Optimal Residential Model Predictive Control Energy Management Performance with PV Microgeneration

Radu Godina a, Eduardo M. G. Rodrigues a, Edris Pouresmaeil b,c, João P. S. Catalão a,b,d,*

a C-MAST, University of Beira Interior, R. Fonte do Lameiro, 6201-001 Covilhã, Portugal
b INESC-ID, Instituto Superior Técnico, University of Lisbon, Av. Rovisco Pais, 1, 1049-001 Lisbon, Portugal
c Department of Electrical Engineering and Automation, Aalto University, 02150 Espoo, Finland
d INESC TEC and Faculty of Engineering of the University of Porto, R. Dr. Roberto Frias, 4200-465 Porto, Portugal

Abstract—The energy demand of the residential sector and the adjacent option for fossil fuels has negative consequences by both greenhouse gases (GHG) and other air pollutants emissions. Since home energy demand consists mainly of energy requirements for space and water heating along with the energy dedicated for appliances, different strategies that aim to stimulate an efficient use of energy need to be reinforced at all levels of human activity. In this paper, a comprehensive comparison is made between the thermostat (ON/OFF), proportional-integral-derivative (PID) and Model Predictive Control (MPC) control models of a domestic heating, ventilation and air conditioning (HVAC) system controlling the temperature of a room. A power interface that adjusts the MPC dynamic range of the output command signal into a discrete two level control signal is proposed, as a new contribution to earlier studies. The model of the house with local solar microgeneration is assumed to be located in a Portuguese city. The household of the case study is subject to the local solar irradiance, temperature and 5 Time-of-Use (ToU) electricity rates applied on an entire week of August 2016. The purpose of the optimisation is to achieve the best compromise between temperature comfort levels and energy costs and also to assess which is the best electricity ToU rate option provided by the electricity retailer for the residential sector. Also, for each electrical load of the HVAC system, the energy and cost are calculated and the results are presented by varying the different MPC weight combination in order to obtain the best possible solution and increase the quality of the model. Finally, after the best tariff and controller are determined, the impact of the solar generation is assessed.

Index Terms—Energy optimization; Model predictive control; Energy management controller; Photovoltaic microgeneration; Residential building.

1. Introduction

Concerns regarding climate change tend to grow when confronted with the damaging consequences of rapid and uncontrolled urbanisation. To cope with the current energy consumption growing rate several efforts are necessary to oppose environmental threats [1] [2]. Results published by the Intergovernmental Panel on Climate Change (IPCC) emphasised the requirement to preserve the GHG below 450 ppm CO₂ equivalence by 2050 in order to maintain the increase of the temperature of the planet under 2°C [3] [4].

Countries gradually concentrate their initiatives and environmental policies on reducing the negative environmental repercussions of careless energy consumption [5]. The energy sector is experiencing substantial transformation driven by legislation with the purpose to reduce energy consumption and the consequently related environmental impacts [6] [7].

* Corresponding author at: Faculty of Engineering of the University of Porto, R. Dr. Roberto Frias, 4200-465 Porto, Portugal. Email address: catalao@ubi.pt (J.P.S. Catalão).
Presently, the consumption of energy in buildings is responsible for circa 33% of the final energy consumption on the planet [8] [9]. In the case of primary energy consumption, the building sector embodies about 40% in most of the IEA (International Energy Agency) nations and is accountable for 36% of the European Union (EU) CO₂ emissions [10]. Also, the same sector absorbs 40% of EU final consumption and 60% of electricity consumption [11].

As part of the building sector, the residential sector is accountable for 60% of the final energy consumption and presents the highest prospective to decrease the peak demand which is described by the volatility of energy utilisation [12].

The tendency of the electricity production and supply system, still deeply rooted in electricity generated by centralised fossil fuel and nuclear power plants, is to change into the direction of a distributed electricity generation system comprising small scale renewable energy producers such as edifices and houses equipped with photovoltaic panels (PV) and even storage systems. The enormous structural change that is being witnessed during the last decade and complemented by the recent regulatory policies unveils new challenges concerning how the energy is consumed [13].

Even though the flexibility related with home appliances and distinct electricity ToU rate options can accomplish palpable positive results for consumers, the present residential load control operations are still operated manually, thus signifying demanding challenges to consumers in planning the activity of their appliances in an optimal manner. Several clients might not have enough time to plan such type of scheduling operations and at times when the price variation is quick and recurrent the scheduling might be perceived as significantly complex. Therefore, an energy management system (EMS) could be a solution to optimise the operation of appliances [14].

As a whole, two methodologies currently exist aiming to achieve energy savings: the inclusion of more energy efficient equipment in buildings, or the efficient management of the energy consumption through an EMS [15]. In the last ten years, the price of storage, data processing, and communication diminished while the incorporation of EMS has become increasingly effective. Such types of solutions offer additional possibilities for the project and implementation of forefront control methods [16].

However, it is not enough. As soon as the communications and distributed capabilities will be easily available, new ways to involve and control different utility-owned and third-party assets in a reliable and sustainable mean will be required [17]. Transactive energy could pave the way for the accomplishment of such a goal by utilising value as a common ground with the purpose to integrate control and economic methods and to adjust value streams for every party involved by stimulating actively managed systems that can coordinate their behaviour with the requirements of the rest of the elements of the system [18] [19].
The development of numerous control techniques have been proposed for HVAC systems and are classified into classical control, soft control, hard control, hybrid control, and other control methods. Yet, due to their simplicity, ON/OFF and PID controls are still utilised in several HVAC systems even though such settings might not be adequate for the entirety of the house. Thus, through the definition of set points for local controllers, the regulatory control is utilised to enhance the global system performance such as costs or energy consumption [20].

Driven by the recent improvements in data storage, communication devices, and computing, it is currently possible to materialise a suitable control technique to surmount the characteristic weaknesses in HVAC control. The implementation of EMS control strategies could be an auspicious solution for reaching improved results in HVAC systems when compared to other common control methods. By utilizing embedded EMS control units in HVAC systems, several improvements in energy efficiency could be obtained without the changes affecting the heating and cooling systems. Such types of controllers are found to be a reliable improvement for dwellings and can be without much effort installed, ran and replaced [21].

Several methods based on the MPC have been created and tested with the purpose to optimise the operation of HVAC systems. The MPC is, in essence, an optimisation based approach in which a clear model is used to predict the performance of the controlled plants over a receding horizon. The popularity of the MPC ascended ever since its first application in the process industry in 1970s [22].

Presently, MPC is broadly utilised in numerous applications that range from railway traffic management [23], improving energy efficiency of hybrid and electric vehicles [24] [25], to supply chain systems [26] and switching frequency regulation in power converters [27]. The common research of MPC is mostly dedicated to a centralised implementation. On the other hand, with the accelerated improvement of energy efficiency technology and the required improvement for the economic behaviour, large scale systems, such as EMS, are becoming more complex [28]. Thus, new means are required to engage and control various utility-owned and third-party assets in a reliable and sustainable way [29].

The MPC is considered to be superior to the classical control techniques such as ON/OFF and PID controllers. PID controllers are inferior due to the reason that they offer low accuracy in processes which are either non-linear or have a large time delay. For instance, PID controllers only manage efficiently single input single output (SISO) systems. On the other hand, the MPC is able to manage multiple input-multiple output (MIMO) systems, to show a greater accuracy, to operate with constraints, is robust when facing disturbances and has the capacity to predict the performance of the controlled plants over a receding horizon. However, such advantages are counterweighted with greater computational requirements [7] [30] [31] [32].

Academics have been researching a vast range of control techniques with the goal to achieve an efficient energy utilisation of HVAC systems and other domestic appliances [20].
In [33] is showed the application of MPC to HVAC units and demonstrated that the closed-loop performance and energy efficiency can be improved over conventional methods, but the authors focus only on the comparison between the MPC and the proportional-integral controller (PI) without addressing the costs. A model predictive and genetic algorithm-based optimisation of residential temperature control of a HVAC unit and an electric heater in the presence of time-varying electricity prices was presented in [34]. However, in this case the pricing scheme is not specified. In [16] the authors develop a MPC model with the purpose to minimise the energy consumption of the air temperature and flow rate of an air-handling-unit for multi-zone variable air volumes. Yet, this study lacks of a proper economic analysis since no costs reflecting the energy that was saved are given in this study. In [35] the MPC was combined with conventional local loop PID controllers in a hierarchical structure in order to minimise the energy consumption using current energy sources and minimal retrofitting and using weather predictions. This study, however, also does not support the obtained results with an economic analysis. A MPC architecture design for the optimal temperature control of a real commercial building was presented in [36]. However, the pricing scheme was not specified in this study.

In [37] a retrofitted and inexpensive HVAC testbed was built with the purpose to reduce the transient and steady state electricity consumption in HVAC systems using learning-based MPC. Still, a more flexible approach was made since within the objective of this study the upper and lower limits of the room temperature were moderately disrupted. An economic MPC to a commercial building HVAC system in Milwaukee WI that described the effectiveness of the method in closed-loop load shifting and demand reduction was applied in [15], but the authors use only one price scheme. In [38] an economic MPC operated building aggregator was proposed and it combines a significant amount of the flexible power consumption of a group of commercial building HVAC fans as fast regulation reserve to the grid. In this study also only one electricity rate is provided. A hierarchical distributed MPC algorithm for HVAC Systems in order to regulate the temperature of buildings is proposed in [39]. Even though in this study some sort of energy efficiency is reached the associated costs of the proposed hierarchical distributed MPC was not addressed. In [9] the authors propose a detailed multi-input-multi-output (MIMO) model predictive control (MPC) for a direct expansion (DX) air conditioning (A/C) system but nor address how this controller performs in comparison with other controllers nor the associated costs.

The aim of this paper is to compare the MPC performance with the ON/OFF and PID control of a domestic HVAC system controlling the temperature of a room under five different electricity ToU rates. These rates are the ones currently applied by the Portuguese electricity retailer [40]. A model of a house with local solar microgeneration is proposed for this study and the location is assumed to be in the city of Covilhã, Portugal. The study covers an entire week of August of 2016 in which very high levels of ambient temperature and solar irradiance were observed.
According to the specialists it was the second hottest August in Portugal since 1931 [41]. Thus, a comparison is made of the HVAC performance in maintaining the temperature of a room controlled by ON/OFF, PID and MPC steady while influenced by 5 types of electricity ToU rates and then the impact of the PV generation is assessed. The new contribution of the paper is a power interface that adjusts the MPC dynamic range of the output command signal into a discrete two level control signal. Also, in order to increase the quality of the model, an optimisation of the MPC weights is performed with the purpose of finding the lowest cost.

The remainder of the paper is organised as follows: in Section II the MPC proposed control methodology is formulated. Then, the novelty and the modelled test case are presented in Section III. The obtained results are thoroughly discussed in Section IV. Finally, conclusions are drawn in Section V.

2. Methodology

The MPC is known in the literature by several alternative designations, one of the most recognised ones being the receding-horizon control, rolling-horizon planning, dynamic matrix control, and dynamic linear programming [42]. The MPC is an optimal control technique which is intended to optimise a series of manipulated variable adjustments bound by a prediction horizon and has been demonstrated to be very effective given its capability to deal with multiobjective problems and handle hard constraints explicitly. The MPC utilises a control model with the purpose of optimising the predictions of the process behaviour based on a linear or quadratic objective, restrained by equality or inequality constraints.

In such a control technique the optimisation is performed repeatedly on-line – the receding horizon which is the inherent contrast between MPC and other control methods. In perfect circumstances only the suboptimal result for the total solution can be achieved, this is the restriction of such finite-horizon optimisation. Yet, the optimisation of the receding horizon can efficiently include the uncertainties suffered by the model and also the time-varying disturbances and behaviour [43].

2.1 Elemental MPC Controller Formulation

The broadest state-space representation of a linear system with $p$ inputs [44], $q$ outputs and $n$ state variables is represented as follows:

$$x(k + 1) = Ax + Bu(k)$$

$$y(k) = Cx(k)$$
where
\[ x(k) \in \mathbb{R}^n, \ u(k) \in \mathbb{R}^p, \ y(k) \in \mathbb{R}^q \] (3)

The consequence of such representation is that the system is observable and controllable.

As stated above, in the case of the MPC, the control action at each time step is achieved by solving an online optimisation problem, and in an elemental MPC controller the performance measure is nearly each time a quadratic cost. Through the representation of the positive definite matrices as:
\[ M = M^T > 0 \] (4)

and the performance weights is given by:
\[ W = W^T > 0 \] (5)

The optimal control input has to be identified in order to minimise the infinite horizon performance cost:
\[ J(k) = \sum_{j=k}^{\infty} x^T(j \mid k)Mx(j \mid k) + u^T(j \mid k)Wu(j \mid k) \] (6)

In an unconstrained scenario, the solution to this equation is given by the linear quadratic (LQ) controller. Yet, in a constrained scenario, no analytic solution exists. As an alternative, the objective in the MPC is to establish a prediction horizon \( N \) and approximate the problem with a finite horizon cost:
\[ J(k) = \sum_{j=k}^{k+N-1} x^T(j \mid k)Mx(j \mid k) + u^T(j \mid k)Wu(j \mid k) \] (7)

The finite horizon is essential and without a defined one the overall problem cannot be solved. By utilizing the model from (1) and (2), it is possible to predict the state \( x(k+j \mid k) \), given a future control sequence \( u(\cdot \mid k) \) and the current state \( x(k \mid k) \). In such a case, no state estimation is obligatory and it is assumed that \( C=I \), therefore, \( x(k \mid k) = x(k) \). Consequently, the prediction is represented as:
\[ x(k + j \mid k) = A^j x(k \mid k) + \sum_{i=0}^{j-1} A^{j-i-1}Bu(k+i \mid k) \] (8)

By utilizing such predictions, it is possible to define the following optimisation equation:
\[ \min_u \sum_{j=k}^{k+N-1} x^T(j \mid k)Mx(j \mid k) + u^T(j \mid k)Wu(j \mid k) \] (9)
which is subject to:
\[ u(k + j | k) \in \Upsilon \]  

and
\[ x(k + j | k) = Ax(k + j - 1 | k) + Bu(k + j - 1 | k) \]  

and thus, it is possible to design an elemental MPC controller.

### 2.2 Quadratic programming formulation of MPC

The quadratic programming (QP) methods are considered to be a significant section in the application of the MPC since the control action at each time step is acquired by solving an online optimisation problem [42]. The application of the MPC for processes with restricted signals needs the resolution of a QP problem thus turning it into an optimisation problem including a quadratic objective function and linear constraints. This is justified by the reason that the majority of the MPC applications need a linear model to characterise the process of interest under a moving time horizon with a quadratic objective function with the purpose of driving the controlled variables back to their setpoints [45].

With the purpose to transform the optimisation problem in a system fitting for QP, it is now feasible to insert arranged vectors with future states and control inputs:

\[
X = \begin{bmatrix}
x(k | k) \\
x(k + 1 | k) \\
\vdots \\
x(k + N + 1 | k)
\end{bmatrix},
U = \begin{bmatrix}
u(k | k) \\
u(k + 1 | k) \\
\vdots \\
u(k + N + 1 | k)
\end{bmatrix}
\]  

By defining the predicted states by:
\[ X = \Gamma x(k | k) + \Psi U \]  

where:
\[ \Gamma \in \mathbb{R}^{Nn \times n}, \ \Psi \in \mathbb{R}^{Nn \times np} \]  

Larger versions of matrixes \( M \) and \( W \) can now be obtained as following:
\[ \bar{M} = \text{diag}(M, M, \ldots, M) \in \mathbb{R}^{Nn \times Nn} \]  

and
\[ \bar{W} = \text{diag}(W, W, \ldots, W) \in \mathbb{R}^{Np \times Np} \]
However, linear inequalities define the constraint on the input and can be defined as:

\[ \Phi U \leq \nu \]  \hspace{1cm} (17)

Consequently, the optimisation equations (9)-(11) can now be rewritten as:

\[ \min_U (\Gamma x(k | k) + \Psi U)^\top \mathcal{M} (\Gamma x(k | k) + \Psi U) + U^\top \mathcal{W} U \]  \hspace{1cm} (18)

and subjected to (17).

3. Case Study

The model of the house with local solar microgeneration is assumed to be located in Portugal, specifically in the city of Covilhã. The modelling tool employed in this study is Simulink, developed by MathWorks, Inc. The PV panel chosen for this study is model X21-345 of the SunPower® with the module efficiency of 21.5% [46]. The dwelling of the case study is subject to the local solar irradiance, temperature and electricity ToU rates of a specific week of summer – from 8th to 14 August, 2016. Five different electricity ToU rate options are used in this study and the prices of the electricity tariffs can be observed in Table 1. The aforementioned five electricity ToU rate options are the ones currently applied by the Portuguese electricity retailer and can be observed in Figure 1. Option B is the standard flat tariff of Portugal, option A and C are three tier Tariffs, and options D and E are both two tier Tariffs. Electricity ToU rates and price information was taken from [47] which were recently made available for the Portuguese residential market by the electricity retailer – EDP – Energias de Portugal. As it can be observed in Table 1 the highest price is naturally at critical peak hours with 0.27 €/kWh for three tier ToU rates and the lowest one is at valley hours for both two and three tier ToU rates with 0.12 €/kWh.

"Figure 1 can be observed at the end of the document”.
"Table 1 can be observed at the end of the document”.

A very hot day of August was chosen for this study with the intention to assess the behaviour of the HVAC system under extreme conditions. The weekly exterior air temperature of the aforementioned city is presented in Figure 2. Also, the existence of a PV panel was considered with a capacity of 335 W since the location of the city in the studied time period is in an area with a high insolation rate, thus, reducing part of the electric bill. The local solar irradiance is shown in Figure 3.

"Figure 2 can be observed at the end of the document”.
"Figure 3 can be observed at the end of the document”.
3.1 HVAC Model

The HVAC system cools the temperature of the room through a cooling capacity of 2.608kW. The heat exchange with the exterior occurs through the outer wall of the room and it is the key cause of disturbance of the preferred thermal comfort level of the room. With the purpose of testing all control strategies, the rate of heat loss/generation through the model of the external wall of the room is simulated using a temperature based time series with significant wide thermal amplitude variation upon 24 hours. The ON/OFF, PID and the MPC are set with a limit of +/1 ºC and having as reference 23ºC.

In order to achieve pleasant thermic home indoor surroundings additional energy needs to be consumed for the process of addition and/or removal of heat. As a consequence, the preferred comfort target is fixed by defining a guiding temperature and through the measurement of the air temperature. The comfortable fixed temperature level centred on temperature is disrupted by the thermal mass of the room, all the inhabitants present in the studied room and the temperature exchange through the outer walls with the outside temperature. Hence, the dynamics of the temperature of a dwelling’s room is a by-product of energy balances among the HVAC that adds or removes heat from the room, the exterior environment temperatures, and the internal thermal mass as represented in Figure 4.

"Figure 4 can be observed at the end of the document".

With the purpose of evaluating and comparing the performance of the controller a model of thermal mass is utilised for this study through the use of a resistance-capacitance circuit analogy. The model contains the heat flow balance between the windows and external wall of a dwelling room and the thermal capacitance concerning the air from the interior of the room. The linear transient thermal model of the room is given by the expressions (11-13) which were taken from [48]:

\[
\frac{dT_{wd}}{dt} = \frac{Q_{ac}}{C_{wd}} + \frac{T_{in} - T_{wd}}{R_{wd}C_{wd}}
\]

\[
\frac{dT_{in}}{dt} = \frac{Q_{ac} \times S(t)}{C_{in}} + \frac{T_{wd} - T_{in}}{C_{in}R_{wd}} + \frac{T_{wd} - T_{in}}{C_{in}R_{wd}}
\]

\[
Q_e = A_w h_o (T_{out} - T_i)
\]

in which \(Q_{ac}\) represents the entering cooling flow into the room, the ambient temperature is represented by \(T_{out}\), the temperature of the room is given by \(T_{in}\), \(T_{wd}\) represents the temperature of wall, the wall's thermal capacitance is given by \(C_{wd}\), the wall's thermal resistance of is expressed by \(R_{wd}\), the windows’ thermal resistance is given by \(R_{wd}\), the indoor air’s thermal capacitance is represented by \(C_{in}\), \(Q_e\) represents the heat flow incident on an external surface of the dwelling exposed to solar radiation, the linked radiation and convection heat transfer coefficient is given by \(h_o\), \(A_w\) represents the area of the wall, the wall’s surface
temperature is represented by $T_s$ and $S(t)$ represents a binary variable capable of reproducing the “on” and “off” switching of the ON/OFF. The values of the physical parameters are taken from [49].

The typical thermostatic and PID control function as a reference to the MPC. As a result, the energy needed to control the HVAC, the cost of consuming the energy throughout the off-peak, mid-peak, and on-peak along with the temperature variation are intended as function of weight selection related to the manipulated variable and process output as a portion of the cost function. In this specific case, the HVAC is activated and deactivated by a power switch block without internal losses.

The calculation of the energy cost depends on the electricity ToU rate of the Portuguese residential market employed during a period of one week. For the assessment of the time horizon, $P$ control moves number is set to 4 and the $N$ predicted outputs is set to 20.

3.2 The MPC design for the load control of the dwelling

a) The architecture proposed in this paper

The proposed architecture in this paper and one of the main contributions of this study is the application of the MPC as shown in Figure 5. Usually, the configuration of the MPC consists of the blocks preceding the 2-level signal modulator. The improved MPC adaptation proposed in this paper and shown in Figure 5 comprises another signal processing block and is depicted by the indicated area in grey.

"Figure 5 can be observed at the end of the document”.

b) The two-level signal modulator

In the case of the output every signal that has to be transformed by the actuator block has to be susceptible to the natural limitations of the equipment. A standard actuator is able to give linear responses with the condition of being restricted by superior and inferior limits. Usually, the manipulated variable response is limited in line with the natural limitations of the actuator. An attempt to minimise all possible costs have to be done with the purpose of reducing the price of the MPC implementation as an alternate controller. Therefore, the actuator should be perceived as a costly element. A proper power switch is required for linear power management. For this purpose solid state relays with a decent performance continue to be too costly for the control of home appliances. Instead of employing a linear power switch, a system of two-level control signals is projected in this study. The proposed system in operation alters a finite continuous string of manipulated variables into a discrete array of integers. The proposed two-level input vector $\Psi(k)$ is represented as follows:

$$\Psi(k) = \{0, 1\}$$ (22)
With the purpose of codifying the dynamic range signal $u(k)$ into a two-level power control signal of a discrete nature, a unsophisticated comparison operator is considered displaying the operational performance as in equation (23):

$$
\Psi(k) = \begin{cases} 
1, & \frac{U_{\text{max}}}{2} < u(k) \leq U_{\text{max}} \\
0, & 0 < u(k) \leq \frac{U_{\text{max}}}{2} 
\end{cases}
$$

(23)

The dynamic range of manipulated variable is split in two equal zones. All the $u(k)$ values over fifty percent of its dynamic range will be interpreted by the two-level control signal as 1. Therefore, full power is provided to the HVAC system. Alternatively, no power is received by the HVAC system if the $u(k)$ signal emerges on the bottom zone which is under fifty percent of the dynamic range. Figure 6 shows the operation technique with the aforementioned two-level modulation proposed in this paper.

"Figure 6 can be observed at the end of the document".

4. Results and Discussion

By taking into account the exterior temperature of Covilhã of the studied week of August, 2016, the solar radiation of the same day, the five distinct ToU rates during the studied time frame and the transient thermal model of the room equations, several results can be obtained.

As mentioned before, the comparison of the HVAC performance between the control options of ON/OFF, PID and MPC is made with the purpose of maintaining the temperature of the room steady. Naturally, since these control techniques are quite distinct the room’s temperature behaviour that they influence is quite different as well.

The control regulation of the temperature of the room by the aforementioned three control options is presented in the Figures 7-9. Monday, Saturday and Sunday were the chosen days for the comparison due to the fact that two electricity ToU rate options, A and D, vary during weekends. The representation of Monday as the only day of the working week in Figure 7 is justified by two reasons: (1) it is the hottest day of the week and (2) the room temperature of the remaining working days shows very similar behaviour and thus is redundant.

"Figure 7 can be observed at the end of the document".

"Figure 8 can be observed at the end of the document".

"Figure 9 can be observed at the end of the document".
4.1 The consumed energy

After the HVAC performance in maintaining the temperature of a room controlled by ON/OFF, PID and MPC is assessed, the results show that if controlled by ON/OFF the consumed energy in kWh is higher, except Monday, 8\textsuperscript{th} August.

The results have also shown that the best HVAC performance is by employing the MPC control option in terms of consumed energy during the studied week. The energy consumption can be observed in Figure 10. The lower consumption verified on 10\textsuperscript{th} of August occurs due to the reason that is the coolest day of the week, thus HVAC system is less utilised.

"Figure 10 can be observed at the end of the document".

4.2 The consumed energy cost

Once the energy consumption of the HVAC performance during the entire week was assessed, the consumed energy cost can be calculated by taking into account all the available ToU rates by the electricity retailer for the residential sector. The consumed energy cost in cents of ON/OFF, PID and MPC of the controlled HVAC system by employing the 5 ToU rates is shown in the Figures 11-15.

"Figure 11 can be observed at the end of the document".

"Figure 12 can be observed at the end of the document".

"Figure 13 can be observed at the end of the document".

"Figure 14 can be observed at the end of the document".

"Figure 15 can be observed at the end of the document".

By observing in Figures 11 and 14 of options A and D it can be witnessed how the weekends are less costly when compared to Figures 13 and 15 of options C and E since options A and D have valley prices for most of the Saturdays and entire Sundays. The cost of ON/OFF, PID and MPC for every option of ToU rate for the entire week can be seen in Table 2.

"Table 2 can be observed at the end of the document".

The option that offers the lowest cost (28.73 €) is option A followed by option B (30.70 €) and by employing the MPC method to control the HVAC system while the most expensive option is E with ON/OFF which corresponds to 41.71 €. Also, by analysing Table 2 it can be observed that the MPC solution offers the lowest costs for all the available options while the ON/OFF is the most expensive solution. By choosing the MPC and option A the client saves 6.6% per week when compared to the flat tariff for the same control method and saves as nearly as 31.12% when compared with the most expensive option.
4.3 Varying the MPC weights and Impact of PV Microgeneration

Given that a MPC controller design frequently requires the tuning of the cost function weights. The weights associated to the output are utilised to scale the control variables and guide additional control efforts in the direction of the main controlled outputs in order to reach a firm control of the aforementioned controlled outputs [50]. Therefore, tuning such weights could improve significantly the performance of the controller [51]. The majority of the present MPC tuning techniques are performed offline and often employ a trial and error method [52].

In this study the output variable reference tracking $w^y$ and manipulated variable suppression $w^{du}$ were taken into account. These $w$ weights are nonnegative performance weights and those utilised in the MPC have distinct goals and have to be defined accordingly [53]. Meanwhile, scale factors have to be specified for every plant input and output variable and they have to be constant as the controller is tuned. Thus, the aforementioned set of weights was optimised in order to obtain the optimal solution which entails the lowest cost possible for the customer. By tuning the controller with the optimal solution of the set of weights the lowest cost is achieved and consequently less energy is consumed. Naturally, if the chosen set of weights is not the best a poor result will be reached and the quality of the MPC solution will be compromised. Thus, as a condition to obtain the best result possible a good tuning is critical to the performance of the MPC controller.

For each electrical load of the HVAC system the energy and cost are calculated and the results are presented by varying the different MPC weight combination. This was accomplished by employing the best tariff option - option A, as proven by the previous results in subsection 4.2. In this manner, a wide range of results was obtained with all possible weight combinations. The result of the total energy consumption of the HVAC system during the entire week with and without PV microgeneration is presented in Figures 16 and 17. The obtained results related to the overall cost of the consumed energy during the entire week are given by Figures 18 and 19.

"Figure 16 can be observed at the end of the document".

"Figure 17 can be observed at the end of the document".

"Figure 18 can be observed at the end of the document".

"Figure 19 can be observed at the end of the document".

By observing the Figures 16-19 it is possible to observe the best set of weights $w^y$ and $w^{du}$ for each studied case. It can also be noticed that the PV microgeneration has a positive impact on the overall energy consumption. In the case of the energy consumption without the PV microgeneration the worst set of weights translates into extra 5.4% kWh in comparison with the best set of weights. The PV microgeneration allows savings of circa 3.5% of energy consumption when compared with the scenario without the PV (solar energy included).
However, the highest difference is witnessed in the cost. The worst set of weights worsens the cost in 4.6% when compared to the optimal solution. Also, the overall cost of the entire week by employing the PV microgeneration is 75% lower than the scenario without the PV. Such a large reduction is explained by the fact that the PV generates energy during day time which is when the outside temperature is higher since it is when the HVAC system is operating at the highest capacity, thus shifting the source of consumption of the energy during the most demanding period from the grid to the PV.

5. Conclusion

In this paper an alternative MPC strategy with the purpose of stimulating an efficient use of home heating energy has been presented. Instead of utilising a linear power switch, the new contribution of this paper was a power interface that adjusts the MPC dynamic range of the output command signal into a discrete two level control signal. A comprehensive comparison was made between a MPC model and the ON/OFF and PID control models of a domestic HVAC system controlling the temperature of a room. The model of the house with local solar microgeneration is assumed to be located in Portugal, namely in the city of Covilhã. The household of the case study was subject to the local solar irradiance, temperature and five ToU rates made available by the electricity retailer for the residential sector and applied on an entire week of August, 2016. Consequently, the energy consumption of the HVAC performance during the entire week was assessed as well as the consumed energy cost. The results showed that the MPC solution offers the lowest costs for all the available options, while the ON/OFF is the most expensive solution. It can also be assessed from the results that Option A, a three tier ToU rate with adjusted prices during weekends, is the less costly ToU rate for the customer. Also, if the customer selected the MPC and option A, 6.6% per week could be saved when compared to the flat tariff for the same control method, saving as nearly as 31.12% when compared with the most expensive option - option E with ON/OFF control. Finally, using the PV microgeneration allowed savings around 3.5% of energy consumption, including the solar generation, when compared with the scenario without the PV. It is in the bill that the biggest impact of the PV is observed since the customer can save up to 75% by using the MPC controller with the optimal set of weights. This result is only achievable if the MPC controller is configured with an optimal set of weights. Thus, the optimization of these parameters has to be followed in order to obtain the lowest cost possible.

Acknowledgments

This work was funded in part by Fundação para a Ciência e Tecnologia (FCT), under project UID/EMS/00151/2013 C-MAST, with reference POCI-01-0145-FEDER-007718. This work was also supported by FEDER funds through COMPETE 2020 and by Portuguese funds through FCT, under Projects SAICT-PAC/0004/2015 - POCI-01-0145-FEDER-016434, POCI-01-0145-FEDER-006961, UID/EEA/50014/2013, UID/CEC/50021/2013, UID/EMS/00151/2013, and SFRH/BPD/102744/2014. Moreover, the research leading to these results has received funding
from the EU Seventh Framework Programme FP7/2007-2013 under grant agreement no. 309048.

References


List of Tables

Table 1 - The prices of the electricity tariffs in €/kWh.

<table>
<thead>
<tr>
<th></th>
<th>Without VAT (in €)</th>
<th>With VAT € (in €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Tariff</td>
<td>0.1634</td>
<td>0.2010</td>
</tr>
<tr>
<td>Two tier ToU rate Valley</td>
<td>0.1002</td>
<td>0.1232</td>
</tr>
<tr>
<td>Two tier ToU rate Non-Valley</td>
<td>0.1909</td>
<td>0.2348</td>
</tr>
<tr>
<td>Three tier ToU rate Valley</td>
<td>0.1002</td>
<td>0.1232</td>
</tr>
<tr>
<td>Three tier ToU rate Peak</td>
<td>0.1716</td>
<td>0.2111</td>
</tr>
<tr>
<td>Three tier ToU rate Critical Peak</td>
<td>0.2169</td>
<td>0.2668</td>
</tr>
</tbody>
</table>

Table 2 - The cost for ON/OFF, PID and MPC of every option of tariff scheme for the entire week.

<table>
<thead>
<tr>
<th></th>
<th>Cost in €</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PID</td>
</tr>
<tr>
<td>Option A</td>
<td>31.69</td>
</tr>
<tr>
<td>Option B</td>
<td>33.79</td>
</tr>
<tr>
<td>Option C</td>
<td>36.68</td>
</tr>
<tr>
<td>Option D</td>
<td>33.88</td>
</tr>
<tr>
<td>Option E</td>
<td>38.00</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1 – The options of ToU electricity rates.

Figure 2 – The exterior air temperature from 8th to 14th August, 2016.
Figure 3 – The solar irradiance in Covilhã from 8\textsuperscript{th} to 14\textsuperscript{th} August, 2016.

Figure 4 – The temperature control of the indoor environment.

Figure 5 – The overall summary of the two-level control signal MPC scheme.
Figure 6 – The operation technique with a two-level modulation.

Figure 7 – The temperature of the room by using ON/OFF, PID and MPC on 8th August, 2016.
Figure 8 – The temperature of the room by using ON/OFF, PID and MPC on 13th August, 2016.
Figure 9 – The temperature of the room by using ON/OFF, PID and MPC on 14th August, 2016.

Figure 10 – The energy consumption of ON/OFF, PID and MPC of the controlled HVAC system.
Figure 11 – The consumed energy cost in cents of ON/OFF, PID and MPC of the controlled HVAC system by employing the three tier tariff Option A.

Figure 12 – The consumed energy cost in cents of ON/OFF, PID and MPC of the controlled HVAC system by employing the flat tariff Option B.
Figure 13 – The consumed energy cost in cents of ON/OFF, PID and MPC of the controlled HVAC system by employing the three tier tariff Option C.

Figure 14 – The consumed energy cost in cents of ON/OFF, PID and MPC of the controlled HVAC system by employing the two tier tariff Option D.
Figure 15 – The consumed energy cost in cents of ON/OFF, PID and MPC of the controlled HVAC system by employing the two tier tariff Option E.

Figure 16 – The energy consumed in kWh without the PV microgeneration according to the set of weights.
Figure 17 – The energy consumed in kWh with the PV microgeneration according to the set of weights.

Figure 18 – The overall cost without the PV microgeneration given the set of weights.
Figure 19 – The overall cost with the PV microgeneration given the set of weights.