Short-Term Scheduling of Head-Dependent Hydro Power Systems: A Quadratic Optimization Approach

JOÃO CATALÃO¹, SÍLVIO MARIANO¹, VICTOR MENDES², LUÍS FERREIRA³
¹Department of Electromechanical Engineering
University of Beira Interior, R. Fonte do Lameiro, 6201-001 Covilhã
²Department of Electrical Engineering and Automation
Instituto Superior de Engenharia, R. Conselheiro Emídio Navarro, 1950-062 Lisbon
³Department of Electrical Engineering and Computers
Instituto Superior Técnico, Technical University of Lisbon, Av. Rovisco Pais, 1049-001 Lisbon
PORTUGAL
catalao@ubi.pt, sm@ubi.pt, vfmendes@iscte.pt, lmf@ist.utl.pt

Abstract: - The purpose of this paper is to analyze the short-term behavior of a head-dependent hydro power system. We propose a quadratic optimization approach to account with the nonlinear hydropower generation characteristic. Under profit-based environment, the optimal scheduling of the hydropower facilities available is essential for generating companies to face competitiveness. Moreover, also responds to climate change contributing to reduce fossil fuels energy dependency. Results based on a realistic cascaded hydropower system are computed to show the optimal storage trajectories for the reservoirs.

Key-Words: - Hydro scheduling, head dependency, quadratic optimization

1 Introduction
As the traditional monopolistic scenery for the electric energy makes way to a competitive energy market, an improved operational planning is crucial for generating companies to face competitiveness toning to the best profit perspective [1].

In the new profit-based environment, a generating company with hydropower facilities faces the optimal trade-off problem of how to make the best present profit by the management of the water available for power generation without compromising future potential profit. The goal is to maximize the value of total hydropower generation throughout the time horizon considered, satisfying all physical and operational constraints, and consequently to maximize the profit of the generating company from selling electric energy into the energy market. This problem is known as hydro scheduling.

Short-term hydro scheduling is concerned with the operation during a time horizon of one to seven days, usually discretized in hourly intervals. This problem is treated as a deterministic one, due to the small time horizon. Where the problem includes stochastic quantities, such as the energy prices, the corresponding forecasts are used [2].

Hydropower plants with a small storage capacity available are known as run-of-the-river. Typically, run-of-the-river hydropower plants are considered to operate under stationary conditions with constant head and at the maximum water level in the reservoirs, corresponding by design to the optimum efficiency operating point. However, it is often desirable to change this policy, thus incurring into head changes.

The operating efficiency is sensitive to the head, head change effect, due to the small storage capacity. Significant loss of efficiency can occur in operating hydropower plants away from their most efficient operating points. Therefore, hydropower generation has to be considered as a function of water discharge and also of the head in order to obtain accurate and better realistic results.

A natural approach to short-term hydro scheduling is to model the system as a network flow model [3], because of the underlying network structure subjacent in hydro chains. This network flow model is often coded as a linear or piecewise linear one [4]. Linear programming is a well-known optimization method and standard software is available. However, linear programming treats the hydropower generation as linearly dependent on water discharge, thus ignoring head dependence which can lead to leading to inaccuracies.

Mixed-integer linear programming is becoming frequently used for hydro scheduling [5-6], where binary variables allow modeling of start-up costs to avoid unnecessary start-ups, leading to less stress in
the windings and hence less maintenance cost. However, the discretization of the nonlinear dependence between hydropower generation, water discharge and head, used in mixed-integer linear programming to model head variations, augment the computational burden.

A nonlinear model expresses the hydropower generation characteristic more accurately, taking into account the head change effect [7-8]. Head-dependent hydropower plants, due to the hydropower generation characteristic as a nonlinear function of water discharge and head, are not proper modeled by linear methods.

The Portuguese fossil fuels energy intensity is among the highest in the European Union. Hence, enhancements in the exploitation of the hydro resources are increasingly important in nowadays, contributing to reduce fossil fuels energy dependency and responding to climate change [9-10].

Quadratic optimization compromises one of the most important areas of nonlinear programming [11]. We propose a quadratic optimization approach to solve the short-term hydro scheduling problem in light of market conditions as a new contribution to earlier studies, studying the behavior of the reservoirs according to their position in the cascade and according to the parameterization defining the data for the hydro system. The approach is illustrated by a realistic hydro system with three cascaded head-sensitive reservoirs, considering a time horizon of 168 hours.

2 Problem Formulation

The short-term hydro scheduling problem is dependent on the forecasted data for the energy prices over the time horizon. The objective function for this problem is the profit of the generating company with the hydro schedule conversion of the potential energy of water into electric energy. In what follows we use the following notation.

\[ I \] - Set of indices i for the reservoirs.

\[ K \] - Set of indices k for the hours in the time horizon.

\[ a_{ik} \] - Inflow to reservoir i in hour k.

\[ s_{ik} \] - Water spillage by reservoir i in hour k.

\[ q_{ik} \] - Water discharge of plant i in hour k.

\[ v_{ik} \] - Water storage in reservoir i at end of hour k.

\[ l_{ik} \] - Water level in reservoir i at hour k.

\[ h_{ik} \] - Head of plant i in hour k.

\[ \lambda_k \] - Forecasted energy price in hour k.

\[ \eta_{ik} \] - Efficiency of plant i in hour k.

\[ p_{ik} \] - Power generation of plant i in hour k.

\[ \Psi_i \] - Future value of the water stored in reservoir i.

\[ V_{ik} \] - Set of admissible \( v_{ik} \) for plant i.

\[ Q_{ik} \] - Set of admissible \( q_{ik} \) for plant i.

\[ S_{ik} \] - Set of admissible \( s_{ik} \) for plant i.

\[ x \] - Vector of the flux variables corresponding to the arcs of the network structure.

\[ J \] - Quadratic formula.

\[ H \] - Hessian matrix.

\[ f \] - Vector of coefficients for the linear term.

\[ A \] - Node-arc incident matrix.

\[ b \] - Right hand side vector.

The objective function to be maximized can be expressed as

\[ J = \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_k p_{ik} + \sum_{i=1}^{I} \Psi_i (v_{ik}) \]  \hspace{1cm} (1)

This objective function is composed of two terms. The first term represents the profit with the hydro system during the time horizon. The last term expresses the future value of the water stored in the reservoirs at the end of the time horizon.

The optimal value of the objective function is determined subject to equality constraints and inequality constraints, simple bounds on the variables:

\[ v_{ik} \in V_{ik} = [v_{\min}, v_{\max}] \] \hspace{1cm} (5)

\[ q_{ik} \in Q_{ik} = [0, q_{\max}] \] \hspace{1cm} (6)

\[ s_{ik} \in S_{ik} = [0, v_{ik} - v_{\min}] \] \hspace{1cm} (7)
The water balance equation for each reservoir is given in (2), assuming that the time required for water to travel from a reservoir to a reservoir direct downstream is not significant compared to the scheduling time period. In (3) power generation is considered as a function of water discharge and efficiency, which depends on the head. In (4) the head is considered as a function of the water levels in the upstream and the downstream reservoirs to the plant. The water level in the reservoirs depends on the water storage in the respectively reservoirs. In (5) and (6) water storage and water discharge have lower and upper bounds. In (7) spillage can only occur when without it the water storage exceeds its upper bound, so spilling is necessary to avoid damage. The initial water storages and the inflows to reservoirs are assumed known data.

3 Quadratic Optimization Approach

Quadratic optimization can be stated as to maximize:

$$J(x) = \frac{1}{2} x^T H x + f^T x$$  \hspace{1cm} (8)

subject to:

$$Ax = b$$  \hspace{1cm} (9)

$$\text{x min} \leq x \leq \text{x max}$$  \hspace{1cm} (10)

The equality constraints for the water balance in (2) are modeled as in (9). The inequality constraints in (5), (6) and (7) are modeled as in (10).

Efficiency for plants depends on the head, assuming a linearization of this efficiency we have:

$$\eta_{ik} = \alpha_i h_{ik} + \eta_{10}$$  \hspace{1cm} (11)

Water level depends on the water storage, assuming a linearization of the water level for the reservoirs we have:

$$l_{ik} = \beta_i v_{ik} + l_{10}$$  \hspace{1cm} (12)

Substituting (11), (4) and (12) into (3), hydropower generation becomes a nonlinear function of water discharge and water storage, given by:

$$p_{ik} = \alpha_i \beta_i q_{ik} v_{ik} - \alpha_i \beta_{i+1} q_{ik} v_{i+1,k} + \delta_i q_{ik}$$  \hspace{1cm} (13)

The parameters given as the product of $\alpha$'s by $\beta$'s in (13) are the most important coefficients for the hydropower generation nonlinear relationship in our problem, considering head dependence. These parameters will have a crucial importance on the short-term behavior of head-dependent reservoirs in a hydro chain, affecting optimal reservoirs storage trajectories according to the reservoirs position in the cascade and according to the physical data defining the hydro system.

The parameters given as the product of $\alpha$'s by $\beta$'s multiplied by the respectively hourly period's forecasted energy prices appear in the Hessian matrix. Also, the parameters $\delta$'s multiplied by the respectively hourly period's forecasted energy prices appear in the vector of coefficients for the linear term.

The Hessian matrix is a symmetric matrix, hence all its eigenvalues are real numbers. The sum of the eigenvalues for our Hessian matrix is null because the trace, the sum of the main diagonal elements, is null, i.e., the Hessian matrix has positive and negative or null eigenvalues, meaning that the Hessian matrix is an indefinite matrix.

Quadratic optimization for a problem formulated as in (8), (9) and (10) can be classified due to the nature of the Hessian matrix into the well-known concave and the less well-known nonconcave quadratic optimization [11], where the indefinite quadratic optimization is the toughest and still is a research topic among specialists in global optimization, being less general and taking advantage of the special mathematical structure exhibited in the applications.

The objective function of our model is neither a concave nor a convex function; our new model for the short-term hydro scheduling problem is an indefinite quadratic optimization problem. Therefore, the optimal solution is not necessarily attained at a vertex of the feasible region, the polytope defined by the network constraints in (9) and box constraints in (10).

In the past, much research has been devoted to models where the Hessian matrix is definite or at least semidefinite. However, only limited attention has been devoted to exploiting the structure of an indefinite quadratic objective function. For instances, it is recommended a Hessian regularization given by:

$$H^* = H - \mu I$$  \hspace{1cm} (14)

where $\mu$ is not too much larger ideally than the smallest value that will make, in (8), the Hessian matrix a negative definite matrix and acceptable well conditioned.

Historically, this approach for modifying the indefinite Hessian matrices preceded and is in fact formally equivalent to trust region methods. From spectral decomposition it is known that every indefinite quadratic function can be expressed as the
sum of its concave and convex parts corresponding to negative and positive eigenvalues, given rise to an objective function, a DC function, the difference of two concave functions. Hence, this problem can be solved by a DC programming method, exploiting the structure of our model for the short-term hydro scheduling problem, network constraints and the higher sparsity of the Hessian matrix.

4 Case Study

The proposed quadratic optimization approach, considering the head change effect, has been applied on a real case based on one of the main Portuguese hydro systems with three cascaded head-dependent reservoirs.

The hydro system is shown in Fig. 1.

![Fig. 1 Hydro system with three reservoirs.](image)

This case study considers only inflow on the first reservoir of the hydro system. The computational simulation was performed on a 1.6-GHz-based processor with 512 MB of RAM. The scheduling time horizon is 168 hours.

The initial water storage in each reservoir is equal to 80% of maximum storage in the respectively reservoir. Also, the final water storage in each reservoir is constrained to be equal to the initial water storage in each reservoir. Consequently, the future values of water stored in reservoirs are not necessary for this case study.

The forecasted energy price considered is shown in Fig. 2, where $S$ is a symbolic quantity.

![Fig. 2 Forecasted energy price.](image)

In the next figures, the solid, dashed and dash-dot lines denote respectively the results for the first, second and third reservoirs. The case study includes three instances. The first instance considers $\alpha_{1}\beta_{1}>\alpha_{2}\beta_{2}>\alpha_{3}\beta_{3}$ which corresponds to a real case. The results are shown in Fig. 3.

![Fig. 3 Optimal reservoir storage trajectories for the case study first instance.](image)

The quadratic optimization results in Fig. 3 show a delay in power generation at the initial periods on all plants in order to quickly achieve appropriated
reservoir storage levels, pulling up the storage trajectories of the first and second reservoirs, in order to benefit the generation's efficiency of these plants, opposing to the change in the storage trajectory of the third reservoir, constrained to a minimum storage of 20% maximum storage. The second instance assumption is $\alpha_1\beta_1 < \alpha_2\beta_2 < \alpha_3\beta_3$. The results are shown in Fig. 4.

![Fig. 4 Optimal reservoir storage trajectories for the case study second instance.](image)

Finally, the last instance considers Hessian regularization again with $\alpha_1\beta_1 > \alpha_2\beta_2 > \alpha_3\beta_3$. The results are shown in Fig. 5.

![Fig. 5 Optimal reservoir storage trajectories for the case study third instance.](image)

In the second instance, the quadratic optimization pulls up the storage trajectory of the third reservoir in order to benefit the generation efficiency of this plant, due to the assumption. The second reservoir has a storage trajectory similar to the first realistic instance. The first reservoir has the lowest coefficient for the power generation nonlinear relationship of all reservoirs. Hence, it does not assume the behavior observed in the first instance, nevertheless due to its position in the cascade the storage level is kept at a relatively high level.

The Hessian regularization provides no parameterization sensitive storage trajectories. Hence, we conclude that it is not adequate for the hydro scheduling.

5 Conclusion
This paper deals with the short-term hydro scheduling problem viewed as an indefinite quadratic optimization problem with continuous variables.

A better short-term management of the conversion of the potential energy available in the reservoirs into electric energy, considering head dependency, is an important improvement for hydropower utilities to face competitiveness in a profit-based environment.

Results based on a realistic cascaded hydro system are computed to highlight the behavior features of the reservoirs and the influence of the hydro system parameters in short-term hydro scheduling along with results considering the Hessian regularization, which seems to be not adequate for this problem.

References:


