Modeling Strategic Behavior of Distribution Company in Wholesale Energy and Reserve Markets

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Abstract—The decision making framework in power system has changed due to presence of distributed energy resources (DERs). These resources are installed in distribution networks to meet demand locally. Therefore, distribution companies (Discos) are enabled to supply energy through these resources to meet their demand at a minimum operation cost. Moreover, in this framework, the Disco will change its role in the wholesale energy market from price taker to price maker as a decision maker. Moreover, DERs can serve reserve in their normal operation. This opportunity facilitates Disco to provide reserve to the wholesale reserve market. The strategic behavior of a Disco in wholesale energy and reserve markets is modeled in this paper as a bi-level optimization problem. The operation problem of the Disco and the Independent System Operator (ISO) are modeled in the upper- and lower-level problems, respectively. Karush-Kuhn-Tucker (KKT) duality theory conditions are employed to transform the proposed nonlinear bi-level problem to a linear single level one. Numerical studies demonstrate the proficiency of the proposed model and the solution methodology.

Index Terms—bi-level optimization problem, Disco operation problem, energy and reserve markets.

I. INTRODUCTION

In traditional distribution networks, distribution companies (Discos) participate in wholesale energy markets as price-taker players and purchase the required energy for their networks. High energy cost, low reliability, and power losses are the major problems of these networks. To mitigate these problems, distributed energy resources (DERs) including distributed generation (DGs), energy storage, and demand side management are used to meet the demand of distribution networks locally. In the presence of these resources, distribution networks are active distribution networks (ADNs). In such framework, the operation problem of the Disco has changed so that the Disco has some other choices to meet its demand including optimal energy trading with DER owners and optimal scheduling of their DERs. Therefore, the role of the Disco in the wholesale energy market has changed and the Disco behaves as a price-maker player.
Moreover, DERs are fast-response resources that enable the Disco to participate in the wholesale reserve market. The aim of this paper is to model the strategic behavior of distribution networks in the presence of DERs in wholesale energy and reserve markets.

In this context, the operation problem of the Disco as a price-taker player has been studied in many works. An appropriate decision making frameworks is presented for the operation problem of the Disco in day-ahead and real-time electricity markets in [1, 2]. The operation problem of Disco in day-ahead energy market is modeled considering its optimal behavior to trade energy with plug-in electric vehicles in [3]. The short-term operation problem of Disco is modeled considering price-based demand response programs in [4]. Optimal decision making of the Disco in wholesale energy and reserve markets is proposed in [5-8] in which, optimal decision of Disco is determined based on optimal scheduling of DERs in its network. The operation problem of Disco in wholesale energy and reserve markets is modeled considering its optimal behavior with microgrids in its network in [9-11]. As stated before, in all these papers, the Disco is modeled as a price-taker decision maker in wholesale markets [1-11]. On the contrary, in [12] the Disco is considered as a price-maker player in wholesale energy markets while it exchanges energy with DER owners. However, the decision making framework of the Disco is different when it participates in both energy and reserve markets. Moreover, the behavior of DER owners change when they can serve both energy and reserve to the Disco. Therefore, the main contributions of this paper are as follows:

- Modeling the decision making framework of the Disco as a price-maker player in wholesale energy and reserve markets.
- Modeling the decision making problem as a bi-level problem and transforming the proposed non-linear bi-level model into a single-level mixed-integer linear programming (MILP) using Karush-Kuhn-Tucker (KKT) conditions and duality theory.

The rest of the paper is organized as follows. Mathematical modeling and its solution methodology are presented in section III. Numerical results are done in IV and conclusion is presented in section V.

II. PROBLEM DESCRIPTION

In this paper, the strategic behavior of the Disco is modeled in both wholesale energy and reserve markets. The problem is cast as a bi-level optimization problem in which the operation problem of the Disco is modeled in the upper level and the model for the simultaneous energy and reserve markets is presented in the lower level. The upper level problem is as follows:

\[
\text{Maximize } \{ \text{Profit}^{\text{Dis,E}} + \text{Profit}^{\text{Dis,R}} - \text{Cost}^{\text{DG}} - \text{Cost}^{\text{IL}} \} \tag{1}
\]

\[
\text{Profit}^{\text{Dis,E}} = \sum_{a} \rho_{o}(p_{\text{Dis,\text{out}}} - p_{\text{Dis,\text{in}}}) \tag{2}
\]

\[
\text{Profit}^{\text{Dis,R}} = \sum_{a} \rho_{o}(\pi_{\text{RM}} + \kappa_{i}^{\text{RM}} p_{\text{Dis,\text{in}}} \Pi_{t}^{\text{IL}} (1 - \psi_{t}^{\text{Dis}}) - \kappa_{i}^{\text{RM}} r_{\text{Dis}} \Pi_{t}^{\text{Pen}} \psi_{t}^{\text{Dis}}) \tag{3}
\]

\[
\text{Cost}^{\text{Dis,DG}} = \sum_{a} \rho_{o} \sum_{j} (C_{i}^{\text{DG}} p_{\text{Dis,\text{in}}} + C_{j}^{\text{DG}} \kappa_{i}^{\text{RM}} r_{\text{Dis}}^{\text{DG}}) \tag{4}
\]

\[
\text{Cost}^{\text{Dis,IL}} = \sum_{a} \rho_{o} \sum_{j} (C_{i}^{\text{IL}} p_{\text{Dis,\text{in}}} + C_{j}^{\text{IL}} \kappa_{i}^{\text{RM}} r_{\text{Dis}}^{\text{IL}}) \tag{5}
\]

III. MATHEMATICAL MODELING

A. Disco’s Operation Problem

In this paper, the strategic behavior of the Disco is modeled in both wholesale energy and reserve markets. The problem is cast as a bi-level optimization problem in which the operation problem of the Disco is modeled in the upper level and the model for the simultaneous energy and reserve markets is presented in the lower level. The upper level problem is as follows:

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\]

\[
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\]

\[
\text{Cost}^{\text{Dis,IL}} = \sum_{a} \rho_{o} \sum_{j} (C_{i}^{\text{IL}} p_{\text{Dis,\text{in}}} + C_{j}^{\text{IL}} \kappa_{i}^{\text{RM}} r_{\text{Dis}}^{\text{IL}}) \tag{5}
\]

![Fig. 1. Decision making framework of the proposed bi-level approach](image-url)
\[
p_{\text{Dis},\text{out}} = \sum_{i} \rho_{i} P_{i,\text{Dis},\text{out}}^{\text{Dis},\text{out}}, \quad P_{\text{Dis},\text{in}} = \sum_{i} \rho_{i} P_{i,\text{Dis},\text{in}},
\]

subject to:
\[
p_{\text{Dis},\text{in}} \eta^{\text{out}} - P_{\text{Dis},\text{out}}^{\text{Dis},\text{out}} \eta^{\text{in}} + \sum_{i} P_{i,\text{Dis}}^{\text{DG}} + P_{i,\text{Dis}}^{\text{PV}} + P_{i,\text{Dis}}^{\text{WT}}
+ \sum_{k} \left( \frac{p_{\text{batt},\text{in}}}{\eta_{\text{batt}}} - \frac{p_{\text{batt},\text{out}}}{\eta_{\text{batt}}} \right) = \sum_{j} \left( P_{j}^{L} - P_{j}^{\text{IL}} \right)
\]

\[
r_{\text{Dis}}^{\text{Dis},\text{in}} = \sum_{i} r_{i,\text{Dis}}^{\text{DG}} + \sum_{j} r_{j,\text{Dis}}^{\text{IL}} + \sum_{k} r_{\text{batt}}^{\text{Dis}}
\]

\[
p_{i,\text{Dis}}^{\text{Dis},\text{in}} + r_{i,\text{Dis}}^{\text{Dis},\text{out}} \leq P_{i,\text{Dis}}^{\text{DG}}
\]

\[
0 \leq p_{i,\text{Dis}}^{\text{DG}}, \quad 0 \leq r_{i,\text{Dis}}^{\text{DG}},
\]

\[
p_{i,\text{Dis}}^{\text{DG}} - p_{i,\text{Dis}}^{\text{DG},\text{int}} \leq RUP_{i,\text{Dis}}^{\text{DG}} \quad \forall t > 1, \omega
\]

\[
p_{i,\text{Dis}}^{\text{DG}} - p_{i,\text{Dis}}^{\text{DG},\text{ext}} \leq RUP_{i,\text{Dis}}^{\text{DG}} \quad \forall t = 1, \omega
\]

\[
p_{i,\text{Dis}}^{\text{DG}} - p_{i,\text{Dis}}^{\text{DG},\text{int}} \leq RDN_{i,\text{Dis}}^{\text{DG}} \quad \forall t > 1, \omega
\]

\[
p_{i,\text{Dis}}^{\text{DG},\text{int}} - p_{i,\text{Dis}}^{\text{DG}} \leq RDN_{i,\text{Dis}}^{\text{DG}} \quad \forall t = 1, \omega
\]

\[
r_{i,\text{Dis}}^{\text{DG},\text{int}} \leq RUP_{i,\text{Dis}}^{\text{DG}}
\]

\[
r_{i,\text{Dis}}^{\text{IL},\text{out}} = p_{i,\text{Dis}}^{\text{IL},\text{in}}, \quad 0 \leq p_{i,\text{Dis}}^{\text{IL},\text{in}}
\]

\[
r_{i,\text{Dis}}^{\text{IL},\text{out}} + p_{i,\text{Dis}}^{\text{IL},\text{in}} \leq P_{i,\text{Dis}}^{\text{IL}}
\]

\[
0 \leq p_{i,\text{Dis}}^{\text{IL},\text{in}}, \quad 0 \leq r_{i,\text{Dis}}^{\text{IL},\text{in}}
\]

Equation (1) presents the objective function of the Disco’s operation problem consisting of four main terms. The first term is the energy trading of the Disco in the wholesale electricity market, which is modeled in equation (2). The reserve provision for the wholesale reserve market is modeled in (3). The reserve interaction includes income from reserve provision and delivering energy after the reserve call including the penalty for not being ready to deliver the required reserve amount. The cost of energy generation and the required energy interruption by DGs and ILs are presented in (4) and (5), respectively. The uncertainties of renewable energy resources power forecasted are modeled in the form of some scenarios and are considered only in upper-level problem and according to the them, the expected value of Disco’ decision variables consists of power purchased/sold to/from energy market and reserve provided to the reserve market is determined as described in (6).

The constraints of the Disco problem including energy and reserve balance and technical constraints of DERs are modeled in (7)-(29). Equations (7) and (8) represent the energy and reserve balance of the Disco. The detailed operational modeling of DGs are described in (9)-(16). The maximum amount of power and reserve provided by DGs is limited by (9). The ramp up and ramp down limits of DGs are described in (12) and (15) as proposed in [9]. The interruptible loads submit their bids to the Disco consists of price of load curtailment and maximum amount of it. Its price is considered in Disco objective function (equation (5)) and its technical constraints is modeled by (17)- (19). The power generation of wind turbine and photovoltaic arrays are limited to their forecasted power as described in (20) and (21). The minimum and maximum limitation of charging/discharging power of battery and its energy to provide energy and reserve is shown in (22)-(27). Energy balance of battery in each time step is modeled as (28) and (29). Based on the proposed model, the decision variables of the upper-level problem used as parameters in the lower-level problem are [\pi_{\text{Dis},\text{offer}}, \pi_{\text{Dis},\text{bid}}, \pi_{\text{Dis},\text{res}}].

B. Modeling Energy and Reserve Markets

In the lower-level problem, the Disco participates in both the wholesale energy and reserve markets, simultaneously. Both markets are operated by the Independent System Operator (ISO) and consist of retailers and generation companies (Gencos). Retailers participate in the energy market to supply their customers’ energy consumption. Moreover, the customers are considered as smart players who decrease their demand based on retailers’ incentive-based demand response programs in the operation period. The Gencos supply the energy required by the system and participate in the reserve market to guarantee the system adequacy in case of contingency. The lower-level optimization problem of the proposed bi-level problem is as follows:

\[
\text{Minimize} \sum_{n} \sum_{i} \prod_{\text{GenCo,Offer}} P_{n,i}^{\text{GenCo,Offer}} - \sum_{m} \prod_{\text{Retailer, Bid}} P_{m,i}^{\text{Retailer}} + \sum_{n} P_{b,\text{Dis},\text{out}}^{\text{Dis},\text{out}} - P_{b,\text{Dis},\text{in}}^{\text{Dis},\text{in}}
\]

\[
+ \sum_{m} \prod_{\text{Retailer, Bid}} P_{b,\text{Dis},\text{out}}^{\text{Dis},\text{out}} - P_{b,\text{Dis},\text{in}}^{\text{Dis},\text{in}}
\]

\[
+ \sum_{n} \sum_{i} \prod_{\text{GenCo,Offer}} P_{n,i}^{\text{GenCo,Offer}} + \sum_{m} \prod_{\text{Retailer, Bid}} P_{m,i}^{\text{Retailer}} (1 - \psi_{\text{Retailer}})
\]

\[
- \sum_{m} \sum_{i} \prod_{\text{GenCo,Offer}} P_{n,i}^{\text{GenCo,Offer}} - \sum_{m} \prod_{\text{Retailer, Bid}} P_{m,i}^{\text{Retailer}} (1 - \psi_{\text{Retailer}})
\]

\[
+ \sum_{n} \sum_{i} \prod_{\text{GenCo,Offer}} P_{n,i}^{\text{GenCo,Offer}} - \sum_{m} \sum_{i} \prod_{\text{Retailer, Bid}} P_{m,i}^{\text{Retailer}} (1 - \psi_{\text{Retailer}})
\]

\[
+ \sum_{n} \sum_{i} \prod_{\text{GenCo,Offer}} P_{n,i}^{\text{GenCo,Offer}} - \sum_{m} \sum_{i} \prod_{\text{Retailer, Bid}} P_{m,i}^{\text{Retailer}} (1 - \psi_{\text{Retailer}})
\]

\[
\sum_{n} P_{\text{Retailer}} - \sum_{n} P_{\text{GenCo}} + P_{\text{Dis},\text{in}} - P_{\text{Dis},\text{out}} = 0 : \lambda_{\text{Dis}}^{\text{Dis}}
\]
The model proposed in the previous section is a non-linear bi-level optimization problem. Since the decision variables of the upper-level problem are considered as parameters in the lower-level problem, they can be replaced with their KKT conditions [9, 13-15]. Therefore, the bi-level problem is transformed into a single-level one named mathematical program with equilibrium constraints (MPEC). Moreover, the non-linear expressions in the upper-level problem are replaced with linear expressions using duality theory [9, 13]. The resulting model is a MILP.

\[
\begin{align*}
\sum_{m} r_{\text{Retailer}, m, \lambda} + \sum_{n} p_{\text{GenCo}, n} + r_{\text{Dis}, \lambda} &= R_{\text{Sys}, \lambda} \quad \lambda = \text{EM} \\
0 &\leq p_{\text{Retailer}, m, \lambda} : \mu_{\text{Retailer}, m, \lambda} \\
p_{\text{Retailer}, m, \lambda} + r_{\text{Retailer}, m, \lambda} &\leq R_{\text{Retailer}, m, \lambda} : \overline{p}_{\text{Retailer}, m, \lambda} \\
0 &\leq r_{\text{Retailer}, m, \lambda} : \mu_{\text{Retailer}, \lambda} \\
p_{\text{GenCo}, n} + r_{\text{GenCo}, n} &\leq R_{\text{GenCo}, n, \lambda} : \overline{p}_{\text{GenCo}, n, \lambda} \\
0 &\leq R_{\text{GenCo}, n, \lambda} : \mu_{\text{GenCo}, \lambda} \\
p_{\text{Dis}, \lambda} + r_{\text{Dis}, \lambda} &\leq \overline{p}_{\text{Dis}, \lambda} : \mu_{\text{Dis}, \lambda} \\
0 &\leq \mu_{\text{Dis}, \lambda} \\
0 &\leq r_{\text{Dis}, \lambda} : \mu_{\text{Dis}, \lambda}
\end{align*}
\]

Equation (30) is the objective function of the ISO to clear both energy and reserve markets, simultaneously. In this model, Gencos sell to the energy and reserve markets their generation in normal and contingency modes. Moreover, retailers bid to buy the required energy in the energy market and offer their interruption capability to the reserve market. Equations (31)-(42) show the market players operational constraints to participate in both energy and reserve markets. \(\lambda\) and \(\mu\) are the dual variables for equality and non-equality constraints of the lower-level problem, respectively that are shown on the right hand side of each equation.

C. MPEC

The simulation results are shown in Figs. 2 to 6. The results show that the demand in hours 1-12, 21, 22, and 24 will be met by Genco 1, Genco 2, and Gencos 5-9 as these Gencos have low marginal costs. Therefore, the prices of the wholesale energy market in these hours are low. In hours 14-20, the demand is increased and Genco 10 is added to the network to meet the demand of system. Due to the high marginal cost of this Genco, wholesale electricity prices are higher in these hours. In hours 13 and 23, the Disco reduces the energy purchased from the market and meets its demand with the optimal scheduling of DERs. This behavior of the Disco decreases the demand of the whole system as well as wholesale energy prices. In fact, if the Disco purchases the required demand in these hours from the market, this demand will be met by Genco 10 which will increase the wholesale energy price from $45/MWh to $48/MWh. Therefore, the wholesale energy price is decreased in hours 13 and 23 due to the strategic behavior of the Disco.

In hours 1-7, the demand of the Disco is low and it utilizes DERs to provide the required reserve for the network. Therefore, the Disco increases the wholesale reserve price to the marginal cost of Genco 7, thus this Genco does not provide reserve for the system and most of the reserve is provided by the Disco. In fact, the wholesale reserve prices in hours 1-7 are determined due to the strategic behavior of the Disco. In other hours, the Disco uses their DERs to provide the required energy and decreases the provided reserve for the system. In these hours, the wholesale reserve prices are determined due to the strategic behavior of Genco 7 and Genco 9.

IV. NUMERICAL RESULTS

To evaluate the effectiveness of the proposed model, a hypothetical network is considered. In the proposed network, 10 Gencos and retailers are considered. Required data for these players are demand bids, production offers, reserve offers, technical constraints of Gencos, maximum demand of retailers, maximum reserve provided by Gencos and retailers, and total reserve required of the network which all of them are described in [16]. Technical and economic data of DERs are required for Disco to determine optimal decisions in wholesale energy and reserve markets is presented in [9]. Moreover, the probability of calling reserve, the incentive price for providing reserve and the penalty for not serving reserve are other required data for decision makers are presented in [9]. The maximum capacity and efficiency of the Disco’s transformer are 200 MW and 0.95.

The uncertainties of PV arrays and wind turbines are modeled with three scenarios for each resource as modeled in [16]. The capacity of these resources are 3.3 MW.

\[
\begin{align*}
\sum_{m} r_{\text{Retailer}, m, \lambda} + \sum_{n} p_{\text{GenCo}, n} + r_{\text{Dis}, \lambda} &= R_{\text{Sys}, \lambda} \quad \lambda = \text{EM} \\
0 &\leq p_{\text{Retailer}, m, \lambda} : \mu_{\text{Retailer}, m, \lambda} \\
p_{\text{Retailer}, m, \lambda} + r_{\text{Retailer}, m, \lambda} &\leq R_{\text{Retailer}, m, \lambda} : \overline{p}_{\text{Retailer}, m, \lambda} \\
0 &\leq r_{\text{Retailer}, m, \lambda} : \mu_{\text{Retailer}, \lambda} \\
p_{\text{GenCo}, n} + r_{\text{GenCo}, n} &\leq R_{\text{GenCo}, n, \lambda} : \overline{p}_{\text{GenCo}, n, \lambda} \\
0 &\leq R_{\text{GenCo}, n, \lambda} : \mu_{\text{GenCo}, \lambda} \\
p_{\text{Dis}, \lambda} + r_{\text{Dis}, \lambda} &\leq \overline{p}_{\text{Dis}, \lambda} : \mu_{\text{Dis}, \lambda} \\
0 &\leq \mu_{\text{Dis}, \lambda} \\
0 &\leq r_{\text{Dis}, \lambda} : \mu_{\text{Dis}, \lambda}
\end{align*}
\]

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provide reserve using its DERs. On the other hand, when the demand of the Disco increases, it has the capability to act in the electricity market as a strategic player, being able to decrease the demand with an optimal scheduling of its DERs.

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