Stochastic Planning and Operation of Energy Hubs considering Demand Response Programs using Benders Decomposition Approach

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Abstract - In this paper, an integrated approach for optimal planning and operation of energy hubs is provided considering the effects of wind energy resources. Inevitable uncertainties of electrical, heating, cooling demands as well as the wind power generation are considered in this study. The proposed model is based on two-stage optimization problems and represented as a stochastic programming problem to address the effects of uncertain parameters. In order to address the uncertain parameters in the model, different scenarios have been generated by Monte-Carlo Simulation approach and then the scenarios are reduced by applying K-means method. In addition, the effects of demand response programs on the operational sub-problem are taken into account. Benders decomposing approach is adopted in this research to solve the complex model of coordinated planning and operation problem. The master problem is supposed to determine the type and capacity of hub equipment, while the operating points of these assets are the decision variables of the operational slave problem. As a result, the proposed mathematical model is expressed as a linear model solved in GAMS. The simulation results confirm that the Benders decomposition method offers extremely high levels of accuracy and power in solving this problem in the presence of uncertainties and numerous decision variables. Moreover, the convergence time is drastically decreased using Benders decomposition method.

Keywords: Energy Hub; Benders Decomposition; Demand Uncertainty; Demand Response Programs; Renewable Energy Resources.

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### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$t$</td>
<td>Time index</td>
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<tr>
<td>$s$</td>
<td>Season index</td>
</tr>
<tr>
<td>$sc$</td>
<td>Scenario index</td>
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<tr>
<td>$em$</td>
<td>Emission</td>
</tr>
<tr>
<td>$TC$</td>
<td>Total cost</td>
</tr>
<tr>
<td>$C_I$</td>
<td>Total investment cost</td>
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<td>$C_O$</td>
<td>Total operation cost</td>
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<tr>
<td>$IC$</td>
<td>Investment cost of each asset</td>
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<tr>
<td>$E$</td>
<td>Each assets capacity</td>
</tr>
<tr>
<td>$P$</td>
<td>Interest rate</td>
</tr>
<tr>
<td>$k$</td>
<td>Life time of asset</td>
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<tr>
<td>$P_{e}^{\text{max}}$</td>
<td>Installed capacity of each asset</td>
</tr>
<tr>
<td>$p_{\text{Install}}^{\text{e}}(c)$</td>
<td>Rated capacity of candidate</td>
</tr>
<tr>
<td>$I_{e}^{\text{up}}(c)$</td>
<td>Binary decision of candidate</td>
</tr>
<tr>
<td>$\rho(sc)$</td>
<td>Probability of demand scenarios</td>
</tr>
<tr>
<td>$w(s)$</td>
<td>Number of days of each season</td>
</tr>
<tr>
<td>$\pi_{\text{Net,Buy}}^{e}(s,t)$</td>
<td>Cost of buying electricity from grid</td>
</tr>
<tr>
<td>$\pi_{\text{Net,Sell}}^{e}(s,t)$</td>
<td>Cost of selling electricity to local grid</td>
</tr>
<tr>
<td>$\pi_{\text{Net}}^{W}(s,t)$</td>
<td>Natural gas price</td>
</tr>
<tr>
<td>$\pi_{e}^{W}$</td>
<td>Produced wind power cost</td>
</tr>
<tr>
<td>$\pi_{\text{ENS}}^{s}$</td>
<td>Energy not supplied cost</td>
</tr>
<tr>
<td>$\pi_{\text{em}}^{s}$</td>
<td>Emission cost for CO$_2$, SO$_x$, and NO$_2$</td>
</tr>
<tr>
<td>$\pi_{\text{h}}^{S}$</td>
<td>ESS operation cost</td>
</tr>
<tr>
<td>$\pi_{\text{h}}^{T}$</td>
<td>TES operation cost</td>
</tr>
<tr>
<td>$p_{\text{Net,Buy}}^{e}(sc,s,t)$</td>
<td>Purchased power from network</td>
</tr>
<tr>
<td>$p_{\text{Net,Sell}}^{e}(sc,s,t)$</td>
<td>Sold power to network</td>
</tr>
<tr>
<td>$p_{\text{W}}^{e}(sc,s,t)$</td>
<td>Power generation of wind turbine</td>
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<tr>
<td>$p_{\text{Net,CHP}}^{g}(sc,s,t)$</td>
<td>Gas consumption by CHP</td>
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<tr>
<td>$p_{\text{Net,Heat}}^{g}(sc,s,t)$</td>
<td>Gas consumption by boiler</td>
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<tr>
<td>$p_{\text{ch}}^{e}(sc,s,t)$</td>
<td>Charge power of ESS</td>
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<td>$p_{\text{dis}}^{e}(sc,s,t)$</td>
<td>Discharge power of ESS</td>
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<tr>
<td>$p_{\text{h}}^{c}(sc,s,t)$</td>
<td>Charge power of TES</td>
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<tr>
<td>$p_{\text{d}}^{h}(sc,s,t)$</td>
<td>Discharge power of TES</td>
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<tr>
<td>$E_{\text{em}}^{\text{CHP}}$</td>
<td>Emission factor for CHP</td>
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<tr>
<td>$E_{\text{em}}^{\text{Boiler}}$</td>
<td>Emission factor for Boiler</td>
</tr>
<tr>
<td>$P_{\text{CHP}}^{\text{eq}}$</td>
<td>Energy not supplied</td>
</tr>
<tr>
<td>$I_{\text{Net,Buy}}^{e}(sc,s,t)$</td>
<td>Binary variable of buying power</td>
</tr>
<tr>
<td>$I_{\text{Net,Sell}}^{e}(sc,s,t)$</td>
<td>Binary variable of selling power</td>
</tr>
<tr>
<td>$p_{\text{Net,CHP}}^{g}(sc,s,t)$</td>
<td>Gas consumption by CHP</td>
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<tr>
<td>$E_{\text{Loss}}^{ec}(sc,s,t)$</td>
<td>Loss power of ESS / TES</td>
</tr>
<tr>
<td>$\Delta_t$</td>
<td>Time step</td>
</tr>
<tr>
<td>$\alpha_{\text{Initial}}^{ec}$</td>
<td>Initial energy stored at EES/TES</td>
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<tr>
<td>$\alpha_{\text{Final}}^{ec}$</td>
<td>Final energy stored at EES/TES</td>
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<tr>
<td>$\alpha_{\text{Loss}}^{ec}$</td>
<td>Loss efficiency of ESS / TES</td>
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<tr>
<td>$\alpha_{\text{ch}}^{ec}$</td>
<td>Discharge efficiency of ESS / TES</td>
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<td>$\alpha_{\text{dis}}^{ec}$</td>
<td>Discharge efficiency of ESS / TES</td>
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<td>$\alpha_{\text{charge}}^{ec}$</td>
<td>Charge efficiency of ESS / TES</td>
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<tr>
<td>$\alpha_{\text{min}}^{ec}$</td>
<td>Minimum charging factor of EES/TES</td>
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<td>$\alpha_{\text{max}}^{ec}$</td>
<td>Maximum charging factor of EES/TES</td>
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<td>$\alpha_{\text{min,dis}}^{ec}$</td>
<td>Minimum discharging factor of EES/TES</td>
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<td>$\alpha_{\text{max,dis}}^{ec}$</td>
<td>Maximum discharging factor of EES/TES</td>
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<tr>
<td>$\eta_{\text{ch}}^{ec}$</td>
<td>ESS/TES discharge efficiency</td>
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<td>$\eta_{\text{dis}}^{ec}$</td>
<td>ESS/TES charge efficiency</td>
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<tr>
<td>$\eta_{\text{eq}}^{sc,sc,sc}$</td>
<td>Dual variables of the constraints</td>
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<td>$\eta_{\text{eq}}^{sc,sc,sc}$</td>
<td>Index of equations</td>
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<tr>
<td>$p_{\text{max}}^{\text{eq}}$</td>
<td>Forced fixed decision variables (*) of assets</td>
</tr>
<tr>
<td>$NT$</td>
<td>Total number of hours</td>
</tr>
<tr>
<td>$\mu_{\text{eq}}^{\text{sc,sc,sc}}$</td>
<td>Dual variables of the constraints</td>
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<td>$\mu_{\text{eq}}^{\text{sc,sc,sc}}$</td>
<td>Index of equations</td>
</tr>
<tr>
<td>$p_{\text{max}}^{\text{eq}}$</td>
<td>Forced fixed decision variables (*) of assets</td>
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</table>
1. Introduction

A. Motivation

Nowadays, the global energy challenges including the climate concerns, the continuously increasing energy demands, and the mutual impacts of different energy carriers have motivated the energy system (ES) planners to take into consideration the coordinated operation strategies for multi-energy carriers, which are known as the energy hubs.

In addition, this structure improves the energy efficiency as compared to the distinct operation of the electric power grids or the gas networks. In this paper, a nonlinear energy hub model consisting of heating, cooling, and electrical assets for the generation, conversion, or storage of energy is proposed. The proposed model is composed of the planning and operation sub-problems. For simplification purposes, the planning horizon is defined as one-year period. To this end, the investment value for one year is defined in the planning section based on the Annual Investment Cost (AIC) model. In the operation stage, the study is conducted for a fixed sample day of four seasons. Finally, the total operational cost for a year is added to the investment cost addressed by the master problem, i.e. planning stage. Benders decomposition method is used to solve this complex model and the planning and operation stages are introduced as the master problem and the sub-problem, respectively. The planning and operation of energy hubs problem is influenced by uncertainties due to the accumulation of the electrical, thermal, and cooling loads and the impossibility of the accurate prediction of these loads in the planning and operation horizon. In addition, power generation by the renewable wind energy resources is carried out stochastically. Therefore, all the mentioned uncertainties are taken into account in the model proposed in this paper due to the significant importance of the aforementioned uncertainties. The scenarios for the wind turbine power generation are created using historical data. The other scenarios are also created using Monte Carlo simulation method. To reduce the computational burden, the number of scenarios is reduced using the K-means technique. The proposed model is mathematically modeled using the mixed integer non-linear programming (MINLP) model including the binary decision variables corresponding to the capacity additions in the planning problem, and the operational binary and continuous decision variables. Moreover, the capacities are selected based on the candidates for each asset to prevent the installation of non-realistic capacities in the planning phase.

B. Literature Review

The concept of energy hub was introduced in 2007 [1]. According to the definitions of this concept, an energy hub is a set of assets capable of generating, converting, and storing different types of electrical, cooling, and heating energies [2]. In fact, an energy hub functions as an interface between the hub input components, including the distribution electrical network, the natural gas pipelines and end-users’ demands (i.e. electrical,
heating, and cooling loads). Due to the growth of the economical energy systems, the mutual dependence of different energy infrastructure has caused changes in the policies of countries for planning for the development of energy networks. In [3], a mixed integer linear programming (MILP) model is proposed for the development of multi-carrier energy systems including the power grid and gas network. In this optimal planning process, the best location (site), time, and type of assets are identified to minimize the investment costs.

In [4], a dynamic capacity calculation model is proposed for the residential energy hub networks that are composed of a Combined Heat and Power (CHP) units, boilers, photovoltaic systems, and electrical energy storages as well as thermal energy storages. In this model, the planning horizon is divided into two sub-periods and the optimal decisions for the installation of assets in each sub-period are determined.

The optimal operation of energy systems is one of the major challenges regarding the economy, flexibility, and sustainable energy supply of energy hubs. The Authors of [5] proposed a linear standard multi-stage model for the energy hubs. In this method, the complicated energy hub model is divided into several simple energy hubs based on the graph theory and the virtual node concept. Considering the notion of the Internet of Things (IoT) and the possibility of partial demand planning, the operation of an energy hub for the satisfaction of the electrical, heating, and cooling demands is studied in [6]. In the mentioned research, the specifications of the energy hub assets including the energy cost and the degradation cost of electrical, heating, and cooling energy storages as well as the operating point of CHP units are fully analyzed.

A two-objective mathematical model is proposed for energy hub planning based on preventive conservation policies has been proposed in [7]. In this framework, the hub assets are exposed to forced outages and therefore, a periodical predictive maintenance model is proposed considering the energy hub planning issue.

Our review of a wide range of planning studies carried out to analyze the effect of the energy hubs on the optimal planning of different energy systems clearly suggests that the number of studies on the energy hubs and operation is several times higher than the planning studies [8].

Furthermore, many developed countries are concerned about the contingent events in the future due to the effects of the energy component on environmental issues. As a result, they are resorting to the more environment-friendly green energy resources [8]. In addition, the power systems play a substantial role in reducing greenhouse gases due to the continuous increase in the power demand. A probabilistic optimization method has been proposed in [9] for planning of energy hubs based on the solar and wind renewable energy resources capacity additions. In this model, the effects of the presence of the renewable energy resources on the profit of the operating energy hubs has been demonstrated.

In [10], the necessity of changing the structure of metropolitan networks for creating an environment that allows for an increase in the number and capacity of distributed renewable energy resources is addressed and the corresponding actions plans are analyzed. Given the possibility of installing solar power stations for the
end users, the direct effect of photovoltaic systems, as renewable energy resources, on the price of power was studied in [11]. In [12], a multipurpose framework has been proposed for the electrical and natural gas networks with the biogas-solar-wind combined renewable system as a multi-carrier energy system. In this framework, the biogas-solar-wind supplements have been used to establish a thermodynamic connection and interaction with the electrical, gas, and thermal uses.

In [13], it is stated that a three-stage model is developed for handling the demand uncertainties in the mixed integer programming (MIP) framework for energy systems. The aforesaid three stages are: scenario creation, scenario reduction, and scenario optimization based on real load information. A scenario-based stochastic programming model with real solar radiation, temperature, and wind speed information is studied in [14] to deal with the inherent volatility of renewable energy resources. Besides, the effects of the real Time-of-Use (TOU) tariffs of power in line with the real price of gas have been assessed. The long-term impacts of different supportive and incentives programs have been demonstrated in [15]. In [16], a more accurate energy hub framework has been developed by introducing a stochastic model for the real-time price of electricity, natural gas price, and power demand. In this paper, objective function of the energy hub aims at minimizing the weighted sum of operational cost and greenhouse gases. A robust optimization approach is presented in [17] for the robust planning of multi-carrier energy systems based on the economic and environmental constraints as well as the market price uncertainties. In this paper, two pricing methods, namely the Real-Time Pricing (RTP) and TOU pricing techniques, are studied. In [18], risk-based stochastic programming for energy hubs has been carried out using the scenario-based stochastic programming technique in the presence of renewable energy resources. In the mentioned paper, the downside risk constraints concerning the minimization of uncertainty risks are recommended for the risk-based planning of energy hubs. A robust two-stage stochastic programming model is proposed in [19] for sizing the energy hub assets with the distribution sustainability guarantee. In this paper, the generation uncertainties of renewable resources and load demands have been modeled using probabilistic distribution functions.

Energy storages are highly important for maintaining the sustainable performance of power systems, especially networks with highly penetrated by renewable energy resources. The energy storage systems can maintain the imbalance and frequency stability preservation issues in power systems. In [20], a robust optimization method is proposed for the coordinated planning and operation of energy hubs based on the precise economic model of energy storage systems. In this model, the costs of operation and battery deterioration are taken into account.

In terms of reliability assessment, there are some outstanding research works have been conducted [21–29] addressing the reliability issues in power system studies. In addition, the impacts of demand response programs on electrical energy storage systems have been investigated in [30]. A comprehensive literature review has been conducted to clarify the role of the electrical energy storage (EES) devices on reliability
improvement in power system [31]. In Ref. [32], a novel model for adequacy assessment of generating system with highly penetrated wind power generations has been investigated. The impacts of demand response programs, as well as the electrical energy storages on the reliability of the overall power system, have been evaluated in the presence of wind power penetrations.

A joint planning and operation framework has been addressed in [33]. Electrical energy storage system, renewable power generation and demand response program have been considered in this study. The renewable power generation integration has been addressed in Hong Kong. The potential benefits of deployment of renewable power generations, especially wind and solar power integration, have been addressed. Another outstanding research has been investigated in [34] and a cooperative planning structure taking into consideration of interconnected microgrids has been assessed. In this research, meteorological data in Hong Kong was deployed. Comparing with the non-cooperative framework, the overall system cost has been reduced by 35.9%.

In the literature, some of the research works addressed the demand response programs considering the electrical [35] and thermal comfort of the end-users [36–39]. A novel control algorithm considering the thermal comfort level of consumers has been presented in [38] addressing the impacts of renewable energy resources and energy storage devices. The main target of the aforementioned research is to balance the load mismatches taking into account the end-user behaviours while the thermal comfort levels would be guaranteed. Moreover, developing a scalable and robust demand response program has been targeted as the second research gap addressed in the research. The simulation results confirm that the proposed model can effectively reduce the operating costs while the thermal comfort issues have been met. A novel data-driven approach has been presented in Ref. [39] quantifying the potential of individual flexible load users for participation in DR. In this research, the active consumers have been clustered according to their load shifting abilities.

An optimization model is developed in [40] to create a local integrated energy system as a participant in the regulation market based on the pay-for-performance model. In the mentioned paper, the PJM regulation market action model is used for the optimal allocation of regulation resources in a local integrated energy system. In this model, the balance between the revenue and operation costs is provided for the maximization of profit. In [41], the flexibility potential of electrical and thermal storages is analyzed. First, the energy exchanges in the study system are monitored and then the optimization of energy systems is carried out to minimize the operation cost. In [42], an energy hub planning model is introduced based on the optimal operating points of the distributed generation units and the electrical storage system using a cost-effective strategy. Moreover, the cost of the reduced lifetime of the storage system is addressed in the planning stage.
A generalized model for optimal energy planning and operation scheduling of microgrids has been introduced in [43]. The impacts of the electricity prices on the optimal operation of an energy hub have been studied in this paper. The mathematical problem has been formulated as a decentralized energy planning approach taking into consideration of emissions as a constraint in the planning problem. A Monte-Carlo simulation (MCS) approach has been investigated in [44] focusing on the representation of the CHP characteristics in the joint planning and operation problem. To deal with the scalability issues, the optimization problem has been formulated as a standard MILP model considering diverse technologies for energy provision and conversion in the energy hub. The thermal energy storage has been considered in the model, as well to deal with the uncertainties of thermal loads. Ref. [45] presented a multi-objective optimization problem taking into consideration of both maximizations of the social welfare and minimization of the emissions in a typical hub. A permissible solution for solving power, heat and gas flow based on standard Newton-Raphson method has been investigated in the mentioned paper. Compared to the single hub research works, an energy management model for optimal operation of the multi-energy hubs framework has been proposed in [46]. In the proposed model, the profit maximization of each hub in the day-ahead market has been investigated. The mathematical optimization problem has been addressed in a single objective optimization problem and the network constraints have been represented in a linear framework. A three-level integrated energy system (IES) has been proposed in [47] aiming at optimal energy scheduling. The main core idea of this paper is to investigate the problem in Stackelberg game theory framework to address the multiple energy hubs and the impacts of the monopoly of energy providers on overall cost reduction have been studied. A novel stochastic model proposed by Heidari, et.al [48] aims at assessment of the ice storage on optimal operation of an energy hub with renewable energy resources and considering the effects of demand response program. The proposed stochastic programming strategy investigated on optimal operation of clean and renewable energy providers like wind and solar generations and the volatilities of such energy resources have been handled by optimal operation of energy storage devices as well as demand response programs. The impacts of DRPs have been investigated by Lotfi, et al, [49] in the day-ahead markets. In addition to their advantages, multi-carrier energy systems generally introduce new challenges to future energy systems. One of these challenges is the interaction between different energy uses and various operating parameters for the satisfaction of different demands in planning problem of multi-carrier energy systems. In [50], the optimal energy hub planning is carried out based on different planning parameters using a two-stage stochastic risk-constrained model. The overarching goal of the aforesaid paper is to determine the minimum operating cost of a typical energy hub, while serving the cooling, heating, and electrical demands of end-users using demand response programs. A multi-stage model is proposed in [51] to simplify the precise model for the operation
of the assets. In this model, the energy hub model is decomposed into several stages, and the master energy hub model is integrated into the demand response system by multiplying the coupling matrix by the solution obtained in each stage. In [52], an optimal two-level optimal programming model is developed for an energy hub network based on the demand response system to establish a balance between energy consumption and costs. In this paper, the Karush–Kuhn–Tucker (KTT) coefficients have been implemented for the linearization of nonlinear equations. Since end-users of energy hubs can take part in the demand response programs, which result in the flexibility and efficient use of resources, the effect of the demand response programs on certain parameters such as economical operation, reliability, and flexibility, was studied.

C. Contribution

According to the literature review, the joint planning and operation problem for new capacity additions and designing the energy hub is a complex optimization problem in which both technical and economical issues need to be addressed. To address this research gap, the aforementioned complex problem is decomposed into two distinct stages. In the first stage, the techno-economical issues related to the optimal size and technology of the new asset in the design phase were addressed, while the second stage of the problem is related to the determination of optimal operating points of the selected assets in the operational horizon. To solve the problem at the second stage, a scenario-based technique for dealing with the uncertainties has been investigated. In this paper, an integrated model is proposed for the optimal planning and operation of energy hubs. In the proposed model, different assets (including CHP, electrical heater, boiler, electrical heat pump, and absorption chiller) are taken into consideration along with the uncertainties in electrical, cooling, and heating demands and the wind turbine power generation. In addition, the effects of TOU demand response program is investigated in the case studies. The nonlinear planning and operation model formulated as a mixed-integer linear programming model and represented in the form of the Benders decomposition method. In this framework, the problem is decomposed to two stages, namely the master problem (planning) and the sub-problem (operation). The CPLEX solver is also used in GAMS to solve the model.

The main contributions of the model proposed in this paper are as follows:

- Addressing the complex problem of integrated operation and planning of energy hubs in a MILP model.
- Improving the convergence time and reducing the computational burden implementing Benders Decomposition approach.
- Representing the uncertain parameters in the sub-problem using rigorous scenario generation and scenario reduction techniques.
- Considering the seasonal demand and generation profiles to address the realistic features of the planning and operation problem.
• Assessment the effects of demand response programs on both planning and operation problems in the integrated model.

**D. Paper Organization**

The rest of this paper is organized as follows: Section 2 presents the conceptual model of energy hub including different generating, converting, and storing energy assets. Section 3 provides the mathematical model of the integrated planning and operation of the energy hubs in two sub-problems, master and slave problems. In this section, the mathematical model for the master problem and the slave sub-problem is addressed according to the Benders decomposition method. The mathematical model is provided along with the complete dual optimality concept equations for the operational sub-problem. In section 4, the simulation results of the proposed model are addressed in different case studies and the performance and effectiveness of the model is investigated. In this section, the results of the proposed method are compared with some commercial solvers. Finally, section 5 presents the conclusion of the paper.

2. **Energy Hub Model**

Figure (1) depicts the conceptual model of the energy hub in this study. In this model, the hub inputs include electricity and natural gas as energy carriers, while the hub outputs are supposed to be three types of load, namely electrical, cooling, and heating loads. In this framework, the hub can have a CHP unit, which is capable of generating both heat and power at the same time. The electric power demand of the hub can be served by the CHP, wind turbine, and purchases from the grid. The cooling loads can be supplied by absorption chiller (AC) and electrical heat pump (EHP) in cooling mode.

In order to fulfil the heating load of the energy hub, there are some other candidates, such as boiler, electrical heater (EH), EHP in heating mode as well as the heat generated by the CHP unit. In addition, electrical energy storage (ESS) and thermal energy storage (TES) systems are used to improve the hub flexibility in the operation horizon. In this model, the energy hub is capable of purchasing electrical energy from and selling the surplus electricity to the upstream grid and purchasing gas from the natural gas pipelines, as well. In this framework, the optimal size of the candidate units has to be determined in such a way that the overall installation and operation cost being minimized.
The primary goal of this paper is to propose an integrated approach for simultaneously solving the energy hub planning and operation problems according to the conceptual model illustrated in Fig. 1 to fulfil end-users’ demands. To achieve this goal, the optimal planning and operation problems are merged into an integrated framework. The master problem is defined as the planning problem in which the size of each device will be set and then will be introduced to be evaluated in different scenarios for each season. Then, the total cost of the investment stage and the expected operational cost of slave problem will be integrated to shape the total cost of the optimization problem. Stochastic programming is carried out using the scenario-based method due to the presence of new renewable wind resources and the inherent volatility of renewable energy resources. In the proposed model, the uncertainties of electrical, cooling, and heating loads are taken into account in addition to the wind turbine power generation.

3. Mathematical Problem Formulation

The objective function of the integrated planning and operation model presented in this paper seeks to minimize the total system costs including the investment costs and the expected operating costs. The objective function is expressed as (1), where $C_I$ and $C_O$ represents the investment cost and the expected operating costs, respectively. It is noteworthy that the dual sub-problem in this paper is convex and it is modelled as a standard mixed-integer optimization problem.
\[ \text{Min: } TC = C_I + C_O \]  

3-1- Energy Hub Planning Problem (Master Problem)

The investment cost is composed of the cost of installing all assets including CHP, boiler, electrical heater, EHP, absorption chiller, transformer, and electrical and thermal energy storages. The cost of investment using the interest rate and the asset lifetime is calculated as follows to express the financial value of installation expenditure based on the first installation year.

\[ C_I = \sum_{ec} r \left( \frac{1+r}{1+r} \right)^k - 1 C_{ec} \]  

(2)

where, \( r \) denotes the interest rate while \( k \) represents the asset lifetime. The investment cost of capacity additions in the energy hub is shown by \( C(ec) \). The investment cost corresponding to the size of each asset can be expressed as:

\[ C_{ec} = I C_{ec} + P_{ec}^{max} \quad \forall ec \in \Omega_{c,s} \]  

(3)

In this framework, \( I C_{ec} \) is the unit cost of installation of a new capacity and \( P_{ec}^{max} \) represents the installed capacity of each candidate.

Since in the proposed model, the candidate capacities are known, a binary variable associated with each asset is used as the decision variable, \( I_{ec}(c) \). Equations (4-5) are also considered to select one asset from each type of candidate list.

\[ P_{ec}^{max} = \sum_{c=1}^{NC} P_{ec}^{install}(c) I_{ec}(c) \quad \forall ec \in \Omega_{c,s} \]  

(4)

\[ 0 \leq \sum_{c=1}^{NC} I_{ec}(c) \leq 1 \quad \forall ec \in \Omega_{c,s} \]  

(5)

3-2- The Energy Hub Operation Problem (Slave Problem)

In this paper, the hub operation problem is solved for four seasons. In addition, given the uncertainty in the input parameters including the wind turbine and electrical, heating, and cooling loads, planning problem is carried out based on different scenarios for each set of data. Equation (6) represents the expected operating cost of the assets for a given year.
where \( w(s) \) and \( \rho(sc) \) denote the number of days per season and the probability of implementation of each scenario, respectively. In this equation, parts one and two deals with the cost of purchasing electrical energy and the revenue obtained from the sales of electrical energy to the upstream grid. The third part shows the expected operating cost of the wind farm. Given the insignificant cost of wind power generation, the cost of wind power generation (\( \pi^W_e \)) is considered to be zero. The fourth and fifth parts refer to the CHP and boiler operation costs, respectively. The sixth and seventh parts represent the operation costs of the electrical and thermal energy storages, respectively. The eighth part shows the CHP and boiler emission costs and the ninth section shows the penalty for the energy not served. It is worth stating in the proposed model, the cost of emissions corresponding to the three gas-fired devices, namely CO\(_2\), SO\(_2\), and NO\(_2\), are taken into account. The corresponding constraints of the slave problem are as follows:

3-2-1- Energy Exchange with the Upstream Grid

Given the possibility of the exchange of energy for both electricity and gas carriers, the energy exchange equations are expressed via equations (a1) to (a4) [53]. It is worth mentioning that the electricity transactions between the distribution network and the hub is supposed to be bidirectional, while the hub can buy natural gas from the corresponding company. Equation (a.3) insures that at the same hour, the hub couldn’t buy and sell electricity.

\[
P^{Net, Buy}_{e, min}(sc,s,t) \leq P^{Net, Buy}_{e}(sc,s,t) \leq P^{Net, Buy}_{e, max}(sc,s,t): \mu^{a1}_{sc,s,t}, \mu^{a1}_{sc,s,t} \]

\( \forall sc,s,t \)
\[ P_{e}^{\text{Net},\min} I_{\text{Net,Sell}} (sc,s,t) \leq P_{e}^{\text{Net,Sell}} (sc,s,t) \]
\[ \leq P_{e}^{\text{Net},\max} I_{\text{Net,Sell}} (sc,s,t): \mu_{ec,s,t}^{2,\min}, \mu_{ec,s,t}^{2,\max} \forall sc,s,t \]  
(a2)

\[ 0 \leq I_{\text{Net,Buy}} (sc,s,t) + I_{\text{Net,Sell}} (sc,s,t) \leq 1 \forall sc,s,t \]  
(a3)

\[ P_{g}^{\text{Net},\min} \leq P_{g}^{\text{Net}, (sc,s,t)} \leq P_{g}^{\text{Net},\max}: \mu_{g,s,t}^{4,\min}, \mu_{g,s,t}^{4,\max} \forall sc,s,t \]  
(a4)

### 3-2-2- Electrical and Thermal Energy Storages

In this section, the constraints on the operation of the electrical and thermal energy storage systems are presented. The detailed of these equations can be found in [42] and [54].

\[ E_{ec}^{s} (sc,s,t) = E_{ec}^{s} (sc,s,t - 1) + P_{ec}^{ch} (sc,s,t) \eta_{ec}^{ch} \Delta t \]
\[ = - \frac{P_{ec}^{dis} (sc,s,t)}{\eta_{ec}} \Delta t - E_{ec}^{\text{Loss}} (sc,s,t): \mu_{ec,s,t}^{1,1} \forall sc,s,t \]  
(b1)

\[ E_{ec}^{s} (sc,s,t) = a_{ec}^{\text{Initial}} P_{max}^{ec} + P_{ec}^{ch} (sc,s,t) \eta_{ec}^{ch} \Delta t \]
\[ = - \frac{P_{ec}^{dis} (sc,s,t)}{\eta_{ec}} \Delta t - E_{ec}^{\text{Loss}} (sc,s,t): \mu_{ec,s,t}^{1,1} \forall sc,s,t = 1, ec \in \Omega \]  
(b2)

\[ E_{ec}^{s} (sc,s,t) = a_{ec}^{\text{Final}} P_{max}^{ec} : \mu_{ec,s,t}^{3,24} \forall sc,s,t = 24, ec \in \Omega \]  
(b3)

\[ E_{ec}^{\text{Loss}} (sc,s,t) = a_{ec}^{\text{Loss}} E_{ec}^{s} (sc,s,t): \mu_{ec,s,t}^{4,ec} \forall sc,s,t, ec \in \Omega \]  
(b4)

\[ \alpha_{ec}^{\text{min}} P_{ec}^{\text{max}} \leq E_{ec}^{s} (sc,s,t) \]
\[ \leq \alpha_{ec}^{\text{max}} P_{ec}^{\text{max}} : \mu_{ec,s,t}^{5,\min}, \mu_{ec,s,t}^{5,\max} \forall sc,s,t, ec \in \Omega \]  
(b5)

\[ \alpha_{ec}^{\text{min,ch}} I_{ec}^{\text{max}} \leq P_{ec}^{ch} (sc,s,t) \]
\[ \leq \alpha_{ec}^{\text{max,ch}} I_{ec}^{\text{max}} : \mu_{ec,s,t}^{6,\min}, \mu_{ec,s,t}^{6,\max} \forall sc,s,t, ec \in \Omega \]  
(b6)

\[ \alpha_{ec}^{\text{min,dis}} P_{ec}^{\text{max}} I_{ec}^{\text{dis}} \leq P_{ec}^{dis} (sc,s,t) \]
\[ \leq \alpha_{ec}^{\text{max,dis}} I_{ec}^{\text{dis}} : \mu_{ec,s,t}^{7,\min}, \mu_{ec,s,t}^{7,\max} \forall sc,s,t, ec \in \Omega \]  
(b7)

\[ 0 \leq I_{ec}^{ch} (sc,s,t) + I_{ec}^{dis} (sc,s,t) \leq 1 \forall sc,s,t, ec \in \Omega \]  
(b8)

The dynamic state of charge of the battery is modelled in (b1), Equations (b2) and (b3) refer to the initial and final state of charge of the battery and in this study, it is supposed that the initial and final level of energy stored in the battery is identical. The operating loss of battery is modelled in (b4) and the minimum/maximum energy can be stored in the battery is provided in (b5). The charging and discharging power limits are addressed in (b6) and (b7), respectively. Equation (b8) states that at each time interval, the battery can work
in one of the charging or discharging states. More details of battery modelling have been explored in [55–57].

3-2-3- Demand Response Program

In general, the demand response programs improve the load profile and reduce the energy supply costs by shifting electrical loads. In this paper, the shiftable load model is used to address the price-based demand response programs. The details of this approach are addressed in [54] and [49].

\[
\sum_{t=1}^{NT} P_{e}^{\text{shup}}(sc, s, t) = \sum_{t=1}^{NT} P_{e}^{\text{shdo}}(sc, s, t) \cdot \mu_{sc,s,t}^{c1} \quad \forall sc, s, t \quad (c1)
\]

\[
0 \leq P_{e}^{\text{shup}}(sc, s, t) \leq LPF_{e}^{\text{shup}} P_{e}(sc, s, t) \cdot I_{e}^{\text{shup}}(sc, s, t) \cdot \mu_{sc,s,t}^{c2} \quad \forall sc, s, t \quad (c2)
\]

\[
0 \leq P_{e}^{\text{shdo}}(sc, s, t) \leq LPF_{e}^{\text{shdo}} P_{e}(sc, s, t) \cdot I_{e}^{\text{shdo}}(sc, s, t) \cdot \mu_{sc,s,t}^{c3} \quad \forall sc, s, t \quad (c3)
\]

\[
0 \leq I_{e}^{\text{shup}}(sc, s, t) + I_{e}^{\text{shdo}}(sc, s, t) \leq 1 \quad \forall sc, s, t \quad (c4)
\]

The demand response program in this paper is organized in such a way that the total energy consumption would be remained fix during the day, while the end-user can shift up and down the electricity consumption according to the hourly price signals. The shifted power must be compensated on the same day. Hence, the total shifted up power should be equal with the shifted down power for the given day (c1). The shifting up and shifting down power are limited to the corresponding load participation factors and these constraints are provided in (c2) and (c3), respectively. It is noteworthy that the associated binary variables have been considered for the power shifting up and down. Equation (c4) refers to this fact that at each time interval, the end-user couldn’t enjoy from both shifting up and down, simultaneously.

3-2-4- Load Balance Constraint

The energy not served criteria are among the most important criteria in the hub operation section. The energy not served constraints are expressed via the following equations [54].

\[
ELF(sc, s) = \frac{1}{NT} \sum_{t=1}^{NT} \frac{P_{e}^{\text{ENS}}(sc, s, t)}{P_{e}(sc, s, t)} \cdot \mu_{sc,s}^{c5} \quad \forall sc, s \quad (c5)
\]

\[
0 \leq ELF(sc, s) \leq ELF^{\text{max}} : \mu_{sc,s}^{c6} \quad \forall sc, s \quad (c6)
\]

According to [54], the expected loss of energy serving is represented in (c5), while the overall loss of energy should be less than the permissible loss of energy determined by the end-user provided in (c6). The less loss of energy results in more investment in capacity additions.
In this paper, the energy consumption carriers used by an energy hub are classified into three categories: electrical loads, heating loads, and cooling loads. The balance equations for each type of load are presented below considering the existing assets [58]:

\[
P_e (sc,s,t) + \frac{P_{Net, sell}^{e\, Net}(sc,s,t)}{\eta_{ee}} + P_e^{ch\,}(sc,s,t) + P_e^{shp\,}(sc,s,t)
+ \frac{P_h^{EHP\,}(sc,s,t)}{\eta_{eh}^{EHP}} + \frac{P_t^{EHP\,}(sc,s,t)}{\eta_{eh}^{EHP}} + P_E^{EH\,}(sc,s,t)
= \eta_{ee}^{CCHP\, P_{Net,CCHP\,}}(sc,s,t) + \eta_{ee}^{CHP\, P_{Net,CHP\,}}(sc,s,t)
+ \eta_{ee}^{Con\, P_{E\, dyn\,}}(sc,s,t) + P_e^{dis\,}(sc,s,t) + P_e^{shd\,}(sc,s,t) + P_e^{ENS\,}(sc,s,t); \mu_{sc,s,t}^1
\forall sc,s,t
\]  

(e1)

\[
P_h (sc,s,t) + P_h^{ch\,}(sc,s,t) + P_h^{AC\,}(sc,s,t)
= \eta_{gh}^{CCHP\, P_{Net,CCHP\,}}(sc,s,t) + \eta_{gh}^{B\, P_{Net,B\,}}(sc,s,t)
+ P_h^{dis\,}(sc,s,t) + P_h^{EHP\,}(sc,s,t)
+ \eta_{eh}^{EH\, P_e^{EH\,}}(sc,s,t); \mu_{sc,s,t}^2
\forall sc,s,t
\]  

(e2)

\[
P_e (sc,s,t) = \eta_{hc}^{AC\, P_h^{AC\,}}(sc,s,t) + P_e^{EHP\,}(sc,s,t); \mu_{sc,s,t}^3
\forall sc,s,t
\]  

(e3)

The energy flow equations for all kind of loads are presented in (e1)-(e3). The electricity can be provided by the grid, CHP, wind power generation, discharged power from the battery, demand response and the surplus can be sold to the grid, charging the battery, increasing the consumption, and can be consumed by EH and EHP in the corresponding cooling or heating modes. The not served energy is modelled in this equation, as well. Equation (e2) states that the heating load can be served by CHP, boiler, EHP, EH and the surplus can be transferred to the AC and the thermal energy storage. The cooling load can be served by AC and EHP in cooling mode (e3).

3-2-5- Technical Constraints

Considering the capacity additions for different assets, the power generation by all assets has to fall into the allowable design range. This constraint, which is also applied to the energy converters in the energy hub, is expressed as follows [54].

\[
0 \leq \frac{P_{Net, sell}^{e\, Net}(sc,s,t)}{\eta_{ee}} \leq P_{T}^{max\,}\cdot \mu_{sc,s,t}^1 \forall sc,s,t
\]  

(f1)

\[
0 \leq \eta_{gg}^{CCHP\, P_{Net,CCHP\,}}(sc,s,t) \leq P_{T}^{max\,}\cdot \mu_{sc,s,t}^2 \forall sc,s,t
\]  

(f2)

\[
0 \leq \eta_{gh}^{B\, P_{Net,B\,}}(sc,s,t) \leq P_{B}^{max\,}\cdot \mu_{sc,s,t}^3 \forall sc,s,t
\]  

(f3)

\[
0 \leq \eta_{hc}^{AC\, P_h^{AC\,}}(sc,s,t) \leq P_{AC}^{max\,}\cdot \mu_{sc,s,t}^4 \forall sc,s,t
\]  

(f4)
0 \leq P^E_{h} (sc,s,t) \leq \eta^E_{h} \cdot P^\max_{EHP} I^E_{h} (sc,s,t) \cdot \mu^5_{sc,s,t} \quad \forall sc,s,t \quad (f5)

0 \leq P^E_{e} (sc,s,t) \leq \eta^E_{e} \cdot P^\max_{EHP} I^E_{e} (sc,s,t) \cdot \mu^6_{sc,s,t} \quad \forall sc,s,t \quad (f6)

0 \leq I^E_{h} (sc,s,t) + I^E_{e} (sc,s,t) \leq 1 \quad \forall sc,s,t \quad (f7)

0 \leq \eta^E_{eh} \cdot P^E_{e} (sc,s,t) \leq P^\max_{EHP} : \mu^8_{sc,s,t} \quad \forall sc,s,t \quad (f8)

3-2-6- Wind Turbine Generations

In general, the power generation by wind turbines is determined by wind speed and the planning specifications. In this paper, the following model is used to calculate the power generation by wind turbines [59].

\[ P^W_{e} (sc,s,t) = 0, \quad v (sc,s,t)v_{ci} \quad and \quad v (sc,s,t)v_{co} \quad \forall sc,s,t \quad (g1) \]

\[ P^W_{e} (sc,s,t) = P_{r} \frac{v (sc,s,t)}{v_{r},v_{ci}}, \quad v_{ci} v (sc,s,t)v_{r} \quad \forall sc,s,t \quad (g2) \]

\[ P^W_{e} (sc,s,t) = P_{r}. \quad v_{r} \leq v (sc,s,t)v_{co} \quad \forall sc,s,t \quad (g3) \]

3-3- The Dual Problem Corresponding to the Sub-Problem

In this section, the dual model equations of the sub-problem for the operation model are presented considering the dual problem variables, which were denoted by \( \mu \) in the previous section. The dual problem objective function is expressed as follows.

\[ \mu^eq_{sc,s,t} \in Z \quad \forall eq \in e_{1,2,3},b_{1,3},c_{1,5} \quad (8) \]
\[
C_O = \sum_{s_c=1}^{\text{SC}} \rho(s_c) \left\{ \sum_{s=1}^{S} \omega(s) \left( \sum_{t=1}^{T} \pi^W_t P_e^W (s_c, s, t) \right) \right\}
\]

\[
\left( P_e (s_c, s, t) - \eta_{\text{con}}^T P_e^W (s_c, s, t) \right) \mu^2 (s_c, s, t) + P_h (s_c, s, t) \mu^2 (s_c, s, t)
\]

\[
+ P_e (s_c, s, t) \mu^2 (s_c, s, t) + P_{\text{Net, max}}^e \mu^1 (s_c, s, t)
\]

\[
+ P_{\text{Net, min}}^e \mu^2 (s_c, s, t) + P_{\text{Net, min}}^e \mu^1 (s_c, s, t)
\]

\[
+ P_{\text{Net, min}}^e \mu^2 (s_c, s, t)
\]

\[
+ P_{\text{Net, min}}^e \mu^2 (s_c, s, t) + P_{\text{Net, max}}^e \mu^4 (s_c, s, t) + P_{\text{max}}^{\text{CHP}} \mu^4 (s_c, s, t)
\]

\[
+ \eta_{\text{EPP}} P_{\text{max}}^{\text{CHP}} \mu^5 (s_c, s, t)
\]

\[
+ \eta_{\text{EPP}} P_{\text{max}}^{\text{CHP}} \mu^5 (s_c, s, t)
\]

\[
+ \alpha_{\text{EES}} P_{\text{max}}^{\text{EES}} \mu^5 (s_c, s, t) + \alpha_{\text{EES}} P_{\text{max}}^{\text{EES}} \mu^5 (s_c, s, t)
\]

\[
+ \alpha_{\text{EES}} P_{\text{max}}^{\text{EES}} \mu^5 (s_c, s, t) + \alpha_{\text{EES}} P_{\text{max}}^{\text{EES}} \mu^5 (s_c, s, t)
\]

\[
+ \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t) + \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t)
\]

\[
+ \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t) + \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t)
\]

\[
+ \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t) + \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t)
\]

\[
+ \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t) + \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t)
\]

\[
+ \alpha_{\text{TES}} P_{\text{max}}^{\text{TES}} \mu^5 (s_c, s, t)
\]

\[
\]
\[ \mu^{a,\text{min}}(sc,s,t) + \mu^{a,\text{max}}(sc,s,t) + \eta^T_e \mu^1(sc,s,t) \]
\[ + \eta^T_e \mu^1(sc,s,t) \leq \rho(sc)\omega(s)\pi_e^{\text{Net,Buy}}(s,t) \quad \forall sc,s,t \] (11)

\[ \mu^{a,\text{min}}(sc,s,t) + \mu^{a,\text{max}}(sc,s,t) - \frac{\mu^1(sc,s,t)}{\eta^T_e} \]
\[ + \frac{\mu^1(sc,s,t)}{\eta^T_e} \leq \rho(sc)\omega(s)\pi_e^{\text{Net,Sell}}(s,t) \quad \forall sc,s,t \] (12)

\[ \eta^{\text{CHP}}_e \mu^1(sc,s,t) + \eta^{\text{CHP}}_e \mu^2(sc,s,t) + \eta^{\text{CHP}}_e \mu^2(sc,s,t) \]
\[ \leq \rho(sc)\omega(s)\left( \pi_g^{\text{Net}} + \sum_{em=1}^{EM} \pi_{em} E_{em}^{\text{CHP}} \right) \quad \forall sc,s,t \] (13)

\[ \eta^{\text{CHP}}_e \mu^2(sc,s,t) + \eta^{\text{CHP}}_e \mu^2(sc,s,t) \]
\[ \leq \rho(sc)\omega(s)\left( \pi_g^{\text{Net}} + \sum_{em=1}^{EM} \pi_{em} E_{em}^{\text{CHP}} \right) \quad \forall sc,s,t \] (14)

### 3-3-2- Electrical and Thermal Energy Storages

The variables in the storage system equations include the state of charge, charging capability, discharging capability, and loss of Energy Storage System (ESS). The dual problem equations corresponding to each of the aforementioned variables for the electrical and thermal energy storages are expressed as:

\[ \mu^{b,\text{EES}}(sc,s,t) - \mu^{b,\text{EES}}(sc,s,t+1) - \alpha^{\text{Loss}}_{\text{EES}} \mu^{b,\text{EES}}(sc,s,t) \]
\[ + \mu^{b,\text{min,EES}}(sc,s,t) + \mu^{b,\text{max,EES}}(sc,s,t) = 0 \quad \forall sc,s,t \] (15)

\[ \mu^{b,\text{EES}}(sc,s,t) - \alpha^{\text{EES}}_{\text{EES}} \mu^{b,\text{EES}}(sc,s,t) + \mu^{b,\text{max,EES}}(sc,s,t) \]
\[ + \mu^{b,\text{min,EES}}(sc,s,t) + \mu^{b,\text{EES}}(sc,s,t) = 0 \quad \forall sc,s,t = 24 \] (16)

\[ -\mu^{e,\text{EES}}(sc,s,t) - \eta^{\text{EES}}_{\text{EES}} \Delta^{b,\text{EES}}(sc,s,t) + \mu^{b,\text{min,EES}}(sc,s,t) \]
\[ + \mu^{b,\text{max,EES}}(sc,s,t) = \rho(sc)\omega(s)\pi_e^{\text{S}} \quad \forall sc,s,t \] (17)

\[ \mu^{e,\text{EES}}(sc,s,t) + \frac{\Delta^{b,\text{EES}}(sc,s,t)}{\eta^{\text{EES}}_{\text{EES}}} + \mu^{b,\text{min,EES}}(sc,s,t) \]
\[ + \mu^{b,\text{max,EES}}(sc,s,t) = \rho(sc)\omega(s)\pi_e^{\text{S}} \quad \forall sc,s,t \] (18)

\[ \mu^{b,\text{EES}}(sc,s,t) + \mu^{b,\text{EES}}(sc,s,t) = 0 \quad \forall sc,s,t \] (19)

\[ \mu^{b,\text{TES}}(sc,s,t) = \mu^{b,\text{TES}}(sc,s,t+1) - \alpha^{\text{Loss}}_{\text{TES}} \mu^{b,\text{TES}}(sc,s,t) \]
\[ + \mu^{b,\text{min,TES}}(sc,s,t) + \mu^{b,\text{max,TES}}(sc,s,t) = 0 \quad \forall sc,s,t \] (20)

\[ \mu^{b,\text{TES}}(sc,s,t) - \alpha^{\text{Loss}}_{\text{TES}} \mu^{b,\text{TES}}(sc,s,t) + \mu^{b,\text{min,TES}}(sc,s,t) \]
\[ + \mu^{b,\text{max,TES}}(sc,s,t) = 0 \quad \forall sc,s,t = 24 \] (21)
- \mu^2(sc, s, t) - \eta_{TES}^h \Delta_h \mu^{h,TES}(sc, s, t) + \mu^{h_{\text{min}, TES}}(sc, s, t) \\
+ \mu^{h_{\text{max}, TES}}(sc, s, t) = \rho(sc) \omega(s) \pi_h^S \\
\forall sc, s, t \tag{22}

\mu^2(sc, s, t) + \frac{\Delta_h \mu^{h_{\text{max}, TES}}(sc, s, t)}{\eta_{TES}^h} + \mu^{h_{\text{min}, TES}}(sc, s, t) \\
+ \mu^{h_{\text{max}, TES}}(sc, s, t) = \rho(sc) \omega(s) \pi_h^S \\
\forall sc, s, t \tag{23}

\mu^{h_{\text{TES}}}(sc, s, t) + \mu^{h_{\text{A,TES}}}(sc, s, t) = 0 \\
\forall sc, s, t \tag{24}

3-3-3- The Dual Problem of Demand Response Program Constraints

The variables in the demand response program equations include an increase or a decrease in the hourly load, the hourly load shedding, and the daily load shedding. The dual problem equations corresponding to each of the aforementioned variables are presented in the following:

- \mu^1(sc, s, t) + \mu^1(sc, s, t) + \mu^2(sc, s, t) \leq 0 \\
\forall sc, s, t \tag{25}

\mu^1(sc, s, t) - \mu^1(sc, s, t) + \mu^2(sc, s, t) \leq 0 \\
\forall sc, s, t \tag{26}

\mu^5(sc, s, t) - \frac{\mu^5(sc, s)}{T \cdot P_e(sc, s, t)} \leq \rho(sc) \omega(s) \pi_{\text{ENS}}^S \\
\forall sc, s, t \tag{27}

\mu^5(sc, s) + \mu^6(sc, s) = 0 \\
\forall sc, s \tag{28}

3-3-4- The Dual Problem of Technical System Constraints

The dual problem equations corresponding to the variables in the EH and EHP equations and the absorption chiller are listed in the following.

- \mu^1(sc, s, t) + \eta_{\text{c}} \mu^2(sc, s, t) + \eta_{\text{hc}} \mu^8(sc, s, t) \leq 0 \\
\forall sc, s, t \tag{29}

\frac{\mu^1(sc, s, t)}{\eta_{\text{EHP}}^c} + \mu^2(sc, s, t) + \mu^5(sc, s, t) \leq 0 \\
\forall sc, s, t \tag{30}

\frac{\mu^1(sc, s, t)}{\eta_{\text{EHP}}^c} + \mu^3(sc, s, t) + \mu^6(sc, s, t) \leq 0 \\
\forall sc, s, t \tag{31}

- \mu^2(sc, s, t) + \eta_{\text{hc}} \mu^3(sc, s, t) + \eta_{\text{hc}} \mu^4(sc, s, t) \leq 0 \\
\forall sc, s \tag{32}

3-3-5 - The Sub-Problem Decision Variables

Considering the decision variables in the mathematical model of the sub-problem, the solution space confinement method was used to apply the effect of the binary variables. If the sub-problem has a number of integer variables, the master problem is expressed as follows [60].
\[
\begin{align*}
\min & \quad C^T x + d^T y \\
\text{s.t.} & \quad Ax + By \geq b \quad \forall \ x \in W, \ y \in \mathbb{Z}_+^p
\end{align*}
\]

The master problem and sub-problem functions in Benders decomposition method are introduced below based on the mathematical model (33) [60].

\[
\begin{align*}
\text{MP} : \min & \quad C^T x + \theta \\
\text{s.t.} & \quad Ax + By \geq b \quad \forall x \in W \\
\text{SP} : \min & \quad d^T y \\
\text{s.t.} & \quad By \geq b - Ax
\end{align*}
\]

Since there are integers in the sub-problem solution space, the relaxed sub-problem (RSP) model is defined as follows in Benders decomposition method [60].

\[
\begin{align*}
\text{RSP} : \min & \quad d^T y \\
\text{s.t.} & \quad Ax + By \geq b \quad \forall y \in \mathbb{R}_+^p \\
y_k \geq a_k & \quad \forall k \in B_k^i \\
y_k \leq a_k & \quad \forall k \in B_k^0
\end{align*}
\]

where \(B_k^0\) and \(B_k^i\) represent the set of binary values associated with variable \(u\) at the master problem solution point. In other words, \(a_k = \bar{y}_\text{Root}\). Hence, the set of \(y\) variables is defined as follows:

\[
y \in \{ I_\text{Net,Buy} , I_\text{Net,Sell} , I_{\text{ch}} , I_{\text{ec}} , I_{\text{shap}} , I_{\text{shdo}} , I_{\text{EHP}} , I_{\text{EHP}} \} \quad \forall \ sc, s, t, ec \in \Omega
\]

3-4- The Flowchart of Proposed Algorithm

Fig. (2) depicts the different steps of the proposed algorithm based on Benders’ inner loop. The presented model was simulated in GAMS 27.2 installed on a computer with a core i5 CPU 2.6 GHz and 6 GB RAM. In stage one, an applicable solution to the master problem, which is the asset capacity problem, serves as the starting point. Afterward, the sub-problem binary variables are searched based on the solution space created by the solution to the master problem, and the best sub-problem solution would be obtained with the integer values of the decision variables. After determining the binary sets for the decision variables, the sub-problem dual problem will be solved and the unbound cuts or feasibility cuts are made. If the dual problem is solved optimally, the upper bound of the Benders’ loop is updated. In this stage, if the difference between the lower
bound and the upper bound of the Benders iteration loop is smaller than a certain value, the solving process stops; otherwise, the master problem is solved using the new values of the operation variables. The different steps of implementing the proposed algorithm and the related equations are presented in the following:

- **Stage 1:** Determining the initial capacity of the assets and initializing the planning problem variables.
- **Stage 2:** Solving the relaxed sub-problem considering the solution space constraints, objective function (36) and constraints (37-38), until the applicable solution including the integers is found to the decision variables.
- **Stage 3:** Solving the dual sub-problem and changing the marginal values of the operation constraints using the objective function (7) and constraints (8-32).
- **Stage 4:** Creating the unbound or feasibility cuts considering the solution to the dual problem and updating the upper bound of the Benders loop using the following equation.
  \[
  \text{Upper Bound (UB)} = \text{Min (UB, TC)}
  \]
- **Stage 5:** Solving the master problem by using the marginal values and changing the installed capacity of the assets based on objective function (2) and constraints (3-5).
- **Stage 6:** Updating the lower bound in Benders loop using the following equation and analyzing the difference between the upper bound and the lower bound.
  \[
  \text{Lower Bound (LB)} = \text{Max (LB, TC)}
  \]
- The convergence process stops when the exit condition is met; otherwise, the loop returns to the second stage.
Fig. 2. The flowchart of the proposed algorithm.
4. Simulation Results

The problem was solved in five case studies as shown in Table (1) to study the effect of the uncertainties and the demand response program on the planning and operation of the energy hub and to analyze the capability of the proposed method. In addition, since the operation problem is carried out in four seasons basis, the electrical, heating, and cooling load profiles are depicted in Fig. (3) along with the power generation by the wind farm in each season according to the historical data [28].

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Without Uncertainty</th>
<th>With Uncertainty</th>
<th>Number of Scenarios in Each Season</th>
<th>Demand Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>5–Scenario</td>
<td>10–Scenarios</td>
</tr>
<tr>
<td>1</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>2</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>3-A</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3-B</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3-C</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

The peak of electrical, heating, and cooling loads and the power generation by the wind farm are set to 800 kW, 600 kW, 350 kW, and 200 kW, respectively.

In Fig. (4), the sale and purchase price of electrical energy for the four seasons are illustrated. Energy is sold to the upstream grid at a price that is 10% higher than the purchase price due to incentive the surplus energy injection to the power grid. The other specifications of the operation criteria for different assets are classified
in Table (2). Table (3) also presents the candidate list of the hub appliances with their corresponding techno-economic features.

Table 2. The data of energy hub assets.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{ch} )</td>
<td>0.9</td>
<td>( \eta_{CHP} )</td>
<td>0.4</td>
<td>( LPP_{Supply} )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \eta_{de} )</td>
<td>0.9</td>
<td>( \eta_{CHP} )</td>
<td>0.85</td>
<td>( LPP_{Supply} )</td>
<td>0.1</td>
</tr>
<tr>
<td>( d_{Init} )</td>
<td>0.3</td>
<td>( \eta_{EH} )</td>
<td>0.85</td>
<td>( ELF_{Max} )</td>
<td>0.05</td>
</tr>
<tr>
<td>( d_{max} )</td>
<td>0.3</td>
<td>( \eta_{EH} )</td>
<td>0.85</td>
<td>( \pi_{CO2} )</td>
<td>0.014</td>
</tr>
<tr>
<td>( a_{loss} )</td>
<td>0</td>
<td>( \eta_{gh} )</td>
<td>0.85</td>
<td>( \pi_{NO2} )</td>
<td>0.99</td>
</tr>
<tr>
<td>( a_{min} )</td>
<td>1</td>
<td>( \eta_{AC} )</td>
<td>0.85</td>
<td>( \pi_{CO2} )</td>
<td>4.2</td>
</tr>
<tr>
<td>( a_{max} )</td>
<td>0.9</td>
<td>( \eta_{EH} )</td>
<td>0.85</td>
<td>( ELF_{CC2} )</td>
<td>1.596</td>
</tr>
<tr>
<td>( a_{min,CH} )</td>
<td>0</td>
<td>( \eta_{ee} )</td>
<td>0.9</td>
<td>( ELF_{SO2} )</td>
<td>0.007</td>
</tr>
<tr>
<td>( a_{max,CH} )</td>
<td>0.25</td>
<td>( \eta_{ee} )</td>
<td>0.9</td>
<td>( ELF_{NO2} )</td>
<td>0.44</td>
</tr>
<tr>
<td>( \pi_{s} )</td>
<td>0.03</td>
<td>( v_{ci} )</td>
<td>4</td>
<td>( ELF_{CO2} )</td>
<td>1.755</td>
</tr>
<tr>
<td>( \pi_{h} )</td>
<td>0.03</td>
<td>( v_{r} )</td>
<td>13</td>
<td>( ELF_{SO2} )</td>
<td>1.011</td>
</tr>
<tr>
<td>( \pi_{ENS} )</td>
<td>5 * ( \pi_{Net, Buy(s,t)} )</td>
<td>( v_{co} )</td>
<td>22</td>
<td>( ELF_{NO2} )</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Table 3 Candidate list of the hub assets with the corresponding techno-economic features

<table>
<thead>
<tr>
<th>Asset</th>
<th>Minimum Capacity (kW)</th>
<th>Maximum Capacity (kW)</th>
<th>Capacity Step (kW)</th>
<th>Installation Cost ($/kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trans.</td>
<td>50</td>
<td>100</td>
<td>50</td>
<td>450</td>
</tr>
<tr>
<td>CHP</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>990</td>
</tr>
<tr>
<td>Boiler</td>
<td>50</td>
<td>800</td>
<td>50</td>
<td>450</td>
</tr>
<tr>
<td>EH</td>
<td>30</td>
<td>600</td>
<td>50</td>
<td>400</td>
</tr>
<tr>
<td>EHP</td>
<td>50</td>
<td>600</td>
<td>50</td>
<td>500</td>
</tr>
<tr>
<td>AC</td>
<td>50</td>
<td>500</td>
<td>50</td>
<td>470</td>
</tr>
<tr>
<td>EES</td>
<td>50</td>
<td>600</td>
<td>50</td>
<td>250</td>
</tr>
<tr>
<td>TES</td>
<td>50</td>
<td>600</td>
<td>50</td>
<td>250</td>
</tr>
</tbody>
</table>

Fig. 4. The price of electrical energy purchased from the grid.

As it was mentioned in the problem formulation section, there is a constraint for the capacity addition associated with the candidates and it is supposed to be one unit can be added to the hub. Therefore, the optimum selection and sizing of the assets must be achieved in the optimization problem.
4-1- Case Study 1

In this case, the optimal planning and operation of the energy hub are carried out under the deterministic conditions without the demand response program implementation. The obtained results are presented in Table (4). In this case, the total investment cost is $ 183411.52. Since the maximum electrical load occurs in summer, 800 kW, the portion of the energy demand is met by the wind farm and CHP units and the rest of the demand must be purchased from the distribution network. In addition, the electrical energy storage system is also involved in the supply of the power demand. The absorption chiller capacity equals to the peak of the cooling load during summer. Moreover, the installed capacity of the heating assets including CHP, boiler, EHP, and EH fits the heating load peak during winter. Figure (5) presents the operating points of the hub assets. Fig. 5 (a) and 5 (b) also show the power and heat generations by the CHP unit, respectively. As it is evident in these figures, during the early hours of the day in spring and summer, the CHP unit is turned off. Since there is no need for the heating power from one side and the electricity price is low during the mentioned hours from the other side, the CHP unit can be out of service for least cost operation during these hours. Moreover, the electricity demand of the hub can be supplied by purchases from the power grid and the wind turbine generation.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Trans.</th>
<th>CHP</th>
<th>Boiler</th>
<th>EH</th>
<th>EHP</th>
<th>AC</th>
<th>EES</th>
<th>TES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Installed Capacity (kW)</td>
<td>350</td>
<td>400</td>
<td>200</td>
<td>-</td>
<td>100</td>
<td>350</td>
<td>150</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operation Cost ($/Day)</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
<th>Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1669.57</td>
<td>2249.58</td>
<td>1554.79</td>
<td>1422.74</td>
<td></td>
</tr>
</tbody>
</table>

| Installation Cost ($/Year) | 183411.52 |

Fig. 5 (c) shows that the AC is turned off during the early hours of the day in spring and summer. This is observed that the boiler and CHP are turned off or generate very little amounts of energy as the suppliers of the AC thermal energy demand during these hours. However, the absorption chiller is active at noon due to the availability of additional CHP and boiler heating energy.

Fig. 5 (d) indicates that EHP is active in spring and summer during the early hours and the ending hours of the day in cooling mode when the AC is turned off. Besides, EHP is activated to take part in the supply of the heating load during the early and ending hours of the day in fall and winter in the heating mode, as well. Since EHP only operates in one of the cooling or heating modes at each time, EHP generated heat during fall and winter and contributed to the supply of cooling load during spring and summer.

Also, Fig. 5 (e) indicates that the boiler contributed to the supply of the heating load to the absorption chiller during spring and summer at afternoon, while it contributes more to the supply of the heating load during the early and ending hours of the day in fall and winter. Fig. 5 (f) shows the exchange of energy using the transformer. Since the price is high at noon, the purchase from the grid is at the minimum level and the hub
power demand is mainly met by the CHP and wind turbine. As seen in this figure, the energy hub tends to sell its surplus energy to the grid due to the electrical demand. In this case, the hub owner reduces the costs by purchasing gas and selling the surplus of electrical energy to the grid, marinating it profitable for the hub owner. The State of Charge (SoC) of both EES and TES are depicted in Fig. 5 (g) and 5 (h), respectively. Since the electricity price is at the minimum level during the early hours of the day, the storage system is charged and it is discharged during the peak hours to supply the peak load and reduce the operating costs. Since the electricity price is almost stable during the fall at different hours, the SoC of EES depicts the lowest level of performance during this season. As regards the TES, the variations of the energy stored are highly determined by the heating load level at different hours. Since the natural gas price is unvarying, the thermal energy storage system is used to store the heat surplus of the heating assets and return it to the hub at the desired hours.
4-2- Case Study 2

In this case, the effects of the demand response program on deterministic programming are investigated. The obtained results in this case study are addressed in Table (5). The total investment cost, in this case, decreased approximately by 7% as compared to the first case study, reaching $171755.28. The main reason for the decrease in the investment cost is the installation of a smaller capacity at the Transformer, AC, and electrical energy storage assets. Moreover, due to the changes in the load profile, the partial shift in the power demand to the off-peak hours, and the lower price, the operation cost decreased in different seasons. The load profile before and after the implementation of the demand response program is shown in Fig. (6).

<table>
<thead>
<tr>
<th>Asset</th>
<th>Trans.</th>
<th>CHP</th>
<th>Boiler</th>
<th>EH</th>
<th>EHP</th>
<th>AC</th>
<th>EES</th>
<th>TES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Installed Capacity (kW)</td>
<td>300</td>
<td>400</td>
<td>200</td>
<td>-</td>
<td>100</td>
<td>300</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Season</th>
<th>Operation Cost ($/Day)</th>
<th>1640.42</th>
<th>2220.71</th>
<th>1543.68</th>
<th>1407.57</th>
</tr>
</thead>
<tbody>
<tr>
<td>Installation Cost ($/Year)</td>
<td>171755.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5. The operating points of the hub assets in Case-1; (a) CHP power generation, (b) CHP heat generation, (c) absorption chiller, (d) heat and cooling of EHP, (e) boiler, (f) power transactions, (g) SoC of EES, (h) SoC of TES
In the presence of demand response programs, the total energy consumption during each day does not change, however, it is shifted from peak hours to off-peak hours, and therefore, the operating cost would be reduced, consequently.

The box plot of the dispersion of the load role in the demand response program is depicted in Fig. (7) to determine the effects of the demand response programs at different hours during each season. According to Fig. (7), the average use of the demand response program is significant in all seasons except for fall.

4-3- Case Study 3

In this section, the results of the three states of energy hub optimal planning and operation considering the uncertainties in the electrical, cooling, and heating loads and the wind turbine generation are presented. The generated scenarios of electrical, heating and cooling loads as well as wind speed are shown in Fig. (8). As stated, in this case study, real wind generation information is used to generate scenarios and Monte Carlo simulations are carried out to generate stochastic load scenarios. The simulation results in this case study are listed in Table (6). Since the initial number of scenarios is considerably high, the k-mean method is used for scenario reduction.
According to Table (6), the increase in the number of scenarios increased the computational burden and affected three factors, namely asset capacity, investment cost, and expected operation cost in each season. Table (6) also reveals that in the case with 5-scenario, the few scenarios cannot fully describe the behavior of the problem parameters.
Fig. 8. The generated scenarios of uncertain parameters, Case 3-C

Hence, a cost analysis indicates that with an increase in the number of scenarios from 5 scenarios to 10 scenarios or from 5 to 20 scenarios in each season, the costs decreased despite the heavier computational burden. The total cost in the 20-scenario case decreased approximately by 10% as compared to the 5-scenario. To wit, with an increase in the number of the scenarios, the solution model can provide a better estimate of the load demand. However, the increase in the number of scenarios increases the problem complexity and the computational burden.
Table 6 The optimum results of planning and operation of the energy hub—Case 3

<table>
<thead>
<tr>
<th>Asset</th>
<th>Case 3-A (5 Scenarios)</th>
<th>Case 3-B (10 Scenarios)</th>
<th>Case 3-C (20 Scenarios)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trans.</td>
<td>350</td>
<td>350</td>
<td>300</td>
</tr>
<tr>
<td>CHP</td>
<td>450</td>
<td>450</td>
<td>450</td>
</tr>
<tr>
<td>Boiler</td>
<td>250</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>EH</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>EH P</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>AC</td>
<td>450</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>EES</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>TES</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Installation Cost ($/Year)</th>
<th>Spring</th>
<th>Summer</th>
<th>Fall</th>
<th>Winter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 3-A (5 Scenarios)</td>
<td>209613.17</td>
<td>200447.57</td>
<td>186599.56</td>
<td></td>
</tr>
<tr>
<td>Case 3-B (10 Scenarios)</td>
<td>200447.57</td>
<td>186599.56</td>
<td>186599.56</td>
<td></td>
</tr>
<tr>
<td>Case 3-C (20 Scenarios)</td>
<td>186599.56</td>
<td>186599.56</td>
<td>186599.56</td>
<td></td>
</tr>
</tbody>
</table>

Table 7 Comparative analysis for different case studies

<table>
<thead>
<tr>
<th>Case Study</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3-A</th>
<th>Case 3-B</th>
<th>Case 3-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to Solve (Second)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed Method</td>
<td>10.42</td>
<td>12.48</td>
<td>32.09</td>
<td>76.67</td>
<td>246.92</td>
</tr>
<tr>
<td>DICOPT Solver</td>
<td>149.72</td>
<td>217.3</td>
<td>5481.25</td>
<td>N/C</td>
<td>N/C</td>
</tr>
<tr>
<td>BARON Solver</td>
<td>41.62</td>
<td>47.994</td>
<td>1173.6</td>
<td>N/C</td>
<td>N/C</td>
</tr>
<tr>
<td>Operation Cost ($/Year)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed Method</td>
<td>629322.07</td>
<td>621629.64</td>
<td>731170.17</td>
<td>687163.59</td>
<td>672119.04</td>
</tr>
<tr>
<td>DICOPT Solver</td>
<td>631532.91</td>
<td>630791.31</td>
<td>757718.09</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BARON Solver</td>
<td>631532.91</td>
<td>629041.04</td>
<td>732056.61</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Installation Cost ($/Year)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed Method</td>
<td>183411.52</td>
<td>171755.28</td>
<td>209613.17</td>
<td>200447.57</td>
<td>186599.56</td>
</tr>
<tr>
<td>DICOPT Solver</td>
<td>187695.44</td>
<td>183274.5</td>
<td>221661.23</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BARON Solver</td>
<td>187695.44</td>
<td>176736.58</td>
<td>215686.56</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Total Cost ($/Year)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed Method</td>
<td>812733.59</td>
<td>793384.92</td>
<td>940783.34</td>
<td>887611.16</td>
<td>858718.6</td>
</tr>
<tr>
<td>DICOPT Solver</td>
<td>819228.35</td>
<td>814065.81</td>
<td>979379.32</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BARON Solver</td>
<td>819228.35</td>
<td>805777.62</td>
<td>947743.17</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

N/C: Not Converged due to out of memory problem

Table (7) presents the results obtained from Benders decomposition approach and two well-known nonlinear commercial solvers, namely DICOPT and BARON. The results of the numerical analysis in this paper also indicates that with an increase in the number of scenarios, the DICOPT and BARON solvers did not result in a satisfactory performance. The results listed in Table (7) also indicate that in the case with 5-scenario, the convergence time using the DICOPT and BARON solvers increased significantly. Besides, when the number of scenarios reached 10 and 20, the problem did not converge using these commercial solvers. This issue is not raised when the integrated problem is solved by Benders decomposition approach proposed in this paper. Therefore, with an increase in the number of scenarios, the problem converges and achieves an
optimal solution within an acceptable time. The numerical results of the convergence time analysis and the costs calculated for different scenarios using the DICOPT and BARON methods and Benders decomposition approach are presented in Table (7).

Fig. (9), illustrates the box plot corresponding to the contribution of demand response programs in this case study. According to the results, the peak load demand was reduced during spring, summer, and fall, while the decrease in load during the winter is not considerable. The main reason that in the winter the power demand is not considerable and the electricity tariffs are not high enough to encourage end-users to participate in the demand response programs. This result could be attributed to the tendency of the grid to maintain the sales of electrical power during winter.

![Box plot of power demand reduction by seasons](image)

**Fig. 9.** The contribution of the demand response programs on daily load reductions, Case 3-C

The box plot of the energy exchange during different seasons is shown in Fig. (10) to determine the performance of the energy hub in the exchange of energy with the upstream grid. As stated, the average exchange of energy with the grid was negative at most hours of the day during winter, reflecting the sales of energy to the upstream grid. The results of studying other seasons revealed that the demand response program implementations not only increased the system resistance to the volatilities but also created an income from the transaction with the distribution grid under some scenarios.

![Energy exchange plot](image)

(a)  (b)
The convergence trend of Benders decomposition method is shown in Figure (11). As illustrated in this figure, the highest number of iterations is 17 in the case of 20 scenarios (Case 3-C). Given the increase in the lower bound and the unvarying upper bound, the master problem, i.e. planning stage, first seeks to find the minimum capacity addition problem and then, the slave problem accordingly achieves the best solution for the operational stage.

5. Conclusion

In this paper, the optimal planning and operation of energy hubs were presented. The mathematical problem was decomposed into a master and slave stages according to the Benders decomposition technique. Besides,
the demand response program has been explored taking into consideration of shifting electrical loads. In the overall planning problem, the seasonal demand changes have been considered in line with the price changes for both electrical and natural gas. The mathematical optimization problem has been investigated in a stochastic framework and the uncertainties in electrical, heating and cooling loads, as well as wind power generation, have been investigated. To validate the proposed model, five case studies have been conducted and the impacts of the demand response program and the number of scenarios were demonstrated. The simulation results confirmed that the implementation of demand response program dramatically reduced the overall investment and operation costs due to reducing the size of hub assets, specifically the size of the transformer, electrical energy storages and absorption chiller and hence the total cost was decreased. Also, the simulation results conducted in this paper demonstrated that by increasing the number of scenarios, from 5 to 10 and more, the total cost was decreased, while the computational burden was exponentially increased. Therefore, the Benders decomposition technique has been developed in this paper to overcome the computational burden in the presence of a considerable number of scenarios. The simulation results stated that the Benders decomposition approach can achieve the best results in a reasonable time for optimal hub planning purposes. Further to this work, it is suggested to consider the following items to develop the model; considering seasonal profiles for loads and energy resources, modelling different profiles for regular and weekend days, and taking into account the multi-year dynamic planning horizon typically 10 years or more and addressing the reliability issues, consequently.

References


