Application of Opportunistic Information-Gap Decision Theory on Demand Response Aggregator in the Day-Ahead Electricity Market

Morteza Vahid-Ghavidel, João P. S. Catalão
FEUP and INESC TEC
Porto, Portugal
mv.ghavidel@gmail.com; catalao@fe.up.pt

Miadreza Shafie-khah
Technology and Innovations
University of Vaasa
Vaasa, Finland
mshafiek@uva.fi

Behnam Mohammadi-Ivatloo
Faculty of Electrical and Computer Engineering,
University of Tabriz
Tabriz, Iran
mohammadi@ieee.org

Nadali Mahmoudi
Ernst & Young
Brisbane, Australia
nadali.m85@gmail.com

Abstract—The proposed model analyzes the profit of a demand response (DR) aggregator from trading DR in the day-ahead electricity market in a way that it tends to gain profit from the favorable deviations of the uncertain parameters. Two types of DR programs are implemented in this model, i.e., time-of-use and reward based DR program. The information-gap decision theory is being employed as a risk measure to address the uncertainties. Two uncertain parameters from both sides of the aggregator have been taken into account in this model, such as the participation rate of the consumers in reward-based DR program in the consumer-side of the aggregator and the day-ahead market prices in the wholesale-side of it. The program is simulated in GAMS software using the available commercial solver. Real data is considered to check the feasibility of the proposed program.

Keywords—Demand response, information-gap decision theory, uncertainty, DR aggregator

NOMENCLATURE

Indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Time horizon index</td>
</tr>
<tr>
<td>$j$</td>
<td>RBDR steps index</td>
</tr>
<tr>
<td>$p$</td>
<td>Period index</td>
</tr>
<tr>
<td>$c$</td>
<td>Consumer index</td>
</tr>
</tbody>
</table>

Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{P}^{DA}(t)$</td>
<td>Expected day-ahead market price [$/MWh]</td>
</tr>
<tr>
<td>$\bar{P}^R(t)$</td>
<td>Demand-side consumers’ Participation rate in RBDR program</td>
</tr>
<tr>
<td>$D_0(c,t)$</td>
<td>Initial demand of consumer $c$ in time interval $t$</td>
</tr>
<tr>
<td>$E(c,t,p)$</td>
<td>Consumer $c$ elasticity in time interval $t$ in period $p$</td>
</tr>
<tr>
<td>$\lambda_0(c,p)$</td>
<td>Consumer $c$ initial price in period $p$</td>
</tr>
<tr>
<td>$\lambda(c,p)$</td>
<td>Consumer $c$ TOU price in period $p$</td>
</tr>
<tr>
<td>$d(t)$</td>
<td>Duration of each period</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Deterministic expected profit of DRA [$]</td>
</tr>
<tr>
<td>$B_e$</td>
<td>Desired target profit of the DRA [$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Profit deviation factor</td>
</tr>
<tr>
<td>$\bar{P}_j^{RBDR}(t)$</td>
<td>Load reduction step in the reward-based DR [MWh]</td>
</tr>
<tr>
<td>$\bar{P}_j^{RBDR}(t)$</td>
<td>Given reward in the reward-based DR [$/MWh]</td>
</tr>
</tbody>
</table>

Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Horizon related to uncertain parameter</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>The function of optimal opportunity value</td>
</tr>
</tbody>
</table>

The traditional solution of the independent system operator to mitigate the power misbalancing matters due to the peak periods was to rely on the generators. However, many solutions have been introduced and even employed in the power system, i.e., demand response (DR). Demand response is being used as one of the main key solutions of the general demand-side methods in the smart grids [1] and [2].

Several studies have been done in DR in order to enhance the participation of the end-user consumers in the electricity market environment. Aggregation of the obtained DR from the demand-side is known as one of these solutions. However, the willingness of the end-user consumers in the DR programs plays an essential role to this end.

Therefore, considering their behavior as one of the uncertain parameters in the model is one of the main motivations of this work. Besides that, in order to increase the effectiveness of the model, the aggregator needs to consider the uncertainty of the electricity market prices too. Since the DR aggregator (DRA) has an intermediary role in trading the obtained DR into the electricity market [3], [4].

Two uncertain parameters are taken into account in this study, the participation rate of the consumers in the DR program and the electricity market prices. That one of these uncertainties belongs to the demand-side, and the other one belongs to the other side.

Several DR programs are implemented in the smart grids which could be classified in two main categories, i.e., incentive-based DR programs and price-based DR programs [5]–[8]. In order to employ a comprehensive model, in this study, one DR program from each category has been defined, i.e., time-of-use (TOU) and reward-based demand response (RBDR). Further, DR programs can be modeled for different types of loads. For instance, in Ref. [9], the residential consumers are considered as the main participators in DR programs.
The authors in [10] investigate the feedback of the commercial and industrial loads participating in DR programs. To observe the effects of the proposed model in the loads, all types of consumers are assumed simultaneously, i.e., industrial, commercial and residential.

To address the uncertain parameters, information-gap decision theory (IGDT) is applied as a risk measure, which its advantages in comparisons with other methods like scenario-based models has studied comprehensively in [11]. Employment of IGDT method in various areas of the power system and smart grid are discussed in [12]. There are two main IGDT functions, robust function and opportunity function. The robust one is used for risk-averse decision makers, and the opportunity function is utilized for risk-seeking purposes. Therefore, the behavior of a risk-seeker DRA is modeled in this work through opportunity IGDT.

The contribution of this work is studying the behavior of the risk-seeker DRA considering two DRP, i.e., TOU and RBDR in the demand-side of the aggregator and day-ahead electricity market on the other side of it. Further, the uncertainty of both side of the aggregator is taken into account. And for the risk management of the problem, IGDT method is applied.

II. Problem Formulation

First, it is supposed that there is not any uncertain parameter. In other words, we assume that the day-ahead market price and participation factor of consumers in RBDR program are determined. This section is considered a deterministic formulation. Then, in the second section of formulation, the uncertainties are considered, i.e., day-ahead market prices and participation rate of the consumers in RBDR program. The opportunistic IDGT model is being used to address the uncertainties.

A. The deterministic formulation

In this section, the deterministic problem formulation is written as follows:

\[ B_0 = \max \sum_{t=1}^{T} \sum_{j=1}^{N_j} P_{t,j}^{DA} \lambda_{0}^{DA} - \]

\[ \sum_{t=1}^{T} \sum_{j=1}^{N_j} PF_{t,j} P_{t,j}^{RBDR} R_{t,j}^{RBDR} \]

s.t.: \[ P_{t}^{DA} = P_{t}^{RBDR} - TOU_t \], \forall t \] (2)

\[ TOU_t = \sum_{c=1}^{P} D_{0}(c,t). \] (3)

\[ \sum_{j=1}^{N_j} V_{t,j}^{RBDR} R_{t,j}^{RBDR} \], \forall t, \forall j \] (4)

It is necessary for DRA to know its schedule for trading in the day-ahead market. It has to be noted that in this part, the DRA can predict the uncertain parameters, i.e., the participation rate of consumers in RBDR program and day-ahead market prices.

The objective function is indicated in equation (1) which is a profit-maximization problem. The 1st term belongs to the revenue which is gained from trading the DR in the day-ahead market. The second term refers to the cost of participation in the RBDR program. The power balance equation is considered in (2). The amount of power which is traded in the day-ahead market must be equal to the amount of obtained DR from consumers for each type of consumer and each time steps. In (3), the TOU program is being defined. In this program, consumers receive different price tariffs during a day, for instance, two tariffs for two time periods: low-peak and high-peak. Thus, the consumers’ power usage is being regulated according to this change in the tariffs. \( E(c,t,p) \) shows the elasticity of the consumer type \( c \) in the time step \( t \) and period \( p \). The RBDR program is indicated in (4). As stated in Fig. 1, the volume of the load reduction will be increased as the aggregator offers higher rewards to the consumers in a stepwise manner.

The total value of the reduced load based on RBDR program is specified by \( P_{t}^{RBDR} \). \( PR_t \) shows the participation rate of the consumers in this program, which used as the uncertain parameter in this model and varies from 0 to 1.

\[ R_{t}^{RBDR} = \sum_{j=1}^{N_j} R_{t,j}^{RBDR} \], \forall t, \forall j \] (5)

\[ \overline{R}_{t(j-1)}^{RBDR} \leq R_{t,j}^{RBDR} \leq \overline{R}_{t,j}^{RBDR}, \forall t, \forall j \] (6)

\[ \sum_{j=1}^{N_j} V_{t,j}^{RBDR} = 1, \forall t, \forall j \] (7)

\[ P^{Min} \leq P_{t}^{DA} \leq P^{Max}, \forall t \] (8)

\[ V_{t,j}^{RBDR} \in \{0,1\} \] (9)
High values in $PR_t$, shows a high rate of participation of consumers in that time step. For instance, $PR_t = 1$ means that all the forecasted DR through RBDR program is attainable. In (5), the total amount of reward in each time step based on the RBDR program is being calculated. The level of the reward in each step $j$ and each time $t$ is shown in (6). Note that according to constraint (7), $V_{i,j}^{RBDR}$ is a binary variable, and the aggregator can choose only one level $j$ in each time step $t$.

As stated in (8), the aggregator can only trade an amount of $\hat{\lambda}_d^D$, which is not less than its minimum or not more than its maximum capacity.

**B. The opportunistic IGDT formulation**

The opportunistic IGDT model is formulated in (10)-(16) as follows:

**Obj Func. :**

$$\hat{\beta} = \min \beta$$

**S.t :**

$$B^* \geq B_0 = (1 + \sigma)B_0$$

$$P_{ata} \leq P_{t}^{DA} \leq P_{Max}^{DA}, \forall t$$

$$B^* = \{$$

$$\max \sum_{i=1}^{N} P_{i}^{DA} \lambda_{i}^{D} - \sum_{i=1}^{N} \sum_{j=1}^{N} PR_{i,j} R_{i,j}^{RBDR}, R_{i,j}^{RBDR}$$

$$- (9)$$

$$- (14)$$

$$- (15)$$

$$- (16)$$

$$\}$$

In this section, the uncertain parameters are taken into account. It is considered that the day-ahead market prices and the participation rate of consumers in RBDR program as the uncertain parameters. In order to address these uncertainties, opportunistic IGDT approach is being implemented.

Note that the forecasted values of the uncertain parameters are available at the moment of modeling, i.e., $PR_i$ and $\hat{\lambda}_d^D$. The aim of this program is to minimize the horizon of the opportunity function value while the requirements are being fulfilled. In constraints (15) and (16), the uncertainties have been addressed.

The framework of this model is depicted in Fig. 2. In the first stage, the model calculates the deterministic value of the objective function (the profit of the DRA). In this step, the forecasted values for the uncertain parameters ($PR_i$ and $\hat{\lambda}_d^D$) is employed to derive the deterministic results. In the next stage, by utilizing the deterministic profit of the aggregator and the profit deviation factor ($\sigma$), the uncertain parameters are being addressed through opportunistic IGDT method.

**III. CASE STUDY**

This problem is a mixed-integer nonlinear programming (MINLP) model. As explained before, its aim is to minimize the horizon of the opportunity function value while the constraints are satisfied or met. Various commercial solvers, i.e., SBB could be used to solve this problem using General Algebraic Modeling System (GAMS) [13].

The model is simulated in a PC with 6 GB RAM and 2.43 GHz CPU speed. The model has 3745 variables and 3827 constraints and simulation running time was less than a second, i.e., 0.9. Reference [14] is used for implementing the load data. High-peak and low-peak periods are considered as periods for each day ($p=2$). The high-peak period is assumed from 08 to 22. Accordingly, from 23 to 07 is assumed as the low-peak period. Industrial, commercial and residential are the types of consumers which are taken into account ($c=3$).

The aggregator can offer the obtained DR from the end-user consumers during the high-peak period to the day-ahead market and vice versa during the low-peak. TOU and RBDR program is modeled on the lower side of the aggregator. The data regarding the elasticity matrix which is required for TOU model is employed from [15].

![Fig. 2. The proposed model framework.](image-url)
As stated before, the profit deviation factor is utilized as the risk measure in the IGDT procedure. As the profit deviation factor increases, our model results become more risk-seeker. And $\sigma = 0$ give the deterministic results of the programming. We change the $\sigma$ from zero to 0.85 and each optimum value of the opportunity value is depicted in Fig. 3. Higher profit deviation factors result in higher $\beta$. 

To investigate more in detail about the results of the problem, an arbitrary value of $\sigma = 0.15$ is chosen. To gain the target profit $B_\omega = (1 + \sigma) \times B_\omega = (1 + 0.15) \times 344,800 = 396,500$, the $\beta$ is 44% or 0.44, which means that if the observed uncertain parameters be 44% more than the forecasted values, the aggregator will gain $396,500$.

The curve in Fig. 4 indicates the results regarding the acquired DR through RBDR program. During high-peak period, the amount of DR which is obtained through this program is at its maximum when they are the usual work starting time ($t=9$ AM) and also in the time that the night starts ($t=7$ PM).

Results from implementing TOU program is demonstrated in Fig. 5. As obvious in the figure, the amount of TOU in industrial consumers is much higher than the other consumers including residential and commercial. The industrial end-user plays the main role in the deployment of TOU program.

The day-ahead traded power through the aggregator is also presented in Fig. 6. During the high-peak period, the amount of power which is offered to the pool market is around 1000 kW in the early hours of the high-peak period. It is easily noticeable that the amount of the acquired DR from the consumers through RBDR and TOU programs are equal to the traded power in the day-ahead market, which proves the accuracy of the simulation.

**IV. CONCLUSION**

The behavior of a DRA was studied in the proposed model which tends to gain higher profits due to the favorable deviations of the uncertain parameters in the day-ahead electricity market. To this end, opportunity IGDT method was applied as a risk measure. Two uncertain parameters from each side of the aggregator (upper-side and down-side) were assumed simultaneously as follows: 1- the day-ahead market prices and 2- the participation rate of the consumers in the RBDR program.
The model was simulated for various values of the profit deviation factors. The direct relation between the profit deviation factor and the optimum opportunity function value was shown in the results. To analyze the model effects more in detail, one arbitrary value of the profit deviation factor was chosen, and the correlated results were demonstrated comprehensively. The amount of electric power which was traded in a day-ahead market through the DRA was equal to the obtained DR from the consumers’ side. Moreover, three types of consumers, i.e., industrial commercial and residential, industrial consumers played the main role in employing the TOU program.

REFERENCES