Optimal Management of Demand Response Aggregators considering Customers’ Preferences within Distribution Networks

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Abstract: In this paper, a privacy-based demand response (DR) trading scheme among end-users and DR aggregators (DRAs) is proposed within the retail market framework and by Distribution Platform Optimizer (DPO). This scheme aims to obtain the optimum DR volume to be exchanged while considering both DRAs’ and customers’ preferences. A bilevel programming model is formulated in a day-ahead market within retail markets. In the upper-level problem, the total operation cost of the distribution system, which consists of DRAs’ cost and other electricity trading costs, is minimized. The production volatility of renewable energy resources is also taken into account in this level through stochastic two-stage programming and Monte-Carlo Simulation method. In the lower-level problem, the electricity bill for customers is minimized for customers. The income from DR selling is maximized based on DR prices through secure communication of household energy management systems (HEMS) and DRA. To solve this convex and continuous bilevel problem, it is converted to an equivalent single-level problem by adding primal and dual constraints of lower level as well as its strong duality condition. The results demonstrate the effectiveness of different DR prices and different number of DRAs on hourly DR volume, hourly DR cost and power exchange between the studied network and the upstream network.

Keywords: Demand response, bi-level programming, distribution network, retail market, stochastic modeling, two-stage programming.

Nomenclature

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<th>Parameters</th>
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<tr>
<td>DG</td>
<td>Index of DG.</td>
<td>Index of linear partitions in linearization</td>
</tr>
<tr>
<td>n,n' (NN)</td>
<td>Index (set) of nodes.</td>
<td>Total DR potential</td>
</tr>
<tr>
<td>s (NS)</td>
<td>Index (set) of scenarios.</td>
<td>Maximum and minimum voltage, nominal voltage.</td>
</tr>
<tr>
<td>scen</td>
<td>Superscripts for wind scenarios.</td>
<td>Upper limit in the discretization of quadratic flow (kVA).</td>
</tr>
<tr>
<td>PV</td>
<td>Superscripts for Solar systems.</td>
<td>Maximum power capacity of each σ.</td>
</tr>
<tr>
<td>WF</td>
<td>Superscripts for wind farms.</td>
<td>Distribution lines resistance and reactance.</td>
</tr>
<tr>
<td>PCC</td>
<td>Superscripts for point of connection to upstream network.</td>
<td>DR price for DR aggregator in bus n and hour t.</td>
</tr>
<tr>
<td>t (NT)</td>
<td>Index (set) of hours.</td>
<td>Percentage of the eligible loads for DR trading within same node and different nodes, respectively.</td>
</tr>
<tr>
<td>U ∈ PV,WF, PCC,DG</td>
<td>Superscripts for power.</td>
<td>Probability of scenarios</td>
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<tr>
<td>Pm,DRm</td>
<td>Quantity of DR for customers at bus n.</td>
<td>Quantity of DR for customers at bus n sold to DRA at bus n’.</td>
</tr>
<tr>
<td>Pm,DRm'</td>
<td>Scheduled DR for DRA at bus n.</td>
<td>Scheduled DR for DRA at bus n bought from customers at bus n’.</td>
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A. Indices (sets) and abbreviations

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<tr>
<td>τ</td>
<td>Index of linear partitions in linearization</td>
<td></td>
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<tr>
<td>DG</td>
<td>Index of DG.</td>
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B. Parameters

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>C_m DG</td>
<td>Production cost of generation units.</td>
</tr>
<tr>
<td>C_m^exe,DG</td>
<td>Regulation cost for day-ahead and real-time market.</td>
</tr>
<tr>
<td>MCP</td>
<td>Market clearing price.</td>
</tr>
<tr>
<td>IDR_m^exe, MED_m^exe</td>
<td>Maximum potential of demand response trading for the same node and for different nodes, respectively.</td>
</tr>
<tr>
<td>LD_m^exe, LD_m^ext</td>
<td>Forecasted active and reactive load.</td>
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C. Variables
1. Introduction

1.1 Motivation

Demand Response (DR) trading within distribution network and retail markets plays a key role to overcome the intermittent nature of renewable energy sources (RESs) such as photovoltaic systems (PVs) and wind farms (WFs). Hence, a specific scheme and structure should be designed to implement demand-side management in retail market. According to [1], there is no established market for DR in Nordic countries, where some pioneer countries in market design, especially reserve market, exist in Finland.

However, distribution system operators (DSOs) have the allowance of making direct agreements on DR with customers only in Norway and Denmark. Moreover, no business model has been designed for demand response aggregators (DRAs) in Nordic countries. For example, Denmark is still in a regulatory discussion phase. Sweden has introduced an actor similar to the so-called balance service provider (BSP) who is able to place bids on the retail market without taking the balance responsibility.

Thus, according to these examples, there is the lack and gap of such structures for DR implementation while considering important and active players like DRAs and customers in the most electricity markets. Moreover, the new introduction of DSOs as Distribution Platform Optimizer (DPO) [2] beside smart grid capabilities will enable the operators to provide new optimization mechanism to manage congestion and run the electricity market in the distribution market. As a result, some profound and practical studies to make a suitable structure for DPO are required.

1.2 Literature review

The literature contains several studies addressing DR in different markets. Operation scheduling of microgrids has been studied in [3] neglecting network constraints. Nevertheless, network constraints were considered in [4]-[8] but without DR employment.

Authors in [9] have employed different DR options to exploit the profit of DRA. A DR trading framework was tested in [10] with a Time-Of-Use (TOU) DR program. Competitive scheduling for DRAs has been carried out in [11]. In [12], DRA has been adjusted in a two-stage framework. Another work [13] has maximized the retailers’ profit while implementing an incentive-based DR.

All above-mentioned papers mainly have been modeled the problem from distribution system viewpoint disregarding customers’ preference. On the other hand, some investigations have been focused on how customers take advantage of their DR capability using household energy management (HEMs) and detail information regarding appliances for end-users [14]-[16]. For instance, in [14], authors have considered a new index called response fatigue in addition to customer satisfaction and electricity trading to minimize the billing cost for one customer.

TOU DR has been applied in [15] for customers to optimize dispatch of source-load-storage in a HEMS framework. Likewise, a multi-objective approach has been engaged in [16] to manage flexible loads and minimize not only the electricity bill but also the customers’ dissatisfaction. In another work [17], authors solved the distribution grid congestion by DR while considering customers’ preferences. DR is optimized for smart houses by particle swarm algorithm in [18]. Households in microgrids are also optimized DR with nonlinear auto-regressive neural network in [19]. These papers fail to model how likely utilities will be interested in their DR.

There are a few papers which model DR in a bilevel programming model. In [20], [21], DR scheduling from the independent system operator (ISO)’s viewpoint considering two different levels has been performed.

In [20], a market scheme has been designed within the wholesale market considering DRAs. The DR quantity for trading between the ISO and the DRA has been optimized in a bilevel program. In [22], authors have modeled the interaction among retailers and customers to define a real-time price for customers for their DR participation. Some uncertainties like the price have been considered in a stochastic bilevel model to maximize retailers’ profit while consumers optimize their consumption. The similar problem has been addressed in [23] via bilevel robust optimization with uncertain coefficients in the objective function. Authors in [24] proposed a bilevel optimization model to optimize DR contracts for DR while considering customers satisfaction factor. Where in the upper-level problem, DRA profit is maximized; in the lower-level problem, ISO runs a real-time market process to obtain aggregators’ bid and optimum power for DRA. In other research works, such as [25], [26], bilevel modeling has been performed to optimize dynamic tariffs according to which customers participate in a DR program. To this end, retailers in the upper level aim to maximize the profits and customers in the lower level intend to manage their loads based on the price signal and their comfort needs by minimizing the billing cost.

Retailers and prosumers in another work [27] aim to reach their own targets in a bilevel model while scheduling the DR. These papers have not modeled a comprehensive approach in which DSO, DRAs and customers preferences are addressed and at the same time competitions among DRAs to attract customers DR is modeled.

To obtain the optimum volume of DR trading in the short-term scheduling of distribution networks, the operator should minimize the total operation cost in which buying DR by DRA is a part of operation costs. At the same time, customers would like to minimize the electricity bill through maximization of income from DR selling. Meanwhile, all network constraints,
as well as stochastic variables like PV and WF generation, should be considered to achieve more practical and precise results. To the best of our knowledge, no study has been conducted to schedule DR volume to trade among DRAs and customers with all above-mentioned features.

1.3 Contributions

In this paper, a bilevel programming model is proposed to optimize privacy-based DR trading among DRAs and customers in a competitive way and through HEMSs. DRAs are assimilated as artificial aggregators for DR trading managed completely by DPO. The upper-level problem aims to minimize the operation cost from DPO’s viewpoint. In this level, the purchased DR volume from customers, as a part of operation cost for DPO, is optimized. The lower-level problem intends to minimize electricity bill through income increment from customers’ viewpoint through increment of income. In the upper-level problem, a two-stage stochastic program model is used to capture day-ahead alongside real-time market decisions with all network constraints and handle uncertainties of renewable generations.

The uncertainties are modeled by a Monte-Carlo Simulation (MCS) method and scenario generation (Fig. 1). The bilevel problem is turned into a single-level problem with equilibrium constraints by replacing the lower-level problem with its Karush-Kuhn-Tucker (KKT) conditions.

The contributions of the work are as follows:

− DR is traded among DRAs and customers through HEMS in a privacy-based way and customers can select the proper DRA based on DR prices.

− A business model for obtaining DR quantity is proposed in the distribution network. DRAs, as an artificial aggregator fully managed by DPO, are scheduled to buy DR from all types of customers, while electricity bill from customers’ viewpoint aims to be minimized by increasing the income from DR selling, simultaneously, in a bilevel model.

− Distribution network constraints as linear power flow are implemented in two-stage stochastic programming in which the intermittent nature of PV and WF power generation is considered, as well.

2. Problem statement

The operation strategy applied on a distribution network is outlined. The market for power trading and supporting loads, as well as the market for DR trading, are elaborated. Also, the proposed stochastic bilevel model to operate the network and optimize the traded power and DR is described in this section.

2.1 Operation strategy

This model is in a distribution network framework which consists of different distributed energy resources (DERs), such as gas-fired thermal DG units, PVs, and WFs, along with different types of loads including critical and flexible loads. The network can be in the scale of a large distribution network operated and optimized by the DPO. DRAs in different nodes are in charge of DR trading with customers in the same node or eligible end-users in the other nodes. In fact, DRAs are assimilated to virtual aggregators located at a single node of the network and completely manage by the DPO. In this framework, the network operator can participate in the wholesale market represented by the ISO for power trading and buying regulation.

As Fig. 2 shows, a DPO runs an optimization problem to support the related loads in day-ahead market while considering different decision variables including DR value. The DPO buys/sells energy from/to upstream wholesale market operated by the ISO. Within this framework, all the network constraints, including voltage limitation and line capacity are taken into account in the AC power flow for radial networks.

In the proposed market, DRAs can bid for buying DR from customers, and the DPO checks the preference of customers to sell customers’ DR potential with the proposed DR bids at the same time. To this end, in a bilevel model, customers with the aid of HEMSs decide how much DR they prefer to sell to DRAs in the lower level and based on lower-level decisions, final decisions to deploy DR quantity for DRAs in the upper level are made from network operator’s perspective.

2.2 Stochastic bilevel model

In the proposed bilevel model, the objective of upper-level problem is to minimize the total operation cost in the two-stage stochastic program from DPO’s viewpoint. The first stage is for day-ahead market decisions and second stage is for balancing decisions in the real-time market using MCS method [28]. Generating unit random variables and dedicating to each PDF lead to obtain hourly scenarios for wind speed and solar irradiance. The scenario generation procedure is depicted in Fig. 1 in which $c$ is Rayleigh function parameter, $\Gamma$ is Gamma function and CDF is inverse Cumulative Distribution Function. Moreover, all necessary network constraints for the operation of a distribution system with radial topology are considered in the first stage of the upper level, including voltage magnitude of nodes, reactive and active power flow limitations, and current magnitude for branches are considered. To this end, a linearized branch flow model for radial networks is employed to extract the decision variables as real, precise, and applicable as possible. In the lower level, the objective function aims to minimize electricity bill from customers’ viewpoint, and the optimal DR volume to be sold is obtained. As mentioned, the DR volume to be bought by DRA is achieved in the upper level. Thus, the link between upper-level and lower-level problems consists of the DR quantities traded among DRA and customers.

2.3 DR trading

DRAs play a key role for DR trading in this framework. Through smart facilities of customers i.e., HEMS, they are connected to the eligible customers for DR participation. Indeed, as demonstrates, there is a bi-direction communication among DRA and customers through HEMS which helps to provide a privacy-based connection for customers as Fig. 3 is depicted.

In other words, through HEMS, customers will not need to transfer all the detail information about their consumption to DRA. In this way, DRAs will receive the DR potential of customers collected by HEMS. As depicted in Fig. 4, the model is implemented in day-ahead market; hence, DRAs bid the DR prices the day before for an hourly time step in the next day to buy DR from customers. Based on the DR bidding, eligible customers through HEMSs proceed and decide how many quantities of their DR potential and at which hour they prefer to sell in order to minimize electricity bill followed by increasing income from DR selling. In the proposed DR trading method, DRAs are able to buy DR from all eligible customers in the network. It enables customers to choose the suitable DRA to sell the DR potential. As a result, competition arises among DRAs, who quote different DR prices to cater customers. During this interaction, eligible customers freely change their
DRAs. For example, if DR prices at one hour for DRA-A is lower than DRA-B, the customers prefer to sell more DR to DRA-B in order to make more benefits. Indeed, some of the customers can sell their flexible loads as DR to the DRA at the same node or to DRAs at different nodes in each hour.

Monte-Carlo Simulation method (MCS)

Time Step (t)

**Assumptions**
- Historical data and given data
- Wind speed (Wv) and Solar irradiance (Ir)
- Mean value and variance

**PDF production**
- Rayleigh production
- Beta production

**Scenario generation**
- \( Wv(t) = \text{CDF}^{-1}(URV, c) \)
- \( Ir(t) = \text{CDF}^{-1}(URV, \alpha, \beta) \)

**Scenario reduction**
- \( P_{\text{CC}}^{\text{reg}}(t) = \sum_{s=1}^{S} P_{\text{CC}}^{\text{reg}}(s) \cdot \text{Prob}(s) \)

Fig. 1. Scenario generation method

Fig. 2. The framework of the proposed bilevel model.

3. Problem formulation

3.1 Bilevel model

The upper-level objective function and relative constraints are formulated in (1) – (30). The decision variables are \( P_{\text{CC}}^{\text{reg}}, P_{\text{DR}}^{\text{reg}} \), \( P_{\text{DR}^\text{out}}, \) and \( P_{\text{DR}^\text{in}} \). The first term of the first line includes the cost of buying/selling power from/to upstream wholesale market with MCP. When \( P_{\text{CC}}^{\text{reg}} \) is
negative, DPO exports power to upstream network and vice versa. The second term is the cost of buying power from the local gas-fired DG units. The third term is the regulation cost provided by the upstream network. The second line is total DR cost for all DRAs. The first term in the second line is the DR cost for DRA at node $n$ regarding the buying DR from the customers in the same node and the second term is associated with buying DR from eligible customers to participate in DR in other nodes. The third line is the second stage of the problem.

\[
\text{Minimize } \sum_{n \in N} \left\{ \left[ \sum_{s \in S_{n,DA}} (MCP_s \times P_{s,n}^{CC}) + (\text{DG}_{s,n} \times P_{s,n}^{DG}) + C_{\text{reg}} \times P_{s,n}^{\text{reg}}) \right] + \sum_{m \in M} (P_{m,n} - P_{m,n}^o) \times \lambda_m + \sum_{m \in M} P_{m,n}^o \times \lambda_m \right\} + \sum_{s \in S_{n,DA}} \left[ C_{\text{reg}} \times P_{s,n}^{\text{reg}} \right].
\]

Subject to

\[
\begin{align}
& P_{n,n}^o - P_{n,n}^s = \sum_{m \in M} P_{m,n}^o - \sum_{m \in M} P_{m,n}^s + \sum_{s \in S_{n,DA}} (P_{s,n} - P_{s,n}^o) + R_{n,n}^s \times I_{n,n}^s, \quad \forall t, \forall n, \\
& Q_{n,n}^o - Q_{n,n}^s = \sum_{m \in M} Q_{m,n}^o - \sum_{m \in M} Q_{m,n}^s + \sum_{s \in S_{n,DA}} (Q_{s,n} - Q_{s,n}^o) + X_{n,n}^s \times I_{n,n}^s, \quad \forall t, \forall n, \\
& P_{n,n}^o + P_{n,n}^s = V_{n,n}^o \times I_{n,n}^o, \quad \forall t, \forall n, \\
& Q_{n,n}^o + Q_{n,n}^s = V_{n,n}^o \times I_{n,n}^o, \quad \forall t, \forall n, \\
& P_{n,n}^o + P_{n,n}^s = \sum_{s \in S_{n,DA}} (P_{s,n} - P_{s,n}^o) + R_{n,n}^s \times I_{n,n}^s, \quad \forall t, \forall n, \\
& Q_{n,n}^o + Q_{n,n}^s = \sum_{s \in S_{n,DA}} (Q_{s,n} - Q_{s,n}^o) + X_{n,n}^s \times I_{n,n}^s, \quad \forall t, \forall n, \\
& V_{n,n}^o \leq V_{\text{max}}^o \leq V_{n,n}^o, \quad \forall t, \forall n, \\
& \left| I_{n,n}^o \right| \leq \left| I_{\text{max}}^o \right| \quad \forall t, \forall n, \\
& \left| I_{n,n}^s \right| \leq \left| I_{\text{max}}^s \right| \quad \forall t, \forall n, \\
& P_{n,n}^o \leq P_{\text{max}}^o \quad \forall t, \forall n, \\
& Q_{n,n}^o \leq Q_{\text{max}}^o \quad \forall t, \forall n, \\
& P_{n,n}^s \leq \xi \left| L_{\text{max}}^s \right| \quad \forall t, \forall n, \\
& Q_{n,n}^s \leq \xi \left| L_{\text{max}}^s \right| \quad \forall t, \forall n.
\end{align}
\]

Lower level

\[
\begin{align}
& \text{Min} (\text{Cons. cost}) \sum_{n \in N} \sum_{s \in S_{n,DA}} (P_{s,n}^o \times \lambda_s + P_{s,n}^o \times \lambda_s) \\
& \sum_{n \in N} \sum_{s \in S_{n,DA}} (P_{s,n}^o + P_{s,n}^s) \leq \sum_{n \in N} \sum_{s \in S_{n,DA}} \text{Total}_{DR_{n,s}} \Omega_{n,s} \quad \forall t, \forall n, \\
& P_{n,n}^o - \text{DR}_{n,s}^\text{MRT} \leq \forall t, \forall n, \\
& P_{n,n}^o - \text{DR}_{n}^\text{MRT} \leq \forall t, \forall n.
\end{align}
\]

Equations (2) – (3) indicate active and reactive power balance for the distribution network. The second line of (2) is related to the DR quantity at each node and time period. The first term ($P_{n,n}^o$) is DR quantity that DRA in node $n$ buys from customers at this node in period $t$ and the second term ($P_{n,n}^s$) is DR quantity that DRAs in other nodes buy from customers at node $n$. Voltage drop along distribution line is presented in (4). Active and reactive power limitation are presented in (5) – (6), respectively. Active and reactive power flows in the distribution network are presented in (7) – (14) where linearization of active and reactive power is conducted by (7), and piecewise linearization of constraints is performed by (8) – (14) [6]. Power factor constraint is brought in inequality (15). The limitation of power exchange and power production for different elements in the network is represented in (16). The maximum possible demand response quantity to be bought by each DRA from eligible customers in the same node and other nodes are presented in (17) – (18), respectively.

Equations (19) – (30) indicate the second-stage constraints of the two-stage problem in the upper level. Balancing constraints for active and reactive power for different scenarios in the real-time market are calculated by (19) and (20), respectively. Voltage drop equation for scenarios in the second stage is in (21), and constraints linearization regarding branch power flow for dealing with scenarios in the real-time market are represented in (22) – (26).
Power factor constraint, active and reactive power limitations, and voltage limitation to meet network requirement for scenarios are in (27) – (30), respectively. The lower-level objective function which aims to minimize electricity bills of customers is presented in (31). It includes three terms, the first term \( \text{Cons. cost} \) is the cost of electricity consumption for fixed loads in which here assumed to be constant value. The second term represents the income from selling DR to DRAs in the same node, and the second one is the income from selling DR to DRAs in other nodes. Noted that in this objective function individual loads are not modeled but the group of demand at each node is taken into consideration. The constraints of this problem are in (32) – (34). Inequality (32) indicates that the total sold DR quantity should be lower than the total percentage of the loads. This inequality is the same for DRA and buying DR, the limitation of DR selling for customers in different nodes are given in (33) – (34) where (33) is the capacity of DR selling to DRA in the same node and (34) is the capacity of DR selling to DRAs in other nodes.

### 3.2 Dual form of the lower-level problem

Given DR prices, the lower-level problem (31) – (34) renders a linear program, and strong duality always holds true [29]. Its dual form reads as follows:

\[
\begin{align*}
\text{Max} \left( \text{Cons. cost} \right) & - \\
\sum_{t \in T} \sum_{n \in N} \left( \text{Total}_{DR_n} \times \Omega_w + \text{IDR}_{nt} \times \mu_n + \sum_{t \in T} \text{MDR}_{nt} \times \phi_n \right) \\
\Omega_w + \sum_{t \in T} \psi_{nt} & \geq \sum_{t \in T} \lambda_{nt} \quad \forall t, \forall n \\
\Omega_w, \mu_n, \psi_{nt} & \geq 0 \quad \forall t, \forall n
\end{align*}
\]

where \( \Omega_w, \mu_n \) and \( \psi_{nt} \) are dual variables defined in (32) – (34).

### 3.3 Equivalent single-level problem

To systematically solve the bilevel problem, it should be turned into a one-level problem which can be recognized by off-the-shelf solvers. Indeed, the bilevel problem can reduce to a single-level problem, if the upper-level objective function and the lower-level one point the same direction. In other words, two objective functions should follow the same direction to reach their target which is the same condition for the proposed methodology in the current work.

Moreover, the lower-level problem can be represented by its own constraints, the constraints of dual problem, and strong duality condition, if and only if the problem is convex and continuous which is also followed by the current problem. Therefore, by adding the primal, dual constraints as well as strong duality condition of lower-level problem to the upper-level one, the bilevel problem can be converted to the equivalent single level one. Strong duality condition is based on a theorem stated that feasible solution of the primal problem and the feasible solution of the dual one are optimal if their objective functions are equal as (39).

In other words, if a primal feasible point satisfying (32)-(34) and a dual feasible point satisfying (36)-(38) lead to the same value for the primal objective (31) and dual objective (35), then the pair of primal and dual points solves the respective problem [29].

\[
\begin{align*}
\sum_{t \in T} \sum_{n \in N} \left( \text{Total}_{DR_n} \times \Omega_w + \text{IDR}_{nt} \times \mu_n + \sum_{t \in T} \text{MDR}_{nt} \times \phi_n \right) = \\
\sum_{t \in T} \sum_{n \in N} \left( P_{DR_n} \times \lambda_n + \sum_{t \in T} P_{DR_{nt}} \times \beta_n \right)
\end{align*}
\]

Hence, the lower-level problem can be replaced by its primal-dual optimality condition, which appears in the form of constraints without an objective function to be optimized. It consists of (32)-(34), (36)-(38), and equality (39).

The one-level equivalence of the proposed bilevel problem containing all upper- and lower-level constraints can be formulated as:

\[
\begin{align*}
\text{Minimize} (1) & - \\
\text{Subject to:} & \\
(2) & - (30) \\
(32) & - (34) \\
(36) & - (39)
\end{align*}
\]

Problem (40)-(43) is a linear program and can be easily solved by existing solvers.

### 4. Case study and numerical results

#### 4.1 Case study

A 15-bus IEEE distribution system with nominal power 2300 kW is applied which is in Fig. 5 [30]. It includes four thermal DG units (690 kW each), two PV systems (100 kW each) and two WFs (100 kW each). Market clearing price (MCP), regulation price, and DG production price are shown in Fig. 6.

These prices along with PV systems and WF information are extracted from Spanish market [31]. The proposed model can be easily extended to any realistic-sized network. While we believe the main findings of current work are regardless of a test system.

In this work, two cases are considered.

- In the first case, only two DRAs are taken into account. In this case, the interaction of these two DRAs to run DR trading especially in terms of their competition is studied.
- In the second case, the impact of adding more DRAs in the network on the total operation cost, DR cost, buying power from DGs and the wholesale market as well as selling power to wholesale market are investigated.

The potential of DR participation would be twenty percent, moreover, \( \xi, \xi' \) are 10 and 4.5 percent, respectively. The problem is solved using CPLEX solver in GAMS [32].

#### 4.2 Numerical results

1) Case 1: two DRAs in two nodes

In case 1, two DRAs at node 3 and node 5 are considered. Each one can buy DR from the customers in their node and the customers from the other DRA. It is noteworthy that scenario 1 to scenario 3 consider fixed DR prices for the whole day (Table 1), while time-varying DR prices are considered for scenario 4 (Fig. 7).

The results of the implementation of these scenarios with the proposed bilevel model are demonstrated in different figures (Figs. 8 to 11) and compared with the eligible loads at each node presented as dash line and dot lines named node 3 and node 5, respectively.
**Fig. 5.** 15-bus distribution network.

**Fig. 6.** Hourly different prices.

**Table 1.** Different DR Prices for DRA3 and DRA5 (3 Scenarios)

<table>
<thead>
<tr>
<th>DR price (€/kWh)</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRA-3</td>
<td>0.09</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>DRA-5</td>
<td>0.04</td>
<td>0.04</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Fig. 7.** DR prices for DRA3 and DRA5 in different hours. (scenario 4)

**Fig. 8.** Scenario 1 results.

**Fig. 9.** Scenario 2 results.

**Fig. 10.** Scenario 3 results.

**Fig. 11.** Scenario 4 results.

**Fig. 12.** DR prices for case 2

**Fig. 13.** DR cost for each DRA in different numbers of DRAs in the network.

**Fig. 14.** Hourly total DR cost for different numbers of DRA in the network.

**Fig. 15.** Hourly power generation of DGs and power exchange with the upper network.
As can be seen, the DR capacity in node 3 is larger than those in node 5 during a day, because node 3 has more load consumption than node 5. Once the DR price for DRA5 is higher, a lower DR quantity is scheduled for DRA5 and DRA5-3 based on Fig. 10. This occurs despite the fact that from the customers’ viewpoint, they prefer to sell more DR to DRAs because the objective function from the customers’ viewpoint is to maximize income from selling DR quantity. Yet, this purchase imposes a higher cost on DRAs; thus, the overall decision to make a balance among customers’ and DRAs’ preference is to trade less DR at those time steps with higher DR price. The opposite phenomena happen when DR price for DRA3 is higher according to the Fig. 8.

In scenario 2, with introduction of the equal DR price for two DRAs, the popularity of DRA selection can be compared. On the one hand, these equal DR prices in this scenario are lower than MCP in peak hours; hence, it is cost-efficient to have load reduction as much as possible for both stakeholders (Fig. 9). In this scenario, the DR trading with customers for all DRAs are also relatively equal due to the equivalent DR price. On the other hand, at off-peak hours, just DRA5 and DRA5-3 are scheduled to buy load reduction. Most probably the reason behind this fact is due to the existence of two DERs, one WF and one thermal DG unit, which supply all loads with low cost in node 3 and there is no need for DR (Fig. 9). Hence, availability of other cheaper sources such as RESs to make the balance between supply and demand has a direct impact on buying less DR from customers.

The results shown in Figs. 11 (scenario 4) demonstrate that once the DR prices are defined based on peak hours and MCP, the results are more desirable. In other words, the DR trading is not scheduled for off-peak hours (i.e. 1-6) when the MCP is low enough to cover the demand by wholesale market. Within this scenario at time steps 15 and 16, just DRA5 and DRA5-3 are scheduled for DR trading among DRAs and customer because DR price for DRA5 is lower and more DR trading is not necessary at these time steps. DG production price in period 17 is much higher and not only no DG is committed, but also more DR quantities are scheduled, since the MCP and DR prices have no much difference.

2) Case2: several DRAs

In the second case, multiple DRAs are taken into account, and the impact of adding DRAs on each DRA’s cost and total DR cost are investigated. The DR bids for added DRAs are depicted in Fig. 12.

The impact of the number of DRAs on the cost of DR for each DRA is demonstrated in Fig. 13. As can be seen, after adding DRA in node 2, DR cost for DRA3 becomes lower and for DRA5 becomes higher because of the need to apply DR for node 3 when adding DRA2 declines and on the other side the DR interaction between node 2 and node 5 as DRA5-2 leads to an increase in DR cost for DRA5. Total DR cost for hours 2 to 5 between 2 and 3 DRAs has no difference according to Fig. 14, because DR scheduling does not change at those hours with two DRAs and three DRAs. Moreover, since DRA3 has more DR capacity, a higher DR quantity is scheduled, and the total DR price is higher for DRA3 compared with DRA5 and DRA2. After adding DRA in node 4, DRA4 will have the highest DR cost because of having highest DR capacity followed by the higher level of DR scheduling. Moreover, the hourly total DR cost has a growing tendency after adding DRA4 based on Fig. 14. This trend continues after adding DRA8 and DRA12, because, as it is clear in Fig. 14, the cost of DR increases. Thus, with the existence of more DRAs, the operator prefers to have more DR quantity which is more cost-efficient to provide the demand-supply balance. In other words, scheduled DR is sometimes cheaper for operators and several DRAs make it much cheaper, although the DR cost may increase.

Likewise, the DR cost for each DRA after adding another DRA has an increasing trend, since as mentioned, each DRA will have an interaction with the loads in the new node for DR exchange, which causes extra cost for existing DRAs. Fig. 15 demonstrates the hourly power generation of DG units and hourly exchange power with the upper network in the presence of six DRAs. As it is shown, when the MCP is higher, the operator prefers to supply the loads by inside DGs and sell the extra power to ISO. Therefore, at hours 12 and 19-22, all DGs are committed almost at their maximum capacity to not only supply the consumption but also to make a profit from selling the extra power to the market. At other hours, depending on the MCP and DG production prices, the required power is bought from the market or from DGs. For example, at hours 8-11 and 15-17, since the DG production price is higher than the MCP, the operator buys electric power from ISO instead of using DGs.

5. Conclusions

The proposed innovative method is capable of optimizing the DR quantity to be bought/sold by DRA/customers from/to customers/DRAs, simultaneously, within the distribution network. The upper-level problem was from operator’s viewpoint, considering DRAs, enabled to buy DR, seeking to minimize the total operation cost. The lower-level problem was from customers’ viewpoint and tends to minimize the electricity bill and increase the income from selling DR to DRAs. According to the results, there was a competition among DRAs to buy DR and the main findings of this works are listed as follows:

- The DRA with lower DR prices, especially in the moderate DR prices, was more successful to be selected by customers. On the other hand, however customers could sell more with high DR prices, DRAs disliked buying in this situation and as a result less DR trading has scheduled.
- When the time-dependent DR prices are defined based on a comparison with MCP and peak hours, unnecessary DR trading in off-peak hours and low MCP hours has not occurred.
- The existence of DG or RES in a node has been effective on potential DR trading among DRA and customers in a way that available sources reduces the need for DR trading.
- Adding more DRAs and more eligible customers in different nodes, sometimes, could lead to a decrease in scheduled DR for pre-placed DRAs due to providing the required DR by new potential participants. While, most of the time DR was more cost-efficient to be scheduled even if a new DRA was added.
- The DR cost for each DRA rose after adding another DRA because of adding an interaction DR cost among new potential DR and previous DRA.

For future work, customers will be able to offer DR prices instead of being price taker. Moreover, the uncertain feature of MCP, regulation price as well as loads can be modeled in addition to the current model. In addition, the best DR prices to
earn more profit for DRAs can be achieved by learning algorithms.

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References


