Two-Stage Stochastic Mixed Integer Programming Approach for Optimal SCUC by Economic DR Model

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Abstract—Due to influences by power system restructuring, fuel price uncertainties, future demand forecasting, and utilities and transmission lines availability, demand response (DR) programs for consumers have gained more attention. One important DR scheme is the emergency demand response program (EDRP). This paper focuses on simultaneous implementation of security-constraint unit commitment (SCUC) and EDRP by using an economic model. Moreover, a stochastic optimization method is employed for realistic modelling. Since the combined implementation of SCUC and EDRP results in a complex non-linear optimization problem, a linearization method to ensure computational efficiency is used. The proposed model is formulated as two-stage Stochastic Mixed-Integer Programming (SMIP) model implemented using GAMS. The implemented model is tested on three case studies using the IEEE 24-bus system. Results are analyzed with a focus on the impact of demand elasticity and electricity prices.

Index Terms—Security-constrained unit commitment, mar emergency demand response program, demand response.

I. INTRODUCTION

For the past four decades, demand side management (DSM) strategies have been gaining increasing popularity. By transforming the demand side of the electrical grid to an active element rather than a passive one, governments and utilities adopt DSM programs in order to maximize social welfare and the benefits of utilities and consumers.

DSM is a general expression and includes multiple specific schemes such as: energy efficiency, load management, on-site storage, local energy resources, among others [1].

In more recent years, the restructuring and deregulation of electricity markets lead to the rise of demand response (DR) as a new and important DSM component.

In the past decade, a significant amount of scientific research has been made regarding the development and employment of demand response programs [2-6].

The main aim of DR programs is to motivate end-users to adapt their consumption in accordance with the grid and/or market requirements. The consumers “respond” to either fluctuations in electricity prices during the day (price-based DR), or through direct incentives (incentive-based DR), ultimately resulting in load reduction during peak hours [7]. This in turn results in the overall decrease of electricity prices and greatly enhances the performance of a modern electricity market since available resources from competitive suppliers are coordinated with the required demand.

The active participation of the demand side resulting from DR implementation also has other major benefits such as: large utilities market power reduction and network congestion minimization [8-9].

Numerous papers have investigated various DR implementations.

Reference [10] proposed an integrated DR which combines Time Of Use (TOU) and Emergency Demand Response Program (EDRP), two of the most prominent DR programs, where the first is a time-based program and the latter an incentive-based one. The proposed method was employed for peak load conditions of the Iranian electrical power grid. The study managed to obtain optimal prices for TOU program as well as the optimal incentives for the integrated TOU/EDRP strategy.

In [11], the demand response is being utilized for security enhancement purposes. The paper is mentioned that the demand response has significant role in future smart grids and the physical locations, quantities, and timely responsiveness of load reduction for a DR program directly influence both the efficiency and security of the electric grid.
The paper proposed a multistage optimization approach, which decomposes the nonlinear problem into several linear ones, each of which is then solved at a separate stage. The proposed model was then implemented on three test power systems and its performance is verified.

In [12], an investigation was made to optimize the generation mix with short-term DR bearing in mind the penetration of wind power generation. The relationship between demand elasticity and energy price were studied to adjust the demand profile. The paper shows that the DR model succeeded in the desired peak reduction and valley filling of the load profile, which in turn influences the optimal generation mix. The paper also shows that by increasing the demand elasticity, the installed amount of wind capacity is increased, yielding environmental benefits.

In [8], short-term effects of active demand side participation in were analyzed for the case of the Spanish electricity markets. The paper mentioned that DR resources management is an effective solution to leverage efficiency of electricity markets, with a focus of the customers’ market participation.

This paper presents a new formulation that considers the economic model for emergency DR program (EDRP) in SCUC. A two-stage Stochastic Mixed-Integer Programming (SMIP) optimization model is used to find the optimal solution. The contributions can be listed as follows:

- A new framework for SCUC optimization considering economic DR model is proposed.
- Considering a model of generation units and transmission lines outages. The uncertainties associated with generators and line outages are modeled in different system scenarios.
- Scrutinizing of the impact of economic DR program in SCUC using SMIP optimization.

The manuscript is organized as follows: Section II demonstrates the formulation of the objective function and associated constraints for the problem. In Section III, the proposed solution algorithm is shown. Section IV demonstrates the application of the proposed method to the IEEE test system. Finally, Section IV recaps the conclusions of this work.

II. EMERGENCY DEMAND RESPONSE PROGRAM

This paper uses the bus determination subroutine proposed in [13] with some modifications. The main difference is that calculations related to wind farms are added to the formulation as shown in (2) and (3). After a contingency, in an effort to keep the system. In emergence of the deregulation, the customers usually do not effectively participate in the power market. Rather, major players in the wake of market deregulation are Independent Power Producers (IPPs), Regional Transmission Organizations (RTOs) and regulatory bodies. [14-15] Nowadays, the electricity price impacts customers demand and the customers change their consumption with respect to electricity price. Demand elasticity is a quantification of the demand sensitivity to a price change [16].

\[ e = \frac{\partial DEM}{\partial Price} = \frac{Price_{old}(i)}{DEM_{old}(i)} \times \frac{d DEM}{d Price} \]  

(2)

\[ DEM_{old}(i) \text{ is initial demand value and } Price_{old}(i) \text{ is initial energy price. Demand elasticity in time } i \text{ respect to energy price at time } j \text{ can be calculated by: [17]} \]

\[ e(i, j) = \frac{\partial DEM(i)}{\partial Price(j)} = \frac{Price_{old}(i)}{DEM_{old}(i)} \times \frac{d DEM}{d Price(j)} \]  

(3)

Elasticity shows the variation in energy consumption in period \( i \) with respect to energy price in period \( j \). With the increase of energy price for a given period, the consumers’ tendency for energy consumption is decreased and consumers are henceforth motivated to shift their demand to other periods. As such, self-elasticity is always of a negative value while cross-elasticity is always of a positive one.

\[ e(i, i) = \frac{\Delta DEM(i)}{\Delta Price(i)} \leq 0 \]  

(4)

\[ e(i, j) = \frac{\Delta DEM(i)}{\Delta Price(i)} \geq 0 \]  

(5)

\( \xi(i, i) \) is self-elasticity and \( \xi(i, j) \) is cross elasticity. Due to their natural critical dependence on electrical energy, consumers, especially small ones, are often resistant to modify their consumption, even in response to prices. However, industrial consumers tend to shift their consumption to off-peak and low-peak periods, as their processes are more economically oriented. By such action, the industrial consumers decrease their operational costs. Net Consumer Surplus (NCS) is defined as follow [18]:

\[ \text{NCS} = \left\{ \left\{ \text{GCS}(DEM_{new}(t)) - (DEM_{new}(t) \times Price(t)) \right\} \right\} + \left\{ \left( \text{INCENT}(t) \times (DEM_{old}(t) - DEM_{new}(t)) \right) \right\} \]  

(6)

\( DEM_{new}(t) \) and \( DEM_{old}(t) \) are the demands after and before EDRP, respectively. The maximum consumer net surplus (NCS) occurs when the first derivative is zero:

\[ \frac{\partial NCS}{\partial DEM_{new}(t)} = 0 \Rightarrow \]  

\[ \frac{\partial GCS(DEM_{new}(t))}{\partial DEM_{new}(t)} - Price(t) - INCENT(t) = 0 \]  

(7)

\[ \Rightarrow \frac{\partial GCS(DEM_{new}(t))}{\partial DEM_{new}(t)} = Price(t) + INCENT(t) \]  

Generally, Gross Consumer Surplus (GCS) function is a quadratic function of the demand that shows in equation (7). By derivation of equation (7) and substituting in equation (6), equation (8) is obtained [18]:

\[ GCS(DEM_{new}(t)) = GCS(DEM_{old}(t)) + Price(t) \times \left( DEM_{new}(t) - DEM_{old}(t) \right) \left\{ 1 + \frac{DEM_{new}(t) - DEM_{old}(t)}{2DEM_{old}(t) \sum_{t'} \xi(t, t')} \right\} \]  

(8)
By derivation of equation (7) and substituting in equation (6), equation (8) is obtained [18]:

\[
DEM_{ow}(t) = DEM_{old}(t) \times \left( \frac{\sum_{t'=1}^{N_t} e(t,t')}{Price(t)} \right) \quad (8)
\]

Load curve can be categorized into three segments corresponding to low-, off-, and peak hours, expressed using the \( t' \) parameter. According to equation (8), it is clear that if electricity price in all intervals are equal, the consumers do not have any intention to shift their consumptions. An electricity price in all intervals are equal, the consumers do not have any intention to shift their consumptions. 

The equation corresponds to scenario \( s \). The first term relates to ramp up and down reserve. The second EDRP reserve cost. The third is the involuntary load curtailment cost. all corresponding to scenario \( s \). Generators cost functions are generally non-linear and estimated by quadratic functions [19].

\[
F_i(P(i,t)) = \alpha(i) \times P^2(i,t) + b(i) \times P(i,t) + c(i)
\]

A piecewise linearization is then performed, and the cost–function obtained is as follows:

\[
F_i(P(i,t)) = F_i(P_{\text{min}}(i,t)) \times I(i,t) + \sum_{s=1}^{N_{\text{seg}}} S_{i,s}(i,t) \times P(i,t) \quad (15)
\]

Total output of each generation utility is calculated as:

\[
P_g(i,t) = P_{\text{min}}(i,t) \times I(i,t) + \sum_{s=1}^{N_{\text{seg}}} S(i,t) \times P(i,t) \quad (16)
\]

The base case constraints are modeled in the first stage and are presented henceforth [20]:

- DC power flow constraints

\[
\sum_{i=1}^{N} P(i,t) - \sum_{j=1}^{N} PD_{\text{new}}(j,t) = \sum_{l=1}^{N} P_{\text{old}}(l,t) \quad \forall t
\]

- Lines power flow limits

\[
-LF_{\text{max}}(l,t) \leq I(l,t) \leq LF_{\text{max}}(l,t) \quad (19)
\]

- Generation units startup and shutdown constraints

\[
\alpha(i,t) - \beta(i,t) = I(i,t) - I(i,t-1) \quad (20)
\]

- Active power generation constraints

\[
\left\{ \begin{array}{l}
P(i,t) = P_{\text{max}}(i,t) \times I(i,t) + \sum_{s=1}^{N_{\text{seg}}} P(i,t,seg) \\
0 \leq P(i,t,seg) \leq P_{\text{max}}(i,t,seg) \\
0 \leq P(i,t) + R_{\text{up}}(i,t) \leq P_{\text{max}}(i,t) \times I(i,t) \\
P(i,t) - R_{\text{down}}(i,t) \geq P_{\text{min}}(i,t) \times I(i,t) \\
\end{array} \right. \quad (21)
\]

- Ramping up and down limits

\[
0 \leq R_{\text{up}}(i,t) \leq \tau \times RU(i) \quad \forall i,t
\]

\[
0 \leq R_{\text{down}}(i,t) \leq \tau \times RD(i) \quad \forall i,t
\]

- Mini up / down time constraints

\[
\left\{ \begin{array}{l}
X_{\text{up}}(i,t-1) - T_{\text{up}}^\text{min}(i) \times I(i,t-1) - I(i,t) \geq 0 \quad \forall i,t \\
X_{\text{down}}(i,t-1) - T_{\text{down}}^\text{min}(i) \times I(i,t) - I(i,t-1) \geq 0 \quad \forall i,t
\end{array} \right. \quad (26)
\]

- Ramping up / down constraints

\[
P(i,t) - P(i,t-1) \leq \left( \left[ 1 - I(i,t) \times (1 - I(i,t-1)) \right] \times RU(i) \right) + I(i,t) \times (1 - I(i,t-1)) \times P_{\text{max}}(i) \quad (28)
\]

\[
P(i,t-1) - P(i,t) \leq \left( \left[ 1 - I(i,t) \times (1 - I(i,t-1)) \right] \times RD(i) \right) + I(i,t-1) \times (1 - I(i,t)) \times P_{\text{max}}(i) \quad (29)
\]
EDRP reserve constraints

\[
DEM_{new}(j,t) = \sum_{t=1}^{N_t} \left( INCENT(t) \times \sum_{t'=1}^{N_t} \epsilon(t,t') \right)
\]

Transmission lines flow (DC load flow) constraints in each scenario [9,20]

\[
\sum_{i=1}^{N_i} \chi(i,t,sc) \times P(i,t) + \sum_{i=1}^{N_i} \chi(i,t,sc) \times r_{up}(i,t,sc)
\]

- Ramping up and down reserve in scenarios
- Demand side reserve constraints in scenarios
- EDRP reserve constraints

\[
DEM_{old}(j,t) \times \left( 1 + \frac{\sum_{t=1}^{N_t} \epsilon(t,t')}{\text{Price}(t)} \right)
\]

The second stage constraints are now defined, which are related to the scenarios and shown in following equations:
- Transmission lines flow (DC load flow) constraints in each scenario
- Lines flow limits in scenarios
- Involuntary load curtailment limit

\[
DEM_{new}(j,t,sc) = \sum_{t=1}^{N_t} \left( INCENT(t,sc) \times \sum_{t'=1}^{N_t} \epsilon(t,t') \right)
\]

\[
DEM_{old}(j,t,sc) = \sum_{t=1}^{N_t} \left( 1 + \frac{\sum_{t'=1}^{N_t} \epsilon(t,t')}{\text{Price}(t)} \right)
\]

- Involuntary load curtailment limit

\[
0 \leq LS(j,t,sc) \leq LS_{\text{max}}(j,t)
\]

In order to show the unplanned generation units and transmission lines outage the parameter \( \chi(sc) \) is being utilized which includes two categories \( \chi(sc) = \{ \chi(i,t,sc), \chi(l,t,sc) \} \).

Two state Markov chain is used for each element. Generation units and transmission lines might be out in each scenario; Hereupon, in order to satisfy the active power balance, the base case generation of the units, their spinning reserve and the demand side reserve must be adequate to meet the demands [22]. In the proposed two stage method, the state of the generation units are determined in the first stage and their active and reactive power generations with DC power flow constraints (equations (17) and (18)) are defined in this stage.

The power generation of the generation units are not changed in each scenario, but parameters \( DEM_{new}(j,t,sc), r_{up}(j,t,sc), r_{dn}(j,t,sc) \) and \( LS(j,t,sc) \) are determined such that the scenario based DC power flow is satisfied.

The association between first and second stage variables are defined in equations (33), (34), (37) and (39). Equations (33) and (34) illustrate that in scenario \( s \) only available units can participate in spinning reserve.

**IV. NUMERICAL RESULTS**

The Mixed Integer Linear Programming (MILP) is utilized for the economic-based combined implementation of SCUC and EDRP. To obtain more realistic results, a stochastic optimization method is employed. GAMS 25.1.3 software, which is widely known for being a robust optimization modeling platform, was used for implementation and testing. The proposed approach was validated by testing it on the IEEE 24-bus Reliability Test System, which is illustrated in Fig. 2, and all technical data obtained from [23]. The proposed model was tested using the CPLEX MILP solver [24] on a DELL series laptop computer (Vostro 5470).

All the simulation cases computation times are less than 10 seconds.

The simulations were performed for a 24-hour-ahead decision making horizon. The generation units’ up and down spinning reserves costs are assumed at the rate of 100% and 40% of their uppermost incremental cost of energy. All other required data are extracted from [23].

A summer week day was considered, in which the peak load is set to be 2.85 GW. 8 $/kWh was the cost set for peak-hour (10:00AM to 6:00 PM), which is double the cost for off-peak hours. All load buses are participating in the DR program. Table I shows the demand elasticity values (self- and cross- elasticity).

The three intervals used to segment the load curve were as follows: Low load from 1:00 AM to 9:00 AM, off-peak from 07:00 PM to midnight and peak 10:00 AM to 6:00 PM. The corresponding electricity prices (per MWh) are set as 25$, 45$ and 5$, respectively.
The simulations are categorized in three cases as follow:

- **Case 1**: SCUC.
- **Case 2**: SCUC with DR program.
- **Case 3**: Stochastic SCUC with DR program

The initial load curve and load curve with DR are shown in Fig. 3. It can be observed that DR shifts the load in the peak period to other periods. The load reduction in peak period of case 3 is greater than case 2; this phenomenon is the result of considering unplanned generation and transmission lines outage which increased the need for reserves of the power system.

The usage of the demand response sources is the optimized condition according to spinning reserve and demand response costs. The operation costs and EDRP costs in all three cases are shown in Fig. 4.

Fig. 4 illustrates that operation cost in the second case is lower than the first, which results from EDRP. Also, case 3 operation cost represents generation costs and expected value of various reserves costs in each scenario.

In Fig. 5, a comparison between case 2 and 3 in three indices are depicted. The indices are peak-to-valley (PtV), peak compensation percentage (PC), deviation of peak-to-valley percentage (DPV) and incentive.

Fig. 5 shows that the mentioned indices are improved in the second and third cases. For example, in case 2, 3.59% improvement is achieved in peak compensation percentage index.

The effects of the energy price and elasticity variations are investigated in the third case. The impact of the electricity price variation on operation cost and EDRP cost are shown in Fig. 6.

---

**Table I**

<table>
<thead>
<tr>
<th>Low Load</th>
<th>Peak</th>
<th>OFF Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>01:00-09:00</td>
<td>10:00-18:00</td>
<td>19:00-24:00</td>
</tr>
<tr>
<td>Low Load</td>
<td>-0.1</td>
<td>0.016</td>
</tr>
<tr>
<td>Peak</td>
<td>0.016</td>
<td>-0.1</td>
</tr>
<tr>
<td>OFF Peak</td>
<td>0.012</td>
<td>0.01</td>
</tr>
</tbody>
</table>

---

**Figure 3.** Initial load curve and load curve with demand response program

**Figure 4.** Operation and emergency demand response program costs.

**Figure 5.** Comparison between case 2 and 3 in mentioned indices

**Figure 6.** Impact of the electricity price variation on the operation cost and cost of emergency demand response

**Figure 7.** Influence of the elasticity on the operation cost and cost of emergency demand response
It is observed that energy price and operation cost are directly proportional. Fig. 6 demonstrates that energy price increments cause operation cost increment. Thus, optimized energy price in each time interval is absolutely important. In Fig. 7, the effect of elasticity on the operation cost and EDRP cost are demonstrated. The elasticities represent the customer’s tendency to emergency demand response program participation. Fig. 7 illustrates that elasticities increment results in operation cost reduction.

V. CONCLUSION

An economic model of an emergency demand response program is used in stochastic security constraint unit commitment. For this purpose, the elasticity of price with respect to demand is utilized. According to the behavior of the energy consumers, the proposed model is realistic and it reflects the impacts of the energy price on the demand. The consumers decrease their demand or shift their consumption to other time intervals with high prices, and vice versa. The proposed model was implemented using two-stage stochastic mixed integer programming. The numerical simulation was performed on three cases. The results show that the emergency demand response causes improvements such as operation cost reduction and load factor enhancement. Peak-to-valley and peak load reduction percentage indices are also improved. The impact of demand elasticity and energy price are evaluated and demonstrate that optimal regulation of these factors is crucial.

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