A Block-Coordinate-Descent Robust Approach to Incentive-Based Integrated Demand Response in Managing Multi-Energy Hubs with Must-Run Processes

Mehrdad Aghamohamadi, Student Member, IEEE, Amin Mahmoudi, Senior Member, IEEE, John K. Ward, Member, IEEE, Mohammed H. Haque, Senior Member, IEEE, João P. S. Catalão, Senior Member, IEEE

Abstract—This paper presents a new robust incentive-based integrated demand response (IDR) model for energy hub systems (EHSs). The considered incentive-based demand response (IBDR) schemes are interruptible/curtailable service (I/C) and capacity market program (CAP). The proposed IDR model integrates the arbitrage ability of EHS storages as well as energy conversion into the IDR model. The objective of the IDR optimization problem is to maximize/minimize the allocated incentives/penalties in targeted time periods by IBDR schemes, while supplying must-run processes with no interruption. Uncertainties of load and energy prices are considered through user-defined polyhedral uncertainty sets. A tri-level robust optimization (RO) is developed which includes a tri-level minimax-min problem. To solve the tri-level adaptive robust model, column-and-constraint generation (C&C) technique is employed by means of a decomposition methodology recasting the tri-level model into a single-level min problem and a bi-level max-min problem. Unlike previous RO models which solve the inner max-min problem by duality theory, a Block-coordinate-descent (BCD) methodology is used to solve the max-min problem by means of the first-order Taylor series in this study. The use of BCD technique instead of duality theory enables a recourse-based characterization of integer variables, such as EHS storage status, which was not applicable in previous models (due to use of duality theory). Moreover, Lagrange multipliers are eliminated as no duality is conducted. A post-event analysis is conducted to justify the long-term performance of the robust solutions and determine the optimal settings of the BCD robust approach. Results indicate that the IDR model significantly reduces the EHS input electricity in targeted time periods (four hours per day) by IBDR schemes and covers the required electricity with must-run processes by combined heat and power (CHP) unit, using natural gas. This implies a 2.13% reduction in the operation cost as incentives are obtained through IBDR schemes.

Index Terms—Demand response, Energy hub, Multi-energy system, Robust optimization.

NOMENCLATURE

A. Indices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>f/j</td>
<td>Index of EHS input/output energy carriers.</td>
</tr>
<tr>
<td>i</td>
<td>Index of energy converter units.</td>
</tr>
<tr>
<td>k</td>
<td>Index of energy storage systems.</td>
</tr>
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</table>

B. Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>A_{it}</td>
<td>Allocated incentive to purchased energy reductions.</td>
</tr>
<tr>
<td>A_{it}'</td>
<td>Extra incentive to extra energy reductions.</td>
</tr>
<tr>
<td>e_{it}</td>
<td>Nominal estimated energy f in hour t.</td>
</tr>
<tr>
<td>C_{it}</td>
<td>Maximum positive deviation of energy price f in hour t.</td>
</tr>
<tr>
<td>C_{it}'</td>
<td>Maximum negative deviation of energy price f in hour t.</td>
</tr>
<tr>
<td>d_{it}</td>
<td>Number of EHS input/output energy carriers.</td>
</tr>
<tr>
<td>E_{it}^E</td>
<td>Minimum/maximum value of E_{it}.</td>
</tr>
<tr>
<td>E_{it}^P</td>
<td>Energy loss for storage k.</td>
</tr>
<tr>
<td>I_{it}</td>
<td>Contract level for IBDR schemes.</td>
</tr>
<tr>
<td>I_{it}'</td>
<td>Nominal estimated value of load j in hour t.</td>
</tr>
<tr>
<td>P_{it}</td>
<td>Expected purchased energy in targeted periods.</td>
</tr>
<tr>
<td>P_{it}'</td>
<td>Maximum value of P_{it}.</td>
</tr>
<tr>
<td>P_{it}'^P/P_{it}'^c</td>
<td>Maximum/minimum value of P_{it}'.</td>
</tr>
<tr>
<td>P_{it}'^P/P_{it}'^c</td>
<td>Allocated penalty for breaching IBDR contracts.</td>
</tr>
<tr>
<td>Q_{it}</td>
<td>Set of uncertain prices.</td>
</tr>
<tr>
<td>Q_{it}'</td>
<td>Set of loads/storages.</td>
</tr>
<tr>
<td>Q_{it}'^I</td>
<td>Set of outer/inner loop iterations.</td>
</tr>
<tr>
<td>S_{it}</td>
<td>Start-up cost of converter unit i.</td>
</tr>
<tr>
<td>S_{it}^c</td>
<td>Converters' efficiency between input/output converters.</td>
</tr>
<tr>
<td>S_{it}^{chg}/S_{it}^{dis}</td>
<td>Charging/Discharging efficiency of storage k.</td>
</tr>
<tr>
<td>S_{it}^c</td>
<td>Charge/discharge new robust incentive.</td>
</tr>
<tr>
<td>S_{it}^c</td>
<td>Value of master problem.</td>
</tr>
<tr>
<td>Ψ</td>
<td>Uncertainty budget.</td>
</tr>
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</table>

C. Sets

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>Ξ</td>
<td>Set of converter units.</td>
</tr>
<tr>
<td>Ξ</td>
<td>Set of EHS input energy carriers.</td>
</tr>
<tr>
<td>Ξ</td>
<td>Set of &quot;here-and-now&quot;/wait-and-see variables.</td>
</tr>
<tr>
<td>Ξ</td>
<td>Set of loads/storages.</td>
</tr>
<tr>
<td>Ξ</td>
<td>Set of outer/inner loop iterations.</td>
</tr>
<tr>
<td>Ξ</td>
<td>Set of hours of the scheduling horizon.</td>
</tr>
<tr>
<td>Ξ</td>
<td>Set of uncertain prices/loads.</td>
</tr>
<tr>
<td>Ξ</td>
<td>Set of uncertainties/post-event scenarios.</td>
</tr>
</tbody>
</table>

D. Master Problem variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_{it}^U</td>
<td>Start-up cost of converter unit i, in hour t.</td>
</tr>
<tr>
<td>U_{it}</td>
<td>Status of converter unit i in hour t (1:on, 0:off).</td>
</tr>
<tr>
<td>A_t</td>
<td>Value of master problem.</td>
</tr>
</tbody>
</table>

E. Sub-problem variables

<table>
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<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c/l</td>
<td>Index of outer/inner loop iterations.</td>
</tr>
<tr>
<td>t</td>
<td>Index of hour.</td>
</tr>
</tbody>
</table>
and network participants, a new vision of DR has been introduced which is termed as "Integrated Demand Response" (IDR) [7-8]. IDR can easily make it possible for customers to actively participate in DR programs, even with non-curtailable loads, using energy conversion and storage facilities within EHS [9]. With IDR, it is possible to switch the consumed energy to other forms of energy at the EHS input ports, using converter elements such as CHP or micro turbines, or consume the stored energy by storage systems. As a result, the consumed energy at EHS input ports is leveraged from the upstream network perspective, while the actual energy demand by EHS output ports are met without interruption.

The arbitrage ability of EHS was employed in [10] to increase the benefits of a real world EHS through optimal interactions with upstream network and DR schemes. However, only PBDR schemes were considered in the model of [10]. Refs. [11] and [12] investigated the role of IDR in reducing the customers' payoffs, which is due to the ability of EHS in generating electricity from natural gas and battery systems in electricity peak periods where the electricity price reaches its maximum value. However, the models of [10-12] have only considered PBDR schemes in IDR. In fact, no IBDR schemes was included in the models in [10-12]. A similar PBDR approach was conducted in [13] to minimize the distributed smart energy hubs' payoffs considering only the electricity price. In [14], an optimization model was proposed to reduce energy costs of an EHS within IDR. However, the IDR model of [14] was also based on PBDR schemes and no IBDR scheme including incentive/penalty terms, was considered.

It is seen that, in many studies, IDR has been mostly modelled through PBDR schemes. This is because, PBDR schemes are strong and practical schemes that can motivate customers to participate in such a program and reduce their operation costs regarding upstream energy prices. However, IBDR schemes can also provide benefit recovery opportunities for energy system operators. In particular, IBDR schemes can direct EHS operators to switch the consumed electricity to other forms of energy to maximize their benefit and contribute in leveling the supply pattern from upstream network's perspective.

In [15] an IDR model was proposed for the application of both PBDR and IBDR schemes. However, the study of [15] does not consider the complex interactions among different energy carriers in EHS as it only considers the conventional single-energy demand response model of [15] for PBDR and IBDR schemes. In fact, the EHS responsiveness to IBDR and PBDR schemes are determined based on the DR model in [16], which was developed for single energy infrastructures. The obtained responsiveness patterns were then considered as operational constraints of the EHS. Therefore, the EHS responsiveness to DR schemes in [15] was not based on complex interactions among different energy carriers and/or the arbitrage ability of EHS through storage systems. The ability of EHS in supplying the required demand with multiple parallel paths allows the operator to participate in IBDR schemes flexibly, even with non-curtailable loads. For example, in the case of a considerable incentive allocation, the EHS operator switches the main electricity source to natural gas to cover the required electricity, using CHP or micro turbine. It can also consume the stored energy at the storage systems when directed.

I. INTRODUCTION

A. Problem Description

DEMAND response (DR) is characterized through price-based (PBDR) and incentive-based (IBDR) schemes [1]. PBDR schemes are strong demand response signals that are mostly considered in every sector of energy distribution/delivery, i.e., industrial, residential, etc. [2]. However, IBDR schemes are generally developed through incentive/penalty terms according to which the end users agree to shift/curtail their demand. These schemes generally involve a provisional process interruption for customers as they need to reduce their input electricity [3] to optimize energy costs, directly affecting production procedures. Moreover, customers with must-run processes, i.e., non-curtailable loads, cannot participate in DR schemes properly as their input electricity is dependent on their process which cannot be interrupted [4]. In addition, the associated uncertainties with energy prices and demand poses a noticeable effect on optimal energy management of industrial customers especially during the targeted time periods by DR schemes. These uncertainties can change the operation of EHS such as the level of purchased energy from network and EHS facility operation. Hence, a proper integrated demand response modeling alongside a successful uncertainty characterization leads to higher benefits for multi-energy industrial customers with must-run processes.

B. Review on IDR and IBDR/PBDR applications

With the development of the energy hub system (EHS) [5-6], employed as an interface between different energy infrastructures
Therefore, the responsive load demand is curtailed from the network's perspective, while the actual energy requirements remain unchanged.

Considering the literature of IDR modeling, further studies are required to 1) characterize the incentive-based DR schemes within IDR concept, and 2) provide an integrated demand response model which employs EHS arbitrage abilities of EHS in performing IDR, resulting in a global optimum solution. Although, developing an incentive-based IDR model leads to lower/higher operational costs/benefits for EHSs, the uncertainties associated with EHS loads as well as energy price deviations at the upstream network can still pose significant effects on optimal operation of these systems [17].

C. Review On Uncertainty Modelling in EHS Operation

In the recent study of [18] an incentive-based IDR model was proposed, incorporating complex interactions among different energy carriers in EHS. Nevertheless, the employed IDR model was based on the conventional non-linear EHS model and no uncertainty characterization approach was conducted.

To obtain immunized solutions against uncertainties, the uncertainty sources in EHS operation were modeled through stochastic programming (SP), characterizing RES generation in [19] and demand uncertainty in [20]. SP was also employed in the IDR model of [21] for characterizing wind speed uncertainty. However, SP methods in [19-20] did not consider any IDR in their models. Although IDR was considered in [21], the conducted SP approach faces two main challenges including the lack of tractable methodology and the need of full distributional knowledge of uncertain parameters, which may not be easily available in practice [22].

Hence, the robust optimization (RO) technique as a tractable approach, eliminating the need of having full distributional knowledge of uncertain parameters, has been recently developed and applied to EHS operation in [23, 24]. However, the model in [23] is a single-stage RO with no recourse decisions to be made after uncertainty realizations, which may result in non-exact and non-realistic operational solutions. Moreover, no IDR was considered in [23]. Although, the model in [24] is a two-stage RO with recourse decisions, it has an extensive mathematical burden which is due to the simultaneous employment of decomposition methodology, duality theory, and big-M transformation technique when solving the optimization problem. In addition, no IDR was modeled in [24].

The use of duality theory limits the application area of RO in terms of characterizing mixed-integer models for recourse decisioning. This is because the dual of a mixed-integer model is generally weak, non-tractable and complicated [25]. Therefore, no binary variable can be modelled for recourse decisioning in RO. This issue becomes more important when binary variables, such as the storage charging/discharging status, need to be obtained after uncertainty realizations as recourse decisions. However, in the study of [24], converter start-up decisions and storage charging/discharging status are obtained before the uncertainty realization to ease the implementation of duality theory in the second-stage problem. In [26], a simplified model was considered in which duality theory was used to solve the energy hub operation.

Thus, a proper uncertainty characterization model is required to 1) overcome the non-tractability issues associated with scenario-based models such as SP, and 2) be able to characterize binary variables as recourse decisions after the uncertainty realization to achieve an optimal and realistic solution.

A brief of the conducted literature review has been given by Table I in which each study has been compared to the proposed BCD robust incentive-based IDR model in this paper. Accordingly, it is seen that the proposed model considers both IBDR and PBDR programs while being subject to uncertainties. It also does not require distributional knowledge of the generated uncertainty scenarios, compared to scenario-based and SP models. Moreover, due to the use of BCD technique, it provides recourse-based IDR bids and storage operation. In terms of mathematical burden of the proposed BCD robust model, following highlights are presented:

No duality is conducted in the proposed BCD robust model,
No linearization is required.
Therefore, compared to previous dual-based RO models, the proposed model in this study is subject to a less extensive mathematical burden.

In real-world cases, the arbitrage ability of storage operation provides the most important flexibility in the system when uncertainties arise. Therefore, determining storage operation after uncertainty realization in BCD robust model can reasonably improve the feasibility of the operation, compared to previous dual-based models.

| Reference No. | Uncertainty modelling approach | Price-based IDR | Incentive-based IDR | Uncertainty consideration | Scenario generation required | Recourse-based IDR bids and storage operation |
|---------------|*********************************|----------------|---------------------|--------------------------|----------------------------|---------------------------------------------|
| [9]           | -                             | v              | x                   | No                       | -                          | x                                           |
| [10]          | -                             | v              | x                   | No                       | -                          | x                                           |
| [11]          | Stochastic programing         | v              | x                   | Yes                      | Yes                       | x                                           |
| [12]          | -                             | v              | x                   | No                       | -                          | x                                           |
| [13]          | Stochastic programing         | v              | x                   | Yes                      | Yes                       | x                                           |
| [14]          | Stochastic programing         | v              | x                   | Yes                      | Yes                       | x                                           |
| [15]          | Stochastic programing         | v              | x                   | Yes                      | Yes                       | x                                           |
| [16]          | Dual-based RO                 | v              | x                   | Yes                      | No                        | x                                           |
| [17]          | Dual-based RO                 | v              | x                   | Yes                      | No                        | x                                           |
| [18]          | -                             | v              | x                   | Yes                      | No                        | x                                           |
| [19]          | -                             | v              | x                   | Yes                      | No                        | x                                           |
| [20]          | Stochastic programing         | v              | x                   | Yes                      | Yes                       | x                                           |
| [21]          | -                             | v              | x                   | Yes                      | Yes                       | x                                           |
| [22]          | Proposed model BCD robust     | v              | v                   | Yes                      | No                        | x                                           |
| [23]          | Stochastic programing         | v              | x                   | Yes                      | No                        | x                                           |
| [24]          | Stochastic programing         | v              | x                   | Yes                      | No                        | x                                           |
| [25]          | Stochastic programing         | v              | x                   | Yes                      | No                        | x                                           |
| [26]          | Stochastic programing         | v              | x                   | Yes                      | No                        | x                                           |
| [27]          | Stochastic programing         | v              | x                   | Yes                      | No                        | x                                           |
| [28]          | Stochastic programing         | v              | x                   | Yes                      | No                        | x                                           |
| [29]          | Stochastic programing         | v              | x                   | Yes                      | No                        | x                                           |
As major advantages over previously reported methods, the proposed approach does not require duality-based cuts or linearization scheme. Moreover, some of the dual variables or Lagrange multipliers in dual-based RO models must be bounded through computationally expensive bounding parameter selection procedure which makes the model case-dependent. However, this issue is avoided in the proposed BCD robust model as no duality, and accordingly no dual variables or Lagrange multipliers are presented [30].

D. Motivations of this Study

Compared to the state-of-the-art, the main motivations of this study can be summarized as follows:

1) Current IDR studies have only modeled PBDR schemes, and no study has modeled the arbitrage ability of EHSs i.e., energy integration and storage, in performing IBDR schemes in the presence of uncertainties. Some studies have considered the uncertainty characterization approaches in their multi-energy management models, but no IBDR was modeled. Although PBDR schemes have a strong application in many energy systems today, it is time to go further and provide practical solutions for IBDR schemes in the IDR concept as incentive-based IDR has not been thoroughly investigated in this context.

2) In terms of uncertainty modeling, scenario-based and SP models are subject to non-tractability and non-accurate solutions, which is due to requiring a full distributional knowledge of uncertain parameters and a large number of scenarios (not applicable in practice). Therefore, further uncertainty modelling approaches are required to bypass the need of scenario generation.

3) Although RO has advantages compared to scenario-based and SP models, previous RO models in the literature fail to model binary variables such as EHS converters’ on/off status and storage charging/discharging status as recourse decisions, which is due to the use of duality theory in solving the inner min-max problem.

E. Contributions

This paper is a continuation of an earlier work [26] in which duality theory was used to solve the energy hub operation. Accordingly, following innovative contributions are proposed to significantly extend the existing research and comprehensively meet all issues in previous studies:

Contribution 1: An incentive-based IDR model is proposed to characterize the allocated incentive/penalty terms of IBDR schemes in operation of EHSs with non-curtable loads. The proposed model is conducted from the EHS’s perspective, as an energy customer facing upstream IBDR schemes. Two commonly implemented IBDR schemes in the industrial sector, including interruptible-curtable service (IC) and capacity market program (CAP), are considered. The incentive/penalty terms and contract levels of IC and CAP schemes are characterized to model the exact interactions between EHS and upstream IBDR schemes. Unlike a previous model [15], the proposed incentive-based IDR model in this paper is integrated into the EHS power flow constraints. In particular, it employs the arbitrage ability of storage systems as well as energy conversion throughout the EHS to maximize the incentive recovery and minimize penalty allocation.

Contribution 2: An adaptive robust optimization (ARO) approach is implemented to deal with the uncertainties of load and energy prices in operating EHSs. Uncertain parameters are characterized by bounded intervals in polyhedral uncertainty sets. The ARO model is a tri-level min-max-min problem which is not directly solvable. Therefore, a decomposition methodology is employed to recast the min-max-min ARO problem into two problems including a master problem and a sub-problem. A column-and-constraints (C&C) generation methodology is used to iteratively solve the decomposed problem through primal cutting planes. Several binary variables such as storage charging/discharging status as well as IC/CAP contract bids must be obtained after uncertainty realizations in the sub-problem to be able to compensate the effects of uncertain load/price as recourse decisions. Therefore, instead of using duality theory in solving the sub-problem, Block Coordinate Descent (BCD) method [27] is used in the proposed model (see the next contribution).

Contribution 3: In terms of solution methodology, BCD method [27] is used in the robust approach to iteratively solve the inner bi-level max-min sub-problem. This is implemented by means of Taylor series instead of transforming it into a single-level max-min problem by duality theory in conventional ARO models [28-29]. Therefore, the associated limitation in considering binary variables in the sub-problem is eliminated. In fact, mixed-integer models (even non-linear models) can be solved in the sub-problem through the proposed BCD robust model. As a result, uncertainty-dependent binary variables such as IDR bids and storage charging/discharging status can be obtained after uncertainty realization in the sub-problem as recourse decisions, resulting in more system flexibility in compensating the uncertainty effects such as load and price deviations. Moreover, the linearization of the dualized inner problem is avoided as the Lagrange multipliers are eliminated in this methodology. Thus, the case-sensitivity of the proposed incentive-based IDR model reduces as it does not reflect dual variables.

A post-event analysis has been presented against 25000 uncertainty realizations to validate the effectiveness of the obtained robust solutions against the uncertainties, compared to deterministic model.

II. ENERGY HUB MODEL

Fig. 1 is a general representation of an EHS as per the notations in the nomenclature. An EHS is an interface between energy suppliers at its input ports and energy requirements at output ports. These systems typically consume grid-bound energy carriers such as natural gas, electricity, and heat [31]. Energy inputs are directly consumed, converted to other forms of energy (i.e., using converter elements), and/or stored inside the system to be consumed in other operation time steps [32]. Since the input energy to EHS is consumed by several converters (such as natural gas that can be used in CHP, micro turbine, furnace, etc.), other variables named "dispatch factors" have to be considered in the model [32-34]. The idea of dispatch factors was first introduced by [35]. In this paper, the recent developed EHS model in [36] is employed in which the coupling matrix is replaced by two new matrices \( V \) and \( \mathbf{r} \) alongside a new independent continuous variable \( P_{d}^{t} \) which presents the input energy to each converter element. The relation between EHS input energy and the input energy to each converter is presented as (1a). Interested readers are referred to [36] for further information on the employed EHS model.
Each element of matrix $\eta$ ($\eta_{ji}^{con}$) represents the efficiency of converter unit $i$ converting its input energy to $j^{th}$ EHS load.

$$
\begin{pmatrix}
L_{1t} \\
L_{ft} \\
\vdots \\
L_{mt}
\end{pmatrix} = \begin{pmatrix}
\eta_{11}^{con} & \cdots & \eta_{1n}^{con} \\
\vdots & \ddots & \vdots \\
\eta_{m1}^{con} & \cdots & \eta_{mn}^{con}
\end{pmatrix}
\begin{pmatrix}
P_{ft}' \\
P_{ft}'' \\
\vdots \\
P_{ft}'''
\end{pmatrix} 
\quad (1c)
$$

where $0 \leq \eta_{ji}^{con} \leq 1$, $1 \leq i \leq n$, $1 \leq j \leq m$, $1 \leq t \leq T$; 

Accordingly, (1e) represents the energy flow between each EHS input energy and the input energy to converters.

$$
P_{ft} = \sum_{i \in S} P_{ft}' \cdot v_{ft} \quad \forall f \in S^{f}; \forall t \in S^{T} 
\quad (1e)
$$

Note that, storages can be placed in both inputs and outputs of the EHS. In fact, it is possible to transform storage power flows between inputs and outputs [40]. However, for modelling purposes in this paper (which is in line with previous studies such as [33], [35-36], and [40]), it is assumed that all the energy storages are at the output side. Accordingly, the linearized relationship between EHS inputs and outputs, considering converters and storage systems, would be as (1f) or its equivalent (1g).

$$
L = \eta \cdot P - S \cdot Q^{ch} + S \cdot Q^{dis} 
\quad (1f)
$$

Matrix $S$ describes how changes of the energy stors affect the hub output flows. Each element of $S$ illustrates the existence of energy trade between storage $k$ and output load $j$ ($S_{jk} \in \{0,1\}$). Therefore, in multi-period operation of the EHS, the following linear equation (1h) expresses the equilibrium constraint between demand and supply which is the extended form of (1f).

$$
L_{ft} = \sum_{i \in S} (\eta_{ji}^{con} \cdot P_{ft}') + \sum_{k \in K} (S_{jk} \cdot Q_{kt}^{dis} - S_{jk} \cdot Q_{kt}^{ch}) 
\quad \forall j \in S^{j}; \forall t \in S^{T} 
\quad (1h)
$$

III. DETERMINISTIC INCENTIVE-BASED IDR MODEL

This paper has focused on I/C and CAP schemes which both include incentive and penalty terms. In these schemes, incentives usually consist of up-front reservation payments, and customers face penalties for failure to curtail when called upon to do so. Participants of I/C and CAP schemes should reduce their demand to an acceptable level. This level is known as contract level ($C_{Lft}$) which is obtained as a part of the actual forecasted energy ($P_{ft}'$) in targeted time periods [18]. The values of incentive and penalty are allocated based on long-term contracts between the EHS operator and upstream network operator. However, the activation of DR service is broadcasted to the customers in different time frames such as daily or hourly notice of events [16]. The duration of notice is different from one customer to another and is based on the energy consumption of the customer and its flexibility in load shifting/curtailing.

Some small-scale customers may be flexible enough and can participate in DR schemes by hourly notice. Some other large-scale industrial customers, however, require more time to participate in DR schemes [37]. In this study, it is assumed that the EHS operator is notified about the IBDR activation on a day ahead basis.

Fig. 2 represents IBDR contract’s measures, as per the nomenclature of this paper. According to Fig. 2, participants can respond to the associated upstream IBDR schemes in three different ways [38], which are explained as follows:

**Case 1:** Customer fails to respond to IBDR scheme and cannot fulfill the contract level i.e., the consumed energy is higher than the contract level. In this case, penalties are allocated to the uncurtailed energy. The consumed energy in this case is denoted as $P_{ft}'$.

**Case 2:** Customer fulfills the contract level in the IBDR scheme. The consumed energy is exactly equal to the contract level, and therefore, incentives are fully allocated to the curtailed energy. The consumed energy in this case is denoted by $P_{ft}'$.

**Case 3:** Customer reduces its consumption more than the contract level in the IBDR scheme. In this situation, the curtailed energy is treated the same way as case 2, while the extra curtailed energy is rewarded with an additional rate of incentive.

In this section, a mixed integer linear incentive-based IDR model is presented for an EHS with non-curtailable loads, regarding possible EHS interaction with IBDR schemes, i.e., Cases 1-3. The model aims to a) optimally manage EHS facilities, including converters and storage systems, and b) maximize EHS benefits through IBDR schemes while mitigating actual energy consumptions by must-run processes at the EHS output.

The IDR model measures the EHS input electricity to gain maximum benefit and avoid any breach of contract in targeted time periods. Equation (2), represents the objective function and the associated constraints of the IDR model, considering the EHS operation and possible response to IBDR schemes by the EHS operator, i.e., Cases 1-3 presented by Fig. 2. The objective function (2a) encompasses five terms ($M_1, ..., M_5$).

The start-up cost of EHS converter units (i.e., CHP, heat exchanger, etc.) has been characterized in $M_1$, which is measured by the associated constraints of converter’s on/off binary variables. The cost of EHS purchased energy carriers, regarding the hourly energy prices over the scheduling horizon, is presented through $M_2$. Binary variable $\lambda_{ft}$ adds the third term (i.e., $M_3$) to the objective function (2a) if the EHS fails to fulfill the contract (Case 1 in Fig. 2). In fact, $M_3$ represents the penalty allocations pertaining to any possible breach of contract during the targeted time periods of IBDR schemes. In contrary, if the EHS fulfills the contract (Case 2 in Fig. 2), incentives would be allocated. This is modeled by binary variables $\lambda_{ft}$ in $M_4$ which models the obtained incentives. The extra incentives due to higher values of purchased energy reductions (Case 3 in Fig. 2) are characterized by $M_5$. The considered incentives/penalties in the IDR model (2), i.e., $Pen_{ft}'$, $A_{ft}'$, $A_{ft}''$, are subject to change according to time and energy type.

In fact, they are counted for all values of indices $f$ and $t$. Therefore,
they all can be fixed or time-varying in the operation of IDR (incentives and penalties for each energy type are vectors based on time).

Converters’ start-up constraints are presented by (2b)-(2c). Note that, \( C^\text{SU}_i \) is a variables while \( SU^*_i \) is a parameter reflecting the start-up cost of converter \( i \) (see nomenclature).

\[
\begin{align*}
&\min \sum_{t \in T} \sum_{i \in \mathcal{E}} C^\text{SU}_{it} + \sum_{t \in T} \sum_{f \in F} (C^*_f - P^*_f) \\
&+ \sum_{t \in T} \sum_{f \in F} (P^+_f - (C^*_f \cdot \lambda^+_f) \cdot P^\text{en}_f) \quad \text{(2a)} \\
&- \sum_{t \in T} \sum_{f \in F} (P^+_f - (C^*_f \cdot \lambda^-_f) \cdot A^+_f) \quad \text{(2b)} \\
&- \sum_{t \in T} \sum_{f \in F} \left( (P^+_f - (C^*_f \cdot \lambda^+_f) \cdot A^+_f) \right) \quad \text{(2c)}
\end{align*}
\]

Constraint (2j) limits the converters’ input energy to their allowable ranges.

The energy balance of storage systems, considering charging, discharging, and standby modes is illustratively presented by Fig. 3. As it can be seen, the charging and discharging status, i.e., \( E^c_t \) and \( E^d_t \), respectively, are subject to loss of energy, which is due to the storage efficiency in charging/discharging mode. Moreover, each storage is subject to steady-state mode losses, i.e., \( E^s \). Accordingly, the dynamic energy balance for each storage \( k \) is expressed by (2k). Each storage at the final operational time period must have the same energy level as the first time period, which is known as end-coupling constraint and is expressed by (2l). If this equation is not considered in the model, the optimization still provides the optimal solutions. In fact, considering the end-coupling constraint can be similar to fixing the final value of storage level as its initial value. Ignoring this constraint does not affect the optimality of the solutions. The only difference is that the operation of the storage can be slightly different at the final period as it does not need to be charged again. The energy level, charging rate, and discharging rate for each storage are limited to their allowable ranges through (2m), (2n), and (2o), respectively. Constraint (2p) ensures that each storage operates in one mode only (charge, discharge, or out-of-use) at a time. Finally, the types of variables are specified in (2q) and (2r).

IV. ROBUST APPROACH TO SOLVE THE INCENTIVE-BASED IDR MODEL

A. Uncertainty Set Realization

The uncertainties associated with load and energy prices are considered by bounded intervals through polyhedral uncertainty sets as (3). The uncertain parameters \( \mathcal{L}_f \) and \( C^*_f \) in (3) are allowed to deviate from their nominal estimated values \( \mathcal{L}_{\text{est}} \) and \( C^*_f \) in positive and negative directions, which is shown by (3a) and (3b). The considered deviations are limited to their user-defined allowable ranges through constraints (3c) and (3d). The number of uncertain parameters pertaining to load and energy prices is determined by the uncertainty budget \( \Psi \) in (3e). If \( \Psi = 0 \), no uncertain parameter can deviate from its estimated value, resulting in a deterministic model.

\[
\begin{align*}
\Xi^\text{UL} &= \left\{ \mathcal{L}_{\text{est}} + L^\text{dev} - L^\text{dev}^- \mid \forall j \in \mathcal{E} ; \forall t \in \mathcal{T} \right\} \\
\Xi^\text{UL} &= \left\{ C^*_f + C^\text{dev} - C^\text{dev}^- \mid \forall f \in \mathcal{E} ; \forall t \in \mathcal{T} \right\}
\end{align*}
\]

By increasing the uncertainties are allowed to deviate. The highest value for \( \Psi \) is equal to the total number of uncertain parameters allowing all uncertain parameters to deviate from their nominal estimates. Since robust optimization determines the solution based on the worst-case realization of uncertain parameters, it selects the maximum allowable deviation for each uncertain parameter. In fact, in the optimization \( L^\text{dev}^\pm = L^\text{dev}^- + L^\text{dev}^+ \) and \( C^\text{dev}^\pm = C^\text{dev}^- + C^\text{dev}^+ \).
Thus, equation (3e) illustrates the number of uncertain parameters deviating from their nominal values which is mathematically lower than $\Psi$. However, the optimization selects the exact value of $\Psi$ as the uncertainty budget which is due to the conservativeness of robust optimization. Due to this reason, robust optimization has been considered as a conservative approach in uncertainty modelling, but more immunized against uncertainties. However, it is the post-event analysis which determines the optimal robust setting of the model. In fact, the post-event analysis examines different values of $\Psi$, $\eta^{\text{dev} \pm}$, and $\xi^{\text{dev} \pm}$ against trial scenarios to determine the most suitable robust settings. A more detailed explanation for this procedure is given in Section VI.C, where the post-event analysis is presented.

B. Proposed Adaptive Robust Model

Two main decisions are made in RO, including "here-and-now" decisions, which are obtained before any uncertainty realizations, and "wait-and-see" decisions, which are obtained after the realization of uncertain parameters [39]. As a result, when the uncertainties are realized, these "wait-and-see" recourse decisions can be adaptively made in response to uncertainty variation. This adaptive decision making capability brings an adaptive character into our robust model and makes it superior than a single-stage robust approach which has no "wait-and-see" recourse adaptivity. Due to this adaptive character, the proposed model has been called adaptive robust optimization.

In this study, the converters’ start-up binary variables (i.e., $U_k^m$) are considered as "here-and-now" decisions, while other variables including EHS facilities’ operation and input energy carriers are determined as "wait-and-see" decisions. The adaptive robust model is expressed through a tri-level min-max-min optimization problem as (4).

$$\min_{X \in \mathbb{R}^n} \left( A' \cdot X + \max_{Y \in \mathbb{R}^m} \min_{Y \in \mathbb{R}^m} F'(Y) \right)$$

s.t.

$$\sum \{ X \in \{0,1\}^n | CX \geq D \}$$

$$\sum Y \in \mathbb{R}^n | E(Y, \bar{U}) \geq 0$$

$$\sum \{ U \in \mathbb{R}^m | \bar{U} = \bar{U} + U^{\text{dev} \pm} \}$$

In (4a), the outer min problem minimizes the term $A' \cdot X$ over "here-and-now" variables denoted by vector $X$. This term represents $\mathbb{M}1$ in (2a) as the only dependent term on start-up variables to be obtained before any uncertainty realizations. The outer min problem is subject to constraint (4b) which represents the set of constraints (2b)-(2c) as the start-up characteristics. The inner min problem in (4a) minimizes the term $F'(Y)$ over "wait-and-see" variables, while the inner max problem maximizes it over the uncertain parameters.

The term $F'(Y)$ represents the set of the remaining terms in (2a) (i.e., $M2-M5$) which are dependent on "wait-and-see" variables, including EHS operation and energy flow variables. Therefore, the inner min problem is subject to constraints (2d)-(2p), represented by (4c), while the inner max problem is subject to uncertainty set realizations as (4d) which is the compact form of (3a)-(3b). A decomposition methodology is employed to decompose the tri-level min-max-min problem into a single-level min problem and a bi-level max-min problem by means of column-and-constraint generation technique [29]. The single-level min problem is called "master problem" and the bi-level max-min problem is called "sub-problem", hereafter. The extended version of master and sub-problem is given in the supplementary file.

C. Solution Methodology for the Tri-level Robust Model

The solution methodology includes two iterative loops namely the inner loop and the outer loop as depicted by Fig. 4. The compact mathematical formulations of master problem and sub-problem are given in Fig. 4, as per the notations in nomenclature. The outer loop in Fig. 4 is shown by red arrows while the inner loop is shown by green arrows. Each loop is described as follows:

1) Outer loop

The outer loop is responsible for transporting the obtained "here-and-now" variables in the master problem to the sub-problem (prior to uncertainty realizations), on one hand, and submitting primal cutting planes from sub-problem to master problem, on the other hand. The sub-problem is then solved given the obtained "here-and-now" variables to determine "wait-and-see" variables and the worst-case realization of uncertain parameters to be sent back to the master problem in the next iteration. Therefore, a complete set of primal cuts are added to the master problem in each iteration and "here-and-now" variables are updated to be sent to the sub-problem. This procedure iterates through the outer loop till the convergence criteria is met (values of master and sub-problem become sufficiently close) which terminates the outer loop and the robust solution is obtained.

2) Inner loop

Since the sub-problem is a bi-level max-min problem, it cannot be solved directly. In previous robust models such as [28-29], it was recast into a single-level max-min problem using duality theory which limits the application of RO. This is because no binary variables can be considered in the inner max-min problem as recourse decisions – dual of a mixed integer model is generally weak, non-tractable and complicated [25]. An example of these binary variables is the storage charging/discharging status that needs to be obtained after uncertainty realizations to compensate the shortage/surplus of energy due to uncertainties of load. Another example is the binary variables indicating the status of EHS in terms of responsiveness to the DR program which must be obtained after uncertainty realization to maximize the benefits of EHS. However, none of these are applicable through conventional dual-based robust models. This results in two unfavorable approaches (considering the literature), which are:

1) Removing the binary variables which is not applicable in some studies, like in this study where binary variables are allocated for failing or fulfilling IBDR contracts, i.e., $\lambda_{RT}$, $\lambda_{Rr}$, and must be considered.

2) Characterizing binary recourse variables in the outer min problem (master problem) with no uncertainty characterization, while these variables are meant to be obtained after uncertainty realizations in the inner max-min sub-problem as recourse decisions. This results in non-optimal and non-realistic solutions. Therefore, in the proposed robust model in this paper, the BCD method is used to solve the sub-problem through an iterative methodology instead of transforming it to a single-level problem by duality theory. As a result, there is no limitation in considering binary variables in the inner max-min sub-problem.
Therefore, the storage charging/discharging status variables as well as binary variables for failing or fulfilling IBDR contracts can be obtained after uncertainty realizations. This results in realistic and optimal solutions compared to those of the conventional dual-based robust models.

BCD technique was originally devised to deal with single-level problems. By extending the application of the BCD technique to solve the two-level max-min sub-problem (resulted from the C&C generation technique), it is possible to avoid duality theory in solving the sub-problem. Using the BCD methodology, the sub-problem is recast into a first-stage sub-problem (characterizing the inner min problem), and a second-stage sub-problem (characterizing the inner max problem). The second-stage sub-problem is built upon the first order Taylor series approximation of uncertain parameters in the first-stage sub-problem. The first-stage sub-problem determines the “wait-and-see” variables considering the worst-case realization of uncertain parameters obtained in the second-stage sub-problem, considering the obtained "here-and-now” variables from the master problem. The second-stage sub-problem is then solved to update the worst-case realization of uncertain parameters, considering the obtained "wait-and-see" variables in the first-stage sub-problem. This iterative procedure is executed through the inner loop. Therefore, in each iteration of the outer loop, the inner loop is executed till it converges (the value of first and second-stage sub-problems become sufficiently close).

The extended form of the BCD robust model has been presented in the Appendix at the end of this paper.

V. PERFORMANCE EVALUATION

A. Data Set

Fig. 5 shows the case study. The associated characteristics of the EHS are also presented in Fig. 5. These characteristics are taken from [40]. All inputs and outputs are in per unit (p.u.) with the base value of 100 kW. Table II lists the allocated incentives, penalties, and energy prices associated with upstream electricity market.

In TOU rate, hours 01-06, 07-14/21-24, and 15-20 pertain to valley, off-peak, and peak period, respectively. Note that, both the energy prices and the load demand at the EHS output are considered as uncertain parameters meaning they may deviate in hourly scale. This is due to the fact that, the operation simulations are conducted on an hourly basis. However, the scale of the simulation can be reduced to any other scale. This is in line with many previous studies such as [8], [14-16], [19-21], [23-24], [26], [28], [32], [35-36], and [40]. Input parameters and their allowable ranges are given by Table III. The value of incentives/penalties are taken from [38], while, TOU rates are taken from [41]. Targeted time periods for IBDR schemes are hours 17-20 with 2 p.u. contract level. Four cases are considered with different deviation range of uncertain parameters. These cases include Case 1, Case 2, Case 3, and Case 4 with 5%, 10%, 15%, and 20% deviation, respectively. Since, there are five uncertain parameters (i.e., three input energy prices and two load consumptions), there are 5 x 24 = 120 uncertain parameters during a 24-h scheduling horizon (i.e., 0 ≤ Ψ ≤ 120). Fig. 6 shows the hourly electricity/heat consumption by EHS at output ports, which are uninterruptible. Load data has been taken from [42]. Therefore, the energy consumptions in Fig. 6 are not shiftable/curtailable but subject to uncertainty.
Note that, any deviation in optimization parameters, such as input energy prices, does not affect the optimality of the solution. This is due to the fact that the optimization still reaches the global optima as the model is linear. Simulations have been conducted in GAMS software programming environment through GUROBI solver [43], on a computer with 8GB RAM and a Core-i7 CPU.

B. Robust Solutions

Table IV shows the EHS total operation cost for cases 1-4. It is observed that, $\Psi = 0$ stands as the deterministic representation of the proposed incentive-based IDR model, regardless of the deviation range of uncertain parameters. In fact, when $\Psi = 0$ no uncertain parameter can deviate from its estimate according to (3g).

As the uncertainty budget $\Psi$ increments in Table IV, the total operation cost increases for all cases. This is because, increasing $\Psi$ enlarges the size of the uncertainty set, i.e., a higher number of uncertain parameters can deviate from their nominal values, resulting in more robust operation at a higher expense. For a given value of $\Psi > 0$, the deviation of uncertain parameters in Case 4 > Case 3 > Case 2 > Case 1, which makes the worst-case realization of uncertainties even worst. Therefore, the operation cost of Case 4 > Case 3 > Case 2 > Case 1 for each value of $\Psi$. The computation time for all cases with different robust settings is between 3 and 4.5 seconds.

C. Validation of the Obtained Results

1) Comparison of the Solutions against PBDR scheme only

To illustrate the effectiveness of the proposed model, the studied EHS is operated in two scenarios. In the first scenario, only TOU pricing (which is a PBDR scheme) is considered, while in the second scenario both TOU and incentive-based IDR are considered. Therefore, the first scenario, i.e., TOU only, the objective function includes $M1$, and $M2$, while, the second scenario includes all terms $M1-M5$. The results of this operation are given by Fig. 7. As it is seen, the value of the objective function is lower in all cases 1-4 when considering incentive-based IDR in addition to TOU pricing. This is because the proposed model maximizes the obtained incentives while minimizing possible penalty allocations through IBDR schemes by optimally measuring input energies.

2) Validation of the Robust Solutions by Post-event Analysis

The obtained RO solutions become more immunized against uncertainties as the robustness level (value of $\Psi$, $\Delta_{\text{dev}}^{\text{PE}}$, and $\bar{C}_{\Delta t}^{\text{dev}}$) increases. This feature is called "robustness worth" which is a result of high conservativeness, meaning that the prosumer will face minimum extra costs if uncertainties arise. However, this immunization comes at a higher expense which is called "robustness cost". This is shown in Table IV in which the value of objective function increases as the value of $\Psi$, $\Delta_{\text{dev}}^{\text{PE}}$, and $\bar{C}_{\Delta t}^{\text{dev}}$ increase. Therefore, selecting a very high robustness level leads to over-conservative solutions, resulting in unnecessary robustness cost and impractical robustness worth, and vice versa.

To provide an optimal balance between robustness worth and cost, and to avoid over/under conservative RO solutions, a post-event analysis has been conducted in this study. According to this analysis, the obtained RO solutions for each robustness level (as indicated by Table IV) are examined against a sufficiently large number of uncertainty realizations, while the obtained operational solutions are fixed. The value of objective function is then normalized for all scenarios. The robust settings resulting in the lowest value of post-event value are considered as optimal robust settings of the model.

The energy shortage in each scenario has been modeled by additional free variables in the post-event analysis, to guarantee the feasibility of the solutions. This means, if a power shortage occurs, the additional free variables models the load shed. Therefore, the mixed-integer linear model (2) becomes a simple linear model, only characterizing power shortage. The mathematical model of post-event analysis is given as (8). Note that, only constraints associated with power shortage are considered in post-event model and other constraints are eliminated as they are fixed on the obtained robust solutions (they are constants and have no effect on the post-event value). The subscript $(\cdot)$ in (8), indicates the associated variables in each trial scenario. The post-event value is obtained by (8a) in which the summation of the absolute value of power shortage, i.e., $\sum_{j \in \mathcal{J}_b} y_{j t}^{sh}$, for each trial scenario is normalized over the total number of trial scenarios. The only constraint being affected with power shortage is the energy balance constraint (2i) which is modified and presented by (8b).

$$P_{\text{E}} = \sum_{s \in \mathcal{S}} \sum_{j \in \mathcal{J}_b} \sum_{l \in \mathcal{L}} \left( \frac{|y_{j t}^{sh}|}{\text{Total number of scenarios}} \right)$$

(8a)

$$\bar{L}_{jt} = \sum_{k \in \mathcal{K}} \left( S_{jk} \cdot \eta_k \cdot Q_{dk} \cdot Q_{kt} - \frac{1}{\eta_k} Q_{kt} \right)$$

(8b)

$$\forall s \in \mathcal{S}; \forall \mathcal{J}_b; \forall \mathcal{L}; \forall \mathcal{K}; \forall \eta_k; \forall \mathcal{V}_t \subseteq \mathcal{V}; \forall x_{j t} \subseteq \mathcal{X}; \forall v_{j t} \subseteq \mathcal{V};$$

(8c)
These robust settings, therefore, result in the most practical, feasible, and economical performance of the proposed BCD robust model (see Fig. 8).

D. EHS Operational Solutions Based on the Obtained Optimal Robust Settings by Post-event Analysis

The following operational results are obtained under the optimum robust settings by post-event analysis in previous subsection. Fig. 9 shows the EHS optimal purchased energies, including electricity, natural gas, and heat. It can be seen that, the EHS is more likely to consume input natural gas to generate electricity and heat (using CHP and furnace units) in most of the time. In targeted time periods by IBDR schemes i.e., hours 17 and 19-20, it has optimally purchased 2 p.u. of electric energy, considering the 2 p.u contract level by IBDR schemes. Therefore, incentives have been fully allocated in these hours, according to $M4$ in (2a). However, the input electricity has not been reduced in hour 18, which means the third term in (2a) (i.e., $M3$) has been activated in this hour, allocating penalties to the unreduced purchased electricity.

Fig. 10 illustrates the energy balance and charging/discharging status for both heat and electricity storage systems. These solutions are obtained as recourse decisions due to the employment of BCD method in solving the max–min sub-problem. As it can be seen, the electricity storage has been considerably discharged in hours 17 and 19-20 to increase the EHS obtained incentives in targeted time periods. In targeted time periods by IBDR schemes i.e., hours 17 and 19-20, it has optimally purchased 2 p.u. of electric energy, considering the 2 p.u contract level by IBDR schemes. Therefore, incentives have been fully allocated in these hours, according to $M4$ in (2a). However, the input electricity has not been reduced in hour 18, which means the third term in (2a) (i.e., $M3$) has been activated in this hour, allocating penalties to the unreduced purchased electricity.

Fig. 11 shows the EHS optimal energy balance and charging/discharging status and energy balance after the implementation of the proposed BCD robust IBDR model. Note that, the required heat energy at the EHS output port in hours 15-24 (see Fig. 8), is covered by CHP as the input heat energy and the heat storage energy level is almost zero in these hours. The contribution of electricity storage system is shown in Fig. 11, indicating the variables associated with energy balance constraint (2i), including converters’ output and charging/discharging of electricity storage. Note that, the EHS has supplied the consumed energy with no load shifting/curtailment at the demand side while participating in upstream IBDR schemes by optimal measuring of its input electricity. In fact, the actual energy consumption by non-curtailable loads at the EHS output port has remained unchanged while the input energy at the EHS input ports has been modified to satisfy upstream network IBDR schemes.

The hourly total operation cost of EHS, before and after the implementation of the proposed BCD robust IBDR model, is shown in Fig. 12. As it can be seen, the operation cost significantly reduces in hours 17 and 19-20, when using the proposed IDR model. This is because the EHS reasonably curtails the input electricity to obtain maximum benefits and avoid penalty allocations in these hours. It also consumes the stored electricity in the storage system to cover the required electricity in these hours (see Fig. 11). In hour 18, however, the operation cost under the proposed incentive-based IDR model increases as it purchases a higher value of input electricity, as shown in Fig. 9, resulting in a breach of contract and penalty allocations in this hour.
was not applicable in previous RO applications. This was because of eliminating the need of duality theory in solving the inner max-min problem.

Results showed that the IDR model reasonably reduces the EHS input electricity in targeted time periods (four hours per day) by IBDR schemes and covered the required electricity by CHP unit, using natural gas. Accordingly, no load curtailment was conducted at the EHS outputs, while the input energy is reduced reasonably to participate in upstream DR schemes. This resulted in a 2.13% reduction in the operation cost as incentives are obtained through IBDR schemes. The effectiveness of the incentive-based IDR model was illustrated by comparing the optimal results against a case with no incentive/penalty terms. The robust solutions were also compared to deterministic solutions in the post-event analysis. The value of objective function was considerably increased in the RO model, compared to the deterministic model. However, the RO solutions were more immunized against uncertainties as indicated by the post-event analysis. Accordingly, the post-event cost for the deterministic solutions was $1856, while this value was $1130 for RO solutions.

The proposed model can assist in the optimal operation of EHS facilities to efficiently participate in IBDR programs. The future work will focus on characterizing other types of IBDR schemes, such as emergency demand response.

### APPENDIX: EXTENDED FORM OF THE BCD ROBUST MODEL

#### A. Master Problem

Master problem is solved to determine "here-and-now" decision variables including inverters’ start-up solutions while being subject to start-up constraints only. Therefore, the objective function (9a) includes the term $M_1$ of the deterministic objective function (2a) and is subject to constraints (2b)-(2c) including start-up variables only, i.e., $U_i(t)$ and $C_{it}^U$. Other operational constraints, therefore, are added as primal cuts in each iteration of the outer loop. The epigraph form of the master problem including primal cutting planes given by sub-problem, can be written as (9).

\[
\begin{align*}
\text{Min } & \quad \Lambda_1 \equiv \sum_{t \in T} \sum_{i \in \mathcal{E}} C_{it}^U + \Psi; \\
\text{s.t.} & \quad C_{it}^U \geq S_{uc_i} \cdot \left( U_{i,(t+1)} - U_{i,tf} \right); \quad i \in \mathcal{E}; \quad \forall t \in \mathcal{T} \\
& \quad C_{it}^U \geq S_{uc_i} \cdot \left( U_{it} - U_{i,(t-1)} \right); \quad i \in \mathcal{E}; \quad t \in \mathcal{T}; \quad t > 1
\end{align*}
\]

\[
\Psi \geq \sum_{t \in \mathcal{T}} \sum_{f \in \mathcal{E}} \left( C_{ft} \cdot P_{ftc} \right) \\
+ \sum_{t \in \mathcal{T}} \sum_{f \in \mathcal{E}} \left( P_{ftc} + \left( C_{ftc} \lambda_{ftc} + \lambda_{ftc} \cdot \theta^{+}_{ftc} \right) \cdot \text{Pen}_{ftc} \right) \\
- \sum_{t \in \mathcal{T}} \sum_{f \in \mathcal{E}} \left( \left( C_{ftc} \lambda_{ftc} - P_{ftc} \right) \cdot A_{ftc}^{\theta} \right) \quad \forall \mathcal{E}
\]

where $\lambda_{ftc} \geq 0, \forall f \in \mathcal{F}; \quad t \in \mathcal{T}; \quad \forall \mathcal{E}$

\[
\lambda_{ftc} + \lambda_{ftc} \leq 1; \quad \forall f \in \mathcal{F}; \quad \forall t \in \mathcal{T}; \quad \forall \mathcal{E}
\]

#### B. Robust Sub-Problem

The proposed model reduces the input electricity in targeted time periods by IBDR schemes. Therefore, there would be more incentive allocation than penalty allocation, resulting in higher sensitivity of CoE to incentives.

### VI. CONCLUSION

A BCD robust incentive-based IDR model for multi-energy hub systems with non-curtailable loads was presented in this paper. The aim of the proposed IDR model was to consider incentive/penalty terms in IBDR contracts and provide a reasonable energy purchasing pattern for EHS under short-term operation uncertainties. The uncertainties of EHS of loads and energy prices were integrated into the model through a tri-level min-max-min robust optimization approach. BCD method was employed to solve the inner max-min problem, enabling the proposed IDR model to characterize binary variables after uncertainty realizations, which...
in (2a) and the associated EHS operation and energy flow constraints (2d)-(2p). The first-stage sub-problem is given by (10).

\[
\begin{align*}
\text{Min} N_{\text{H}} & \equiv \sum_{t \in T} \sum_{f \in S_T} (C_{ft} + P_{ft}) \\
& + \sum_{t \in T} \sum_{f \in S_T} \left( P_{ft}^+ - (C_{ft} + \lambda_{ft}) \right) \cdot P_{en,t} \\
& - \sum_{t \in T} \sum_{f \in S_T} \left( P_{ft}^- - C_{ft} - \lambda_{ft} - A_{ft} \right) \\
& - \sum_{t \in T} \sum_{f \in S_T} \lambda_{ft}^+ \cdot P_{ft}^+ + \lambda_{ft}^- \cdot P_{ft}^- \\
& \text{s.t.} \quad \lambda_{ft}^+ + \lambda_{ft}^- \leq 1, \quad \forall f \in S_T, \forall t \in T \\
P_{ft} = \sum_{t \in T} P_{ft}^+ \cdot v_T; \quad \forall f \in S_T, \forall t \in T \\
P_{ft}^- = \sum_{t \in T} P_{ft}^- \cdot v_T; \quad \forall f \in S_T, \forall t \in T \\
C_{ft} = \sum_{t \in T} C_{ft} \cdot v_T; \quad \forall f \in S_T, \forall t \in T \\
0 \leq P_{ft} \leq C_{ft} + \lambda_{ft}; \quad \forall f \in S_T, \forall t \in T \\
\lambda_{ft}^+ \leq \lambda_{ft}^- \leq P_{ft}^+; \quad \forall f \in S_T, \forall t \in T \\
C_{ft} \cdot \lambda_{ft}^- \leq C_{ft} \cdot \lambda_{ft}^+ \leq P_{ft}^+; \quad \forall f \in S_T, \forall t \in T \\
\lambda_{ft}^+ + \lambda_{ft}^- \leq 1, \quad \forall f \in S_T, \forall t \in T
\end{align*}
\]

The objective function (9a) minimizes the converters' start-up costs by determining the start-up solutions as "here-and-now" decisions while being subject to start-up constraints (9b)-(9c). Constraints (9d)-(9q) represent the primal cuts submitted from the sub-problem. The subscript \((\cdot)\) and the superscript \((\cdot)\) in (9), indicate the associated "wait-and-see" variables and the fixed values of the uncertain parameters at iteration \(c\) of the column-and-constraint generation algorithm, respectively. Constraints (9e)-(9q) are equivalent to constraints (2d)-(2p). However, the forecast values of uncertain parameters in (2), i.e., \(\bar{C}_{ft}\) and \(\bar{L}_{ft}\), are replaced with the obtained worst-case realizations from the sub-problem at iteration \(c\), i.e., \(\bar{C}_{ft}\) and \(\bar{L}_{ft}\). The obtained start-up variables in master problem, i.e., \(U_{it}\) and \(C_{SU}\), are then sent to the sub-problem as fixed values to determine both "wait-and-see" decision variables, i.e., EHS operation and energy flow variables, and the new worst-case realization of uncertain parameters.

B. Sub-problem

Sub-problem is solved to determine 1) optimal EHS operation and energy flow variables as "wait-and-see" decisions, and 2) the worst-case realization of uncertain parameters, given the fixed values of start-up variables obtained in master problem. In the conducted BCD methodology, the first-stage sub-problem is responsible for determining "wait-and-see" decision variables, while, the second-stage sub-problem determines the worst-case realization of uncertain parameters. In the following, both first and second-stage sub-problems are presented and discussed.

1) First-stage Sub-problem:

Since, the start-up variables are fixed on their obtained values by master problem, the term of the deterministic objective function (2a), as well as the start-up constraints (2b)-(2c) are not included in the first-stage sub-problem. Instead, it includes the term \(M2-M5\)
including, \( \lambda_{ft}^{r}, \lambda_{ft}^{q}, \sigma_{ft}^{c}_{q} \), and \( \sigma_{ft}^{d}_{q} \), are characterized in the second-stage sub-problem to be obtained as "wait-and-see" variables indicating recourse decisions. This is due to the employment of BCD method which iteratively solves the bi-level max-min sub-problem through the inner loop (see Fig. 5) instead of transforming it to a single max problem by duality theory. Therefore, it is possible to determine the EHS status in fulfilling/failing IBDR stage subproblem as well as storage charging/discharging status after realizing the uncertainties in the sub-problem as "wait-and-see" decisions.

2) Second-stage Sub-problem:
The second-stage sub-problem is built upon the first order Taylor series approximation of the first-stage sub-problem over the uncertain parameters in the previous iteration of BCD method, i.e., \( z = 1 \). Therefore, at iteration \( z \) of the BCD method, the second-stage sub-problem is cast as (11):

\[
\begin{align}
\max \ & \sum_{\epsilon} \sum_{t} \sum_{z} f_{j}^{z}(z) + \sum_{\epsilon} \sum_{t} \sum_{z} f_{j}^{z}(z) - \| z^{r} - z^{q} \|^2 \\
\text{s.t.} & \qquad (3a)-(3g)
\end{align}
\]

(11a)

The second-stage sub-problem determines the worst-case realization of uncertain parameters at each iteration \( z \) of the BCD method, by which the approximated objective function (11a) is maximized subject to the uncertainty sets presented by (11b).

REFERENCES


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