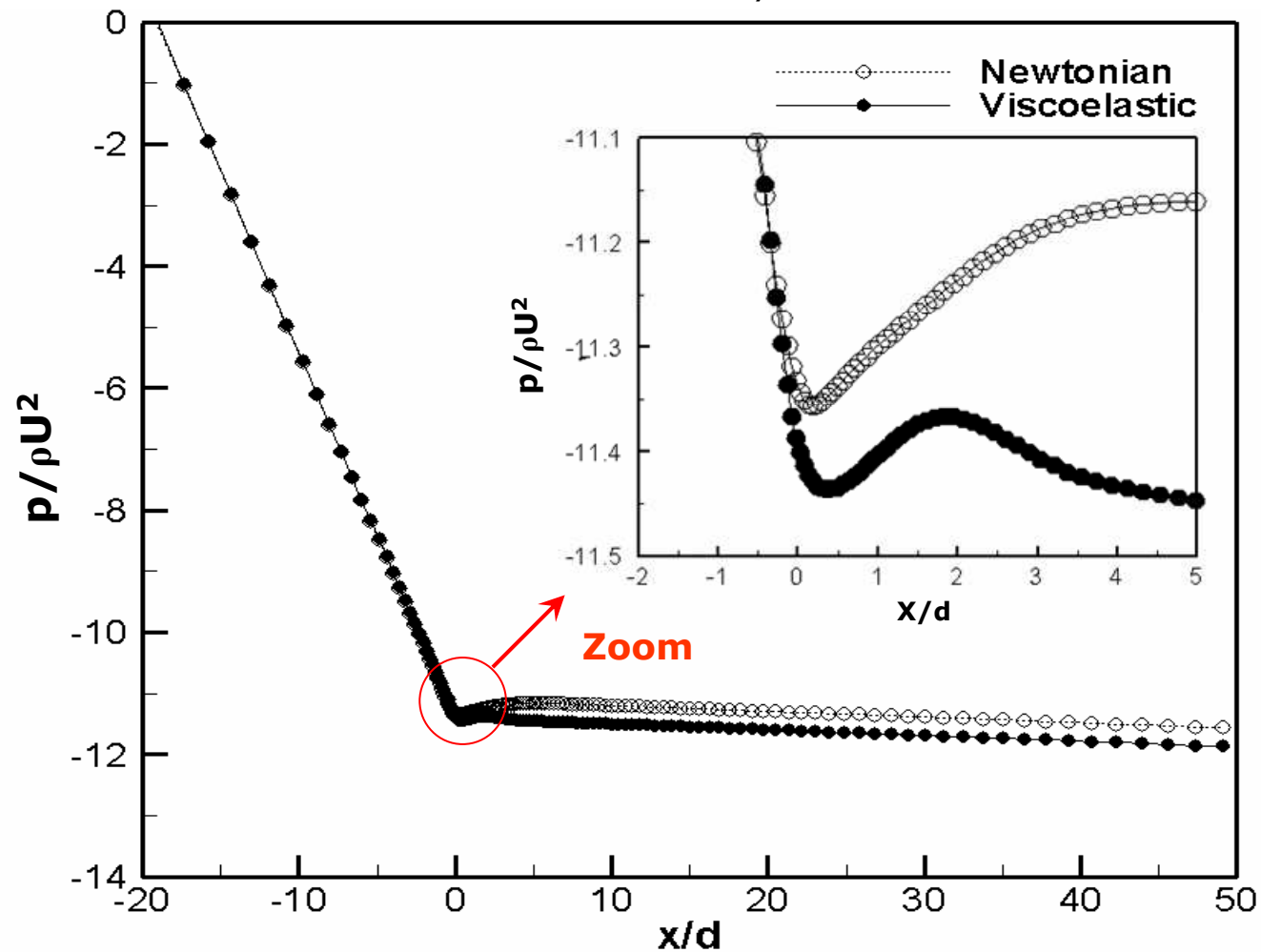


Comparison of Newtonian against viscoelastic flow  
case ( $L^2 = 100$ ,  $We = 2$ ,  $\beta = 0.5$ )

**Pressure profile  
along central plane ( $y=0$ ).**

**$Re = 20$**

$L^2 = 100, We = 2, \beta = 0.5$



# CONCLUSIONS

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- ❑ At low-to-moderate Reynolds numbers (0.01-35) the flow is almost symmetric in respect to the center plane.
- ❑ For the Newtonian fluid: for  $Re$  larger than 1 inertial forces prevail in the flow, while for Reynolds numbers significantly less than 1 the viscous forces dominate.
- ❑ The effect of elasticity is to delay the onset of the bifurcation.
- ❑ Vortex sizes and intensities are smaller for the viscoelastic liquid, compared with the Newtonian fluid.
- ❑ The present results showing a significant reduction in vortex size and intensity with viscoelasticity could not have been obtained with generalized Newtonian fluid because here the shear viscosity was kept constant.
- ❑ The pressure recovery, after the expansion zone, is lower for the viscoelastic fluid.



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