Strategic Behavior of Multi-Energy Players in Electricity Markets as Aggregators of Demand Side Resources using a Bi-level Approach

Maziar Yazdani-Damavandi, Member, IEEE, Nilufar Neyestani, Member, IEEE, Miadreza Shafie-khah, Senior Member, IEEE, Javier Contreras, Fellow, IEEE, and João P. S. Catalão, Senior Member, IEEE

Abstract—The coordination of various energy vectors under the concept of multi-energy system (MES) has introduced new sources of operational flexibility to system managers. In this paper, the behavior of multi-energy players (MEP) who can trade with more than one energy carrier to maximize their profits and mitigate their operational risks has been investigated. The MES is represented based on a multi-layer structure, namely the energy market, MEP, the local energy system (LES), and multi-energy demand. In such environment, a MEP aggregates LES and participates in the wholesale electricity market, simultaneously to maximize its profit. The decision-making conflict of the MEP with other energy players for the aggregation of LES and participation in the electricity market is modeled based on a bi-level approach. Numerical results show the behavior of the MEP as a prosumer in the electricity market to produce smoother demand and price profiles. Results reveal a mutual effect of local and wholesale equilibrium prices by increasing the share of the MEP.

Index Terms—Electricity market, mathematical programming with equilibrium constraints (MPEC), multi-energy player (MEP), multi-energy system (MES).

NOMENCLATURE

A. Acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AB</td>
<td>Auxiliary boiler</td>
</tr>
<tr>
<td>CHP</td>
<td>Combined heat and power</td>
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<tr>
<td>ES</td>
<td>Electric storage</td>
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<td>HS</td>
<td>Heat storage</td>
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<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker conditions</td>
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<tr>
<td>LES</td>
<td>Local energy system</td>
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<tr>
<td>MED</td>
<td>Multi-energy demand</td>
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<tr>
<td>MEP</td>
<td>Multi-energy player</td>
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B. Subscripts

<table>
<thead>
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<th>Subscript</th>
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<tbody>
<tr>
<td>e</td>
<td>Electricity</td>
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<td>g</td>
<td>Natural gas</td>
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<td>h</td>
<td>Heat</td>
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<td>i</td>
<td>Index of LES</td>
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<td>j</td>
<td>Index of retailer</td>
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<tr>
<td>k</td>
<td>Index of Genco</td>
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<tr>
<td>t</td>
<td>Time interval</td>
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<tr>
<td>ω</td>
<td>Set of scenarios</td>
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C. Superscripts

<table>
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<tbody>
<tr>
<td>AB</td>
<td>Auxiliary boiler</td>
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<tr>
<td>Agg</td>
<td>Aggregator</td>
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<tr>
<td>Bid</td>
<td>Bidding of electricity producers</td>
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<tr>
<td>CHP</td>
<td>Combined heat and power</td>
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<td>cha</td>
<td>Heat/electric storage charging</td>
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<td>dcha</td>
<td>Heat/electric storage discharging</td>
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<td>Du</td>
<td>Dual problem</td>
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<td>E</td>
<td>Equality constraints</td>
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<tr>
<td>ES</td>
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<tr>
<td>EM</td>
<td>Energy market</td>
</tr>
<tr>
<td>Forecast</td>
<td>Forecasted amount of RER</td>
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<tr>
<td>Genco</td>
<td>Generation company</td>
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<tr>
<td>HS</td>
<td>Heat storage</td>
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<td>in</td>
<td>Input energy to MEP, HS or ES</td>
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<tr>
<td>LES</td>
<td>Local energy system</td>
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<td>MEP</td>
<td>Multi-energy player</td>
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<td>MED</td>
<td>Multi-energy demand</td>
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<td>N</td>
<td>Non-equality constraints</td>
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<td>Offer</td>
<td>Offering of electricity consumers</td>
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<td>out</td>
<td>Output energy from MEP, HS, or ES</td>
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<td>Primal problem</td>
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<td>PV</td>
<td>Photovoltaic array</td>
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<td>Ret.</td>
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<td>Transformer</td>
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<td>Wind</td>
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D. Parameters and Variables

- $e, E$: The amount of stored energy in ES or HS
- $g, G$: Amount of natural gas supply
- $p, P$: Amount of electricity generation
- $q, Q$: Amount of heat production
- $T$: Time period
- $\rho$: Scenario probability
- $\lambda$: Dual variables for equality constraints
- $\mu, \pi$: Dual variables for the lower and upper limits of non-equality constraints
- $\xi$: Dual variables for equality constraints in specific time intervals
- $\gamma$: Charge/discharge rate
- $\eta$: Efficiency
- $\pi, \Pi$: Energy price
- $\kappa$: Shadow price for energy balance equation of electricity market
- $E$: Vector of equality constraints
- $N$: Vector of non-equality constraints
- $T$: Vector of equality constraints in specific time
- $X$: Vector of decision variables for dual problems

I. INTRODUCTION

A. Motivation and Aim

Merging technologies and change in the business paradigm of the energy sector have introduced new challenges and opportunities to energy system managers supplying future energy needs [1]. The development of distributed energy resource (DER) technologies, e.g., energy converters and storage, has increased the dependency of energy carriers. On the other hand, the establishment of new business environments and the participation of more players in the energy system’s decision-making process have increased the dependency on stakeholders’ decision variables.

In order to address these issues, the concept of multi-energy systems (MES) has been introduced. The MES concept addresses the integration of various energy carriers (e.g., electricity, natural gas, district heating, etc.) and their operation from both technical and economical points of view to enable energy and information interaction in different levels [2]. In such system, multi-energy players (MEP) play a crucial role to aggregate demand side energy resources for enhancing the operational flexibility of the system. A MEP is defined as an energy player who can trade with more than one energy carrier to increase its total profit and mitigate its operational risk [3]. MEP can link different energy markets and substitute energy consumption/production between them. As a result, it can act as a flexible source in the market place. Therefore, increasing the share of the MEP in each energy market brings opportunities for both MEP and different market operators and affects their decision-making parameters in the short- and long-term.

The main aim of this paper is to investigate the role of the MEP as a mediator between demand side resources and the wholesale electricity market. In this market the MEP is a strategic prosumer who can modify the market equilibrium price by changing the amount of its energy exchange with the market. Moreover, to have a realistic behavior for the MEP, it is considered as an energy aggregator who interacts with energy carriers with demand side energy resources based on a leader-follower aggregation framework.

B. Literature Review

In the literature, MES are defined as energy systems with more than one energy carrier [4]. A MES is divided into two main parts, namely, operation centers and interconnectors.

Operation centers represent the integration of energy resources (e.g., energy converters and storage) and interconnectors are energy transmitters between operation centers, such as gas pipelines and power lines. Surveys on MES are concentrated on two areas. In the first area, the management of a single operation center is investigated and new models are developed for integrating new energy elements, uncertain resources, and decision-making frameworks in various time domains. In [5] and [6], optimal operation frameworks for residential and industrial energy hubs are designed, respectively. The integration of renewable energy resources (RER), demand response (DR) programs, plug-in electric vehicles (PEV) and storage is considered in [7]–[10], respectively. Moreover, [11] and [12] evaluate the energy hub approach’s proficiency in the long run.

In the second area, a set of operation centers and their corresponding interconnectors are considered in an interactive environment and the developed models are investigated from economic, technical, and environmental aspects. In [13], an optimal energy scheduling and energy interaction for a set of operation centers is proposed. The model is extended in [14] and an evolutionary method is implemented to increase the accuracy of results and the speed of convergence. Furthermore, in [15], a decentralized control model is proposed for a set of energy hubs to coordinate their operation. A game-based approach among energy hubs for DR provision is suggested in [16]. In order to analyze the impact of a high penetration wind resources on interdependent MES, a robust optimization approach is used in [17]. Numerical results determine the role of the power system to mitigate the uncertainty of wind resources by substituting the energy demand of one carrier with the demand of another energy carrier. On the other hand, authors in [18] have shown that it is possible to increase the utilization factor of wind resources in power system operation with MES facilities. In other words, the power system acts as a link between RER and MES that can help decreasing the uncertainty of these resources by using the inherent flexibility of MES.

Besides MES, the microgrid (MG) concept has been developed in the literature to cover some other aspects of the future energy systems. The main feature of a MG is the stand-alone operation capability in contingency modes that can increase the system’s reliability indices. However, this capability needs the deployment of energy resources with more capacity than the average consumption of MG. These resources provide the opportunity to trade the energy surplus in a normal operation mode. Therefore, MG is able to increase the operator’ total profit as well as their own. Likewise, MG can be considered...
as a MES. However, MG is reliability-oriented while a MES is related to the enhancement of the system’s efficiency. Due to the above-mentioned similarity, MG studies have been also surveyed.

Reference [19] proposes a scheduling framework for a single MG that is equipped with a combination of cooling, heating and power (CCHP) units and RER. Different behaviors of CCHP’s energy carriers and the uncertainty of RER are covered by a multi-time scale framework along with the time horizon. Moreover, for a set of MGs, the authors in [20] have used an agent-based framework to model the cooperation environment among MGs. Regarding this topic, an energy retail market is proposed in [21] to fill the gap between the wholesale market and the demand-side players (i.e., MG). In addition, the authors in [22] have developed a multi-leader multi-follower Stackelberg game to manage energy trading among MGs analyzing its equilibrium point. Employing more than one energy carrier has changed the behavior of MEP compared to MG operators. Thus, new models for evaluating the behavior of MEP in future energy systems are required.

Numerous reports have addressed offering strategies and competition models in electricity markets. A large number of the models present the behavior of market players by means of game theory [23], [24]. A stochastic game-theoretic model based on an adaptive Q-learning algorithm is reported in [25]. In [26], a game-theoretic market model is developed based on multi-agent systems to model the behavior of renewable power producers and DR providers. The ability of the demand to cover wind power imbalances is also addressed in the literature [27]. The optimal offering strategy of a hybrid power plant containing a wind power producer and a DR provider is reported in [28]. The model can increase the correlation among the offers and the load curve and, consequently, can decrease the undischargable nature of wind power and reduce the variability of the renewable-based power systems.

Reference [3] proposes an cooperative framework in a MES for modeling the aggregation of a set of static (without intertemporal constraints) facilities. Although many studies have been oriented to model the MES environment, the aggregation of demand side energy resources under the MEP concept to participate in the electricity wholesale market have not been addressed yet. The aggregation of a set of energy carriers introduces more flexibility to a MEP for the participation in the electricity market. Moreover, using interactive models instead of centralized or tariff based models for aggregation of demand side energy resources, can increase the level of operational flexibility and the utilization of local energy resources on the demand side.

C. Contributions and Paper Organization

In this paper, the behavior of a MEP that is an aggregator of a set of demand side energy resources is studied in an electricity wholesale market. Moreover, the impact of increasing the MEP share in electricity wholesale market is investigated. The contributions of this paper are as follows:

- Modeling the strategic behavior of an MEP in an electricity wholesale market within a bi-level decision making problem;
- Considering the MEP as a medium to allow demand-side resources to participate in the market in an aggregated manner for electricity, gas, and heat energy carriers and model their behavior through a bi-level decision making problem;
- Evaluating the impact of a high penetration of MEP on the equilibrium of the electricity wholesale market and the local aggregation of demand-side energy resources and the cross impact of these two sets of equilibrium points.

The rest of the paper is organized as follows. In Section II the problem and its solution are described. The models for MEP and LES decision making problems are presented in Sections II and III, respectively. The electricity market problem is explained in Section IV. A Numerical study and concluding remarks are presented in Sections V and VI, respectively.

II. PROBLEM STATEMENT

In this paper, the MES is considered as a multi-layer structure and consists of four layers, namely, energy market, MEP, local energy system (LES), and multi-energy demand (MED). The multi-layer structure represents the behavior and scale of each energy player in the proposed MES. The role of each energy player in the proposed framework and their interacting energy variables and parameters are shown in Fig. 1. A short description of each layer follows [3]:

- The energy market consists of individual energy carrier markets linked by the MEP.
- The MEP is an energy aggregator who interacts with a set of LES and participates in energy markets.
- LES is a local energy network equipped with demand side energy resources delivering required energy services to MED.
- MED is the lowest level in this multi-layer structure and can be a set of end-users consuming various energy carriers.

As it is shown in Fig. 1 the main variables that couple all the players are the energy carriers’ prices. It is assumed that, in the long-term, the players will revise their strategies based on the energy carriers prices, which are determined based on the strategies of other players.

In order to investigate the impact of the MEP on the energy market performance, a bi-level programming approach is implemented in this paper (Fig. 2). On the upper level, there is a MEP who is able to trade energy (electricity and natural gas) in the energy markets and also with the LES who serve exogenous demands for energy (electricity, natural gas, and heat) in their own areas. The objective of the MEP is to determine the optimal trading quantities in order to maximize its own profit subject to energy balance constraints.

On the lower level, each LES acts as a prosumer that needs to decide the amount of energy of each carriers to be supplied either from the MEP or distributed resources. In addition to the energy balance constraints, each LES faces physical constraints for the operation of installed equipment. In general, in a leader-follower optimization model, the leader enforces
Fig. 1: Interaction of MEP with LES and the wholesale electricity market.

the price and the follower determines the quantity. In such models the equilibrium price equals to the marginal cost of energy production for the LES.

On the lower level, there are Gencos and retailers whose offers and bids are cleared by a welfare-maximizing independent system operator (ISO). The shadow price of the energy balance constraint of the ISO is the market-clearing price in the electricity market. In this paper both lower-level problems (ISO and LES) are formulated, linearly. Therefore, they may be replaced by their Karush–Kuhn–Tucker (KKT) conditions to turn the bi-level model into a mathematical programming with equilibrium constraints (MPEC). This is further simplified into a mixed-integer linear problem (MILP) by using disjunctive constraints and strong duality to resolve nonlinearities in the constraints and objective function of the MPEC. Numerical results show how market-clearing prices are affected by a greater penetration of MEP. However, the overall energy production is more restricted to the local operations' considerations than to the wholesale electricity market price.

III. MULTI-ENERGY PLAYER’S DECISION MAKING PROBLEM

In the proposed framework, MEP and LES are decision makers who decide about their energy interactions. MEP aggregates LES and exchanges energy with the MEP based on the equilibrium price.

The MEP purchases electricity from the electricity market at the market equilibrium price and natural gas from the gas market at a predetermined price. It also exchanges electricity, gas, and heat at the equilibrium price with LES. The objective function of the MEP is shown in (1). The first two terms are the costs of the MEP in the electricity and gas markets, respectively. The remaining terms are related to the incomes of the MEP at the distribution level from trading electricity, gas and heat with LES at the energy equilibrium prices. Therefore, the decision vector of the MEP for aggregation of LES is:

\[
\pi_{MEP} = [\pi_{MEP}^{Agg}, \pi_{MEP}^{LES}, \pi_{MEP}^{MED}]^T.
\]

\[
\max \left\{ f(x) = \sum_t \left[ - \left( p_{MEP,in}^{MED} - p_{MEP,out}^{MED} \right) \kappa_{EM,t} + \sum_i \left( p_{LES,in}^{MED} - p_{LES,out}^{MED} \right) \pi_{Agg,i,t} + \left( q_{LES,in}^{MED} - q_{LES,out}^{MED} \right) \pi_{Agg,h,i,t} \right] \right\}
\]

(1)
The MEP operator should maintain the energy balance for electricity, gas and heat (2)-(4). For the gas energy carrier, the amount of gas input to the MEP, $g_{t}^{\text{MEP}}$, must be less than $\sum g_{t}^{\text{MEP}}$, which is related to the capacity of its interconnectors with the upstream gas network, as shown in (5). $\sum g_{t}^{\text{MEP}}$ determines the capacity of gas pipelines to supply MEP required natural gas during operational period.

\[
(p_{t}^{\text{MEP,in}} - p_{t}^{\text{MEP,out}}) - \sum_{i}(p_{t,i}^{\text{in}} - p_{t,i}^{\text{out}}) = 0 \tag{2}
\]

\[
g_{t}^{\text{MEP}} - \sum_{i}g_{t,i}^{\text{LES}} = 0 \tag{3}
\]

\[
\sum_{i}(q_{t,i}^{\text{in}} - q_{t,i}^{\text{out}}) = 0 \tag{4}
\]

\[
g_{t}^{\text{MEP}} \leq G_{t}^{\text{MEP}} \tag{5}
\]

Equations (6)-(10) determine the expected amount of energy exchange variables for LES. The expected amounts are calculated based on the amount of energy exchange in each scenario and the scenarios’ probability.

\[
p_{t,i}^{\text{LES,in}} = \sum_{\omega}p_{t,i}^{\text{LES,in}} \tag{6}
\]

\[
p_{t,i}^{\text{LES,out}} = \sum_{\omega}p_{t,i}^{\text{LES,out}} \tag{7}
\]

\[
q_{t,i}^{\text{LES,in}} = \sum_{\omega}q_{t,i}^{\text{LES,in}} \tag{8}
\]

\[
q_{t,i}^{\text{LES,out}} = \sum_{\omega}q_{t,i}^{\text{LES,out}} \tag{9}
\]

\[
g_{t,i}^{\text{LES}} = \sum_{\omega}g_{t,i}^{\text{LES}} \tag{10}
\]

IV. LOCAL ENERGY SYSTEMS’ DECISION-MAKING PROBLEM

A. Operational Problem of the Local Energy System

The LES is equipped with a combined heat and power (CHP) unit, auxiliary boiler (AB), heat storage (HS), RER, and electric storage (ES). Each LES trades at equilibrium prices with the MEP and delivers the required services to the MED to maximize its profit (11). The first three terms of the LES objective function determine the incomes from the energy sold (electricity, gas and heat) to MED. The remaining terms are similar to the ones of the MEP, the costs from trading energy with the MEP in the aggregation equilibrium price. These terms are the coupling variables between the MEP and the LES. The decision vector of the LES is:

\[
\begin{pmatrix}
\pi_{t,i}^{\text{LES,in}} \\
\pi_{t,i}^{\text{LES,out}} \\
\pi_{t,i}^{\text{LES}} \\
\end{pmatrix}
\]

max \[g_{t}(x_{t}) = \sum_{t} [p_{t}^{\text{MED}} + \pi_{t}^{\text{MEP}} \frac{\hat{G}_{t}}{\hat{g}_{t}}] + \sum_{t} \left( (p_{t,i}^{\text{LES,in}} - p_{t,i}^{\text{LES,out}}) \lambda_{i,t} - (q_{t,i}^{\text{LES,in}} - q_{t,i}^{\text{LES,out}}) \lambda_{h,i,t} \right) \]

The LES operational constraints are based on [29] and [30] follows:

1) LES energy balance: Equations (12)-(14) determine the energy balance for electricity, gas, and heat, respectively.

\[
E_{t,i,\omega,t}^{\text{LES}}: p_{t,i}^{\text{LES,in}} - p_{t,i}^{\text{LES,out}} + p_{t,i}^{\text{LES,out}} + p_{t,i}^{\text{LES,in}} + \sum_{i}p_{t,i}^{\text{LES,in}} = 0 \tag{12}
\]

\[
E_{t,i,\omega,t}^{\text{LES}}: q_{t,i}^{\text{LES,in}} - q_{t,i}^{\text{LES,out}} = 0 \tag{13}
\]

\[
E_{t,i,\omega,t}^{\text{LES}}: g_{t,i}^{\text{LES}} = 0 \tag{14}
\]

2) Input energy constraints: Input/output energy carriers to/from LES are limited by their interconnectors, capacities.

\[
N_{t,i,\omega,t}^{\text{LES},1,2} : 0 \leq p_{t,i}^{\text{LES,in}} \leq p_{t,i}^{\text{LES}} \tag{15}
\]

\[
N_{t,i,\omega,t}^{\text{LES},3,4} : 0 \leq p_{t,i}^{\text{LES,out}} \leq p_{t,i}^{\text{LES}} \tag{16}
\]

\[
N_{t,i,\omega,t}^{\text{LES},5,6} : 0 \leq q_{t,i}^{\text{LES,in}} \leq q_{t,i}^{\text{LES}} \tag{17}
\]

\[
N_{t,i,\omega,t}^{\text{LES},7,8} : 0 \leq q_{t,i}^{\text{LES,out}} \leq q_{t,i}^{\text{LES}} \tag{18}
\]

\[
N_{t,i,\omega,t}^{\text{LES},9,10} : 0 \leq g_{t,i}^{\text{LES}} \leq g_{t,i}^{\text{LES}} \tag{19}
\]

3) CHP constraints: CHP produces electricity and heat by consuming natural gas (20), (21). CHP output heat and electricity should be within its operational limits (22), (23). It should be noted that, for the CHP unit, ramp rate, MUT, and MDT constraints are neglected.

\[
E_{t,i,\omega,t}^{\text{CHP}} : p_{t,i}^{\text{CHP}} - p_{t,i}^{\text{CHP}} \eta_{t,i}^{\text{CHP}} = 0 \tag{20}
\]

\[
E_{t,i,\omega,t}^{\text{CHP}} : q_{t,i}^{\text{CHP}} - q_{t,i}^{\text{CHP}} \eta_{t,i}^{\text{CHP}} = 0 \tag{21}
\]

\[
N_{t,i,\omega,t}^{\text{CHP},11,12} : 0 \leq p_{t,i}^{\text{CHP}} \leq p_{t,i}^{\text{CHP}} \tag{22}
\]

\[
N_{t,i,\omega,t}^{\text{CHP},13,14} : 0 \leq q_{t,i}^{\text{CHP}} \leq q_{t,i}^{\text{CHP}} \tag{23}
\]
4) **AB Constraints:** AB produces the required heat by consuming natural gas (24). The output heat of AB should be lower than its maximum capacity (25).

\[
E_{i,ω,t}^{LES,6} = q_{i,ω,t}^{AB} - q_{i,ω,t}^{AB,AB} = 0 \quad : \lambda_{i,ω,t}^{AB} (24)
\]

\[
N_{i,ω,t}, N_{i,ω,t}^{LES,15}, N_{i,ω,t}^{LES,16} : 0 \leq q_{i,ω,t}^{AB} \leq T_{i,ω,t}^{AB} \quad \mu_{i,ω,t}^{AB}, \nu_{i,ω,t}^{AB} (25)
\]

5) **HS Constraints:** In (26) it is determined that the heat balance of HS is based on its energy exchange with LES, while (27) and (28) restrict this exchange based on the charge/discharge rates of HS that is correspondent to the average power values in the relevant hourly intervals. The stored energy in HS should be lower than the maximum capacity (29). To preserve the energy conservation law in the time horizon, in (30) and (31) it is assumed that the stored energy in HS is equal in the first and last time intervals, being half of its maximum capacity.

\[
E_{i,ω,t}^{LES,7} = e_{i,ω,t}^{HS} - e_{i,ω,t}^{HS,t-1} + \eta_{h,i,ω,t}^{HS} \quad : \rho_{i,ω,t}^{HS} (26)
\]

\[
N_{i,ω,t}, N_{i,ω,t}^{LES,17}, N_{i,ω,t}^{LES,18} : 0 \leq q_{i,ω,t}^{HS,in} \leq \gamma_{i}^{HS,in} \quad : \mu_{i,ω,t}^{HS,in}, \nu_{i,ω,t}^{HS,in} (27)
\]

\[
N_{i,ω,t}, N_{i,ω,t}^{LES,19}, N_{i,ω,t}^{LES,20} : 0 \leq q_{i,ω,t}^{HS,out} \leq \gamma_{i}^{HS,out} \quad : \mu_{i,ω,t}^{HS,out}, \nu_{i,ω,t}^{HS,out} (28)
\]

\[
N_{i,ω,t}, N_{i,ω,t}^{LES,21}, N_{i,ω,t}^{LES,22} : 0 \leq e_{i,ω,t}^{HS} \leq \bar{E}_{i}^{HS} \quad : \mu_{i,ω,t}^{HS}, \nu_{i,ω,t}^{HS} (29)
\]

\[
T_{i,ω,t}^{LES,1} : e_{i,ω,t}^{HS} = \frac{E_{i}^{HS}}{2} \quad : \xi_{i,ω,t}^{HS} (30)
\]

\[
T_{i,ω,t}^{LES,2} : e_{i,ω,t}^{HS} = \frac{E_{i}^{HS}}{2} \quad : \xi_{i,ω,t}^{HS} (31)
\]

6) **ES Constraints:** ES constraints are modeled similar to HS. Equations (32)-(37) show the corresponding constraints.

\[
E_{i,ω,t}^{LES,8} = e_{i,ω,t}^{ES} - e_{i,ω,t}^{ES,t-1} + p_{i,ω,t}^{ES,in} \quad : \lambda_{i,ω,t}^{ES} (32)
\]

\[
N_{i,ω,t}, N_{i,ω,t}^{LES,23}, N_{i,ω,t}^{LES,24} : 0 \leq p_{i,ω,t}^{ES,in} \leq \gamma_{i}^{ES,in} \quad : \mu_{i,ω,t}^{ES,in}, \nu_{i,ω,t}^{ES,in} (33)
\]

\[
N_{i,ω,t}, N_{i,ω,t}^{LES,25}, N_{i,ω,t}^{LES,26} : 0 \leq p_{i,ω,t}^{ES,out} \leq \gamma_{i}^{ES,out} \quad : \mu_{i,ω,t}^{ES,out}, \nu_{i,ω,t}^{ES,out} (34)
\]

\[
N_{i,ω,t}^{LES,27}, N_{i,ω,t}^{LES,28} : 0 \leq e_{i,ω,t}^{ES} \leq \bar{E}_{i}^{ES} \quad : \mu_{i,ω,t}^{ES}, \nu_{i,ω,t}^{ES} (35)
\]

\[
T_{i,ω,t}^{LES,3} : e_{i,ω,t}^{ES} = \frac{E_{i}^{ES}}{2} \quad : \xi_{i,ω,t}^{ES}, \forall t = 1 (36)
\]

\[
T_{i,ω,t}^{LES,4} : e_{i,ω,t}^{ES} = \frac{E_{i}^{ES}}{2} \quad : \xi_{i,ω,t}^{ES}, \forall t = T (37)
\]

7) **RER Constraints:** For renewable-based LES, RER generation is limited to its forecasted amount in each scenario, (38) and (39). The scenario generation procedure is explained in Appendix C.

\[
N_{i,ω,t}^{LES,29}, N_{i,ω,t}^{LES,30} : 0 \leq p_{i,ω,t}^{PV} \leq \bar{p}_{i,ω,t}^{PV,Forecast} \quad : \mu_{i,ω,t}^{PV}, \nu_{i,ω,t}^{PV} (38)
\]

\[
N_{i,ω,t}^{LES,31}, N_{i,ω,t}^{LES,32} : 0 \leq p_{i,ω,t}^{Wind} \leq \bar{p}_{i,ω,t}^{Wind,Forecast} \quad : \mu_{i,ω,t}^{Wind}, \nu_{i,ω,t}^{Wind} (39)
\]

B. **MPEC Formulation of the LES Decision-Making Problem**

The MEP’s decision making process as the aggregator form of LES resources may result in different outcomes rather than the individually operation of each LES. As a result, in this study, a bi-level problem is considered where, on the lower level the operation problems of LESs are considered and, on the upper level, the MEP interaction with the market is formulated. To transform the bi-level problem (1)-(5) and (11)-(39), into a single-level MILP problem, we use MPEC ([31] and [32]). The proposed procedure is as follows:

- Transforming the lower-level problem into a convex and linear one;
- Replacing the lower-level problem with its KKT optimality conditions;
- Applying the strong duality theorem to linearize the non-linear terms of the upper-level problem.

The formulation in (12)-(39) is convex and linear; therefore, in (40) it is shown that the Lagrangian of the LES problem and (41)-(44) are its KKT optimality conditions. Equation (41) represents a set of stationary conditions for the LES that represents the first order derivatives of the LES Lagrangian with respect to its primal decision variables. Equations (42) and (43) are the primal optimality conditions for the LES equality conditions in normal and predetermined time intervals, respectively. These conditions are determined according to the first order derivatives of the LES Lagrangian with respect to its dual decision variables. The equation (44) shows the complementarity conditions for the LES lower-level problem. The linearized form of (44) and the upper-level objective function are explained in Appendixes A and B, respectively.
V. Mathematical Formulation of The Electricity Market

The electricity market consists of gencos, retailers, and the MEP, operated by the ISO. The ISO receives the energy producers’ offers and energy consumers’ bids and clears the market in each time interval, obtaining the energy price.

A. Modeling the MEP Strategic Behavior

The MEP is a strategic player that competes with other players in an electricity market environment. This behavior is modeled using bi-level optimization, where the MEP resolves its strategy on the upper level and the impact of its decision on electricity market parameters is determined on the lower level. In the lower level, the ISO receives the market players’ bids/offers and clears the market to maximize social welfare (45). The first two terms of this equation are the offers and bids of the MEP as a simultaneous electricity producer and consumer. The next two terms are the other electricity market players’ strategies that consist of the Gencos’ offers and the retailers’ bids. The decision vector of the MEP is: \([\pi^\text{MEP,Bid}_{t}, \pi^\text{MEP,Offer}_{t}]\) and the decision vector of the lower-level problem is: \([p^\text{Ret}_{t,j}, p^\text{Genco}_{t,k}, p^\text{MEP,out}_{t}, p^\text{MEP,in}_{t}]\).

\[
\max \left\{ h(x) = \sum_{t} \left[ p^\text{MEP,in}_{t} \pi^\text{MEP,Bid}_{t} - p^\text{MEP,out}_{t} \pi^\text{MEP,Offer}_{t} \right. \right.
\left. + \sum_{j} p^\text{Ret}_{t,j} \Pi^\text{Ret}_{j,t} - \sum_{k} p^\text{Genco}_{t,k} \Pi^\text{Genco}_{k,t} \right]\} \tag{45}
\]

The power balance of the electricity market is shown in (46). The dual variable of this equation is the market clearing price. In addition, (47)-(50) show the upper limits of generation/demand, which are equal to the offers/bids.

\[
\sum_{k} p^\text{Genco}_{t,k} - \sum_{j} p^\text{Retail}_{t,j} \Pi^\text{Retail}_{j,t} - p^\text{MEP,Offer}_{t} = 0 : \kappa^\text{EM}_{t} \tag{46}
\]

VI. Numerical Results

In the numerical results the behavior of the MEP to interact with the LES and its participation in wholesale electricity market is investigated. The model has been solved by CPLEX 10 on HP Z800 workstation with CPU: 3.47 GHz and RAM: 96 GB.
A. Input Data Characterization

The MEP aggregates three LES and competes with 10 Gencos and 10 retailers in the electricity market. Table I shows the input data for the LES. Table II contains the bids and offers of the electricity market players. For all the retailers, the offering steps are considered the same as in the base case, where $\alpha_t$ is a correction factor to create the bidding steps (55) changing the amount of the retailers’ bidding in each hour.

$$p_{j,t}^{\text{Ret.}} = \alpha_t (p_{j,t}^{\text{Ret. base}})$$  \hspace{1cm} (55)

The gas market price is $25/MWh. To avoid price spikes in the local energy market, the price caps for electricity, heat and gas are $130/MWh, $150/MWh, and $40/MWh, respectively. In addition, Fig. 3 depicts the total energy consumption of the MED.

B. Equilibrium Price for the Aggregation of LES

Fig. 4 shows the energy carrier prices for the LES and electricity market clearing prices. As shown, due to the small energy exchange of the MEP, it is a price taker in the electricity market and the market price is solely determined based on Gencos’ and retailers’ offers and bids, respectively. Fig. 5 and 6 show the power and heat balance of the MEP, respectively. The MEP trades with three types of energy carriers whose behaviors are as follows:

1) Natural gas: Natural gas is a grid-bounded carrier that cannot be produced locally and the MEP delivers the required amount to LES. Therefore, its price always is equal to the price cap and the MEP maximizes its profit by maximizing the gas price.

2) Heat: Heat is a local energy carrier and is produced only by AB and CHP units. Therefore, its price depends on local operational considerations. In hours 2-13, while the heat production of CHP units does not satisfy MED’s needs and the LES use their AB (CHP units are in heat-lead mode), the heat price is equal to the marginal cost of the AB. On the other
hand, after hour 13, while the price of electricity is high and the CHP units are in electricity-lead mode, the price of heat is almost equal to zero and heat is produced as a supplementary good when generating electricity in the CHP units. As a matter of fact, producing heat is like a bonus for LES helping them to operate their CHP units within their operational limits.

3) Electricity: Electricity can be generated locally or delivered by the MEP. The electricity price has the same behavior as the electricity market price. Note that the aggregator’s equilibrium price of electricity depends on the capability of CHP units to produce cheaper electricity. In general, the electricity price of the CHP units is high but when the LES have large simultaneous heat and electricity demands, their generation will be profitable. However, as these units increase their level of electricity generation, their vacant capacities to compete in the local market decreases. Therefore, the MEP increases the electricity price to maximize its profit. The profit of the MEP depends on two factors: energy quantity and energy price. In the first period (hours 1-13), the MEP increases its profit by decreasing the local electricity price, forcing the CHP units to decrease their generation to increase its own energy delivery share. On the contrary, in the second period (14-24), when the marginal cost of the CHP unit is low, it prefers to increase the electricity price up to the price cap, maximizing its profit by selling electricity to the remaining MED at the highest possible price.

C. Impact of a High Penetration of MEP

Fig. 7 depicts the impact of a high penetration of the MEP on the electricity market. In this paper, the penetration rate of the MEP is defined as the share of the MEP electricity demand with respect to the total demand of the system.

As shown in Fig. 7, by increasing the share of MEP, electricity prices increase in most periods (in particular during the periods 13-19). However, in hour 11, with a penetration of more than 35%, electricity prices decrease. In this hour, the MEP injects its electricity surplus to the grid. Figs. 8 and 9 depict the electricity and heat equilibrium prices for the aggregation of LES for various penetration rates of MEP, respectively. By increasing the electricity market price, the equilibrium electricity price increases and motivates LES to use their internal resources (CHP units and ES) to locally generate electricity. Therefore, the price of heat as a supplementary production in the CHP process decreases in the corresponding hours.

Note that, in general, the MEP’s strategy assures the adequacy of generation by using local energy resources, however, it should be noted that these resources are affected by the local operational constraints and their operation is correlated to their local management. For instance, the electricity production of CHP units and its marginal cost is related to the heat consumption of the MED. However, in case of a contingency, these local resources can protect the system and increase the reliability indices; in a normal operation the local constraints determine their capability to act as rivals of the other market players. Therefore, in comparison with bulk generation, these resources are not beneficial at all times.

Moreover, their marginal costs are not only related to their levels of production but also dependent to their local operational considerations, and change along the time horizon. In the case studied, the lowest marginal cost for CHP production is during hours 11-13, while the MEP has the maximum heat consumption, but the system peak occurs between hours 14-17.

D. General Behavioral Outcomes

The main outcomes of this research regarding the behavior of market players as follows:

1) The main goal to introduce a new player that we call MEP is to release the hidden synergy that is possible between energy carriers. The MEP as an energy aggregator can exchange energy with the LES through different energy carriers. Therefore, as shown in Figs. 5 and 6, in hour 13, the MEP buys electricity at a high price from the LES and sells it to the wholesale market at a low price. For a conventional energy player (as a broker) this energy exchange is not profitable but, for the MEP the situation is different. The MEP trades electricity and gas with the LES, simultaneously. Therefore, in hour 13, when it buys electricity at a high price from the LES rather than from the wholesale market, it sells the required gas for the CHP units at the same time. This means that the net cost of the generated electricity is lower than in the wholesale market. As a matter of fact, there is a hidden flexibility
between gas and electricity due to the use of CHP units in the LES, but conventional players who trade just a certain type of energy carrier cannot release this flexibility. For the MEP who can trade with these two types of energies at the same time this is a good opportunity to employ this flexibility and maximize profit by selling gas to the LES and electricity to the wholesale market at the same time.

2) The MEP participation in the electricity market is modeled as a strategic player who modifies the market price based on its energy interaction amount with the market. In this case, the MEP is a price-maker player using its market power to maximize its profit by changing the price signal, which is related to its energy interaction. As shown in Fig. 7, with a low penetration rate, the MEP is a price-taker player and has no influence on the market price but, after increasing its penetration rate, it can change the energy price during some hours. It should be noted that, due to the role of MEP as an aggregator of LES, the capability of the MEP to exchange energy with the wholesale energy market is completely related to the operational considerations of the LES. Therefore, contrary to conventional power plants, the price-maker capability of the MEP is not as the same at all time intervals.

3) Due to the operational condition of the MEP, it behaves as a marginal player in the wholesale market. Therefore, even a small change in the equilibrium price can change the role of the MEP from an energy producer to an energy consumer and vice versa. As a marginal player, the MEP may have the highest influence on the equilibrium price and its capability is related to its flexibility to use local resources.

4) The market power of the MEP to aggregate LES and participate in the wholesale market is directly related to the local operational considerations of energy resources. For instance, the LES is equipped with CHP units. The proficiency of these units to generate low-price electricity is related to the LES heat consumption and the capacity of the HS to store the excess of heat production. Therefore, if the LES can produce cheap electricity during upstream network peak hours, this means that the MEP is capable to inject local surplus electricity to the wholesale market and change the market price, otherwise it is a price-taker player during the peak hours. Therefore, it is possible that the MEP behaves as an electricity consumer during peak hours increasing the energy price. On the contrary, a suitable local operational condition may lead to a change in the role of the MEP from consumer to energy producer decreasing the energy price during peak hours. In other words, a high electricity price during peak hours is not the only issue that the MEP should consider to produce energy locally. In general, the price of local resource generation units is higher than the bulk generation price in the upstream network but, if the local operators use suitable solutions (e.g. co-generation) its price can compete with bulk resources.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, the behavior of an MEP is investigated for a simultaneous behavior to aggregate a set of LES and participate in the wholesale electricity markets. The impacts of a high penetration of MEP on these two sets of equilibrium prices were studied. Numerical results show that local energy price equilibrium is related to the local energy resources of the LES. Due to the mutual dependency of the energy carriers, LES may have variable marginal costs for the energy production in the operation period. This time-based marginal cost affects local market parameters, especially when the penetration rates of the MEP increase. The MEP also increases the total efficiency of the system in the sense that it assures the adequate use of generation by local resources. Note that the energy produced by the MEP is more related to local operational considerations, rather than the electricity market price.

To model the more realistically energy players and to show the impact of gas price changes on MEP behavior, future research will study the strategic behavior of the MEP in the gas market besides the electricity market. Moreover, the other energy players will be considered as strategic players in an oligopolistic electricity market. The model will be extended in the MEP level to consider the competition among a set of MEPs to maximize their profits while exchanging energy with the same existing LES.
APPENDIX A
COMPLEMENTARITY CONDITIONS LINEARIZATION

The non-linear parts of the mathematical model are as follows:
- Complementarity conditions at the LES level;
- Complementarity conditions at electricity market level;
- Non-linear terms of the MEP objective function:\n  \( p_{t}^{LES,in} \frac{\pi_{Agg}}{p_{i,t}^{LES,in}} + p_{t}^{LES,out} \frac{\pi_{Agg}}{p_{i,t}^{LES,out}} + g_{i,t}^{LES,in} Q_{h,i,t}^{LES,in} + g_{i,t}^{LES,out} Q_{h,i,t}^{LES,out} \), and \( p_{t}^{MEP,in} \frac{\pi_{EM}}{p_{i,t}^{MEP,in}} \) and \( p_{t}^{MEP,out} \frac{\pi_{EM}}{p_{i,t}^{MEP,out}} \).

Equations (56) and (57) show the linear form of (44). Vector \( u_{i,t}^{LES} \) is a set of auxiliary binary variables and \( M \) is a large enough constant to relax the equations in the linearization process.

\[
0 \leq \mu_{t}^{LES} \leq u_{i,t}^{LES} M^{LES,Pr} \tag{56}
\]

\[
0 \leq N_{t}^{LES}(X_{i}) \leq (1 - u_{i,t}^{LES}) M^{LES,Du} \tag{57}
\]

Equations (58) and (59) show the same linearization method for the complementarity constraints of the electricity market.

\[
0 \leq \mu^{EM} \leq u^{EM} M^{EM,Pr} \tag{58}
\]

\[
0 \leq N^{EM}(X) \leq (1 - u^{EM}) M^{EM,Du} \tag{59}
\]

APPENDIX B
OBJECTIVE FUNCTION LINEARIZATION

In order to linearize the objective function, strong duality theory is applied. The strong duality condition states that the gap between the primal and dual optimal values is approximately zero at optimality and the primal and dual objective functions can be equal:

\[
\begin{align*}
\pi_{Agg}^{LES,in} \frac{\pi_{Agg}}{p_{i,t}^{LES,in}} + \pi_{Agg}^{LES,out} \frac{\pi_{Agg}}{p_{i,t}^{LES,out}} + g_{i,t}^{LES,in} Q_{h,i,t}^{LES,in} + g_{i,t}^{LES,out} Q_{h,i,t}^{LES,out} & = p_{t}^{LES,in} \frac{\pi_{Agg}}{p_{i,t}^{LES,in}} + \pi_{Agg}^{LES,out} \frac{\pi_{Agg}}{p_{i,t}^{LES,out}} + g_{i,t}^{LES,in} Q_{h,i,t}^{LES,in} + g_{i,t}^{LES,out} Q_{h,i,t}^{LES,out} \\
& + p_{t}^{MEP,in} \frac{\pi_{EM}}{p_{i,t}^{MEP,in}} + p_{t}^{MEP,out} \frac{\pi_{EM}}{p_{i,t}^{MEP,out}}
\end{align*}
\]

based on the dual formulation of the lower-level optimization problem, equation (63) illustrates the strong duality condition for the lower-level problem.

\[
c_{i}^{T} X_{i} = b_{i}^{T} \Lambda_{i} \tag{63}
\]

The detailed formulation of equation (63), that determines the strong duality condition for the LES-level problem, is represented in (64).

\[
\begin{align*}
\pi_{Agg}^{LES,in} \frac{\pi_{Agg}}{p_{i,t}^{LES,in}} + \pi_{Agg}^{LES,out} \frac{\pi_{Agg}}{p_{i,t}^{LES,out}} & = \pi_{Agg}^{LES,in} \frac{\pi_{Agg}}{p_{i,t}^{LES,in}} + \pi_{Agg}^{LES,out} \frac{\pi_{Agg}}{p_{i,t}^{LES,out}} \\
& + p_{t}^{MEP,in} \frac{\pi_{EM}}{p_{i,t}^{MEP,in}} + p_{t}^{MEP,out} \frac{\pi_{EM}}{p_{i,t}^{MEP,out}}
\end{align*}
\]

Based on the dual formulation of the lower-level optimization problem, equation (63) illustrates the strong duality condition for the lower-level problem.
By substituting these relations using (65), \( \partial L_{MEP,in} / \partial p_t = \nabla_{MEP,in} = \nabla_{MEP,Bid} = \nabla_{MEP,Offer} \) and \( p_t \) are transformed to (72) and the linear form of \( p_t \) as shown in (73).

\[
\begin{align*}
\frac{\partial L_{EM}}{\partial p_t} &= -\nabla_{MEP,Bid} + \kappa_{EM} \\
\frac{\partial L_{EM}}{\partial p_t} &= -\nabla_{MEP,in} + \nabla_{MEP,Offer} = 0 \\
\frac{\partial L_{EM}}{\partial p_t} &= \kappa_{EM} = \kappa_{EM} \\
\frac{\partial L_{EM}}{\partial p_t} &= \kappa_{EM} = \kappa_{EM} \\
\frac{\partial L_{EM}}{\partial p_t} &= \kappa_{EM} = \kappa_{EM} \quad \text{for each scenario.}
\end{align*}
\]

**APPENDIX C**

**MODELING OF THE RENEWABLE ENERGY RESOURCE UNCERTAINTY**

The uncertainty in the inputs from renewable energy sources (wind generation and PV arrays) is modeled by generating appropriate scenarios. In this paper, power generation is modeled according to the hourly historical data of the site under study (i.e., Swift Current [33]), and precise features of the generation units. In order to characterize the random behavior of the renewable energy resources, a typical day with 24-h time periods is considered. The data related to the same hours of the day are utilized to obtain the probability distribution functions (PDFs) corresponding to each time period. Wind speed distributions are often characterized by Weibull distributions [34]. The PDF of the wind speed is represented by (74), where \( c > 0 \) and \( k > 0 \) are the scale factor and the shape factor, respectively.

\[
f_v(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp \left[ -\left( \frac{v}{c} \right)^k \right]
\]

(74)

The probability distribution function is divided into \( N_s \) scenarios, and the probability of each step can be calculated as follows:

\[
prob_{\omega} = \int_{V_{S_{\omega}}} f_v(v) \, dv, \quad \omega = 1, 2, \ldots, N_s
\]

(75)

where \( V_{S_{\omega}} \) is the wind speed of the \( \omega^{th} \) scenario. The power generated, \( P_{GW}(\omega) \), corresponding to a specific wind speed, \( V_{S_{\omega}} \), can be obtained from (75) in which \( A, B, \) and \( C \) are constants calculated according with [34].

\[
P_{GW}(\omega) = \begin{cases} 
0 & 0 \leq V_{S_{\omega}} \leq V_c \text{ or } V_{S_{\omega}} \geq V_0 \\
P_r(A + B \times V_{S_{\omega}} + C \times V_{S_{\omega}}^2) & V_c \leq V_{S_{\omega}} \leq V_r \\
P_r & V_r \leq V_{S_{\omega}} \leq V_0
\end{cases}
\]

(76)

In (76), \( V_c, V_0, \) and \( V_r \) represent the cut-in, cut-out, and rated speeds, respectively. The hourly solar irradiance data for the site under study have been used to generate a Beta PDF [35] for each time period. Therefore, the PDF of the solar irradiance can be calculated as:

\[
f_b(s) = \begin{cases} 
\frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \cdot \Gamma(\beta)} \cdot s^{(\alpha-1)} \cdot (1-s)^{(\beta-1)} & 0 \leq s \leq 1; \alpha, \beta \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

(77)

where \( f_b(\cdot) \) denotes the Beta distribution function and \( \alpha \) and \( \beta \) are the parameters of the Beta function and for each time period, can be determined using historical data.

In the same way, the Beta PDFs are split into several segments which the occurrence probability of each segment during any specific hour can be expressed as follows:
Calculating the output power of the units

\[
\text{prob}_i^s = \int_{S_i}^{S_{i+1}} f_s(s) \, ds \tag{78}
\]

where \(S_i\) and \(S_{i+1}\) indicate the starting and ending points of the interval \(i\), respectively and \(\text{prob}_i^s\) denotes the probability occurrence of interval \(i\).

The uncertainty of the whole system is characterized by the following steps:

- **Generating related PDFs**

  Firstly, the PDFs for solar irradiance and wind speed are obtained using historical data (24 PDFs related to 24-h of a typical day). These continuous PDFs are sliced into several segments for each time period.

- **Developing scenarios with their own probability**

  Next, different realizations of the random variables, i.e., solar irradiance and wind speed are generated using the roulette wheel mechanism (RWM) [36] and Monte Carlo simulation (MCS) [37], separately. In this case, the roulette wheel mechanism (RWM) [36] and Monte Carlo simulation (MCS) [37], separately. In this case, \(N_s\), and \(N_{ws}\) scenarios are generated for solar irradiance and wind speed, respectively. For example, for solar irradiance, each scenario contains 24 values of solar irradiance related to 24-h time period of the typical day. It should be noted that each scenario has its own probability of occurrence.

- **Calculating the output power of the units**

  Then based on the characteristics of generation units, the wind speed and solar irradiance of each state is transformed into the output power of wind and PV-based unit through equations (79) and (80), respectively.

\[
P_{y,t}^{w}(v_{y,t}) = \begin{cases} 0 & \text{if } v_{y,t}^c \leq v_{\text{in}} \text{ or } v_{y,t} \geq v_{\text{out}} \\ P_r^w & v_{y,t} - v_{\text{in}} < v_{\text{in}} < v_{\text{in}} < v_{y,t} \leq v_r \\ P_r^w & \text{otherwise} \end{cases} \tag{79}
\]

where \(v_{\text{in}}, v_{\text{out}}, v_r\) and \(P_r^w\) represent cut-in, cut-out, rated speeds, and rated power of WT, respectively. \(P_{y,t}^{w}\) denotes the output power of WT associated with wind speed \(v_{y,t}\) at time period \(t\) and state \(y\).

\[
P_{y,t}^{s}(s_{y,t}) = N \times \text{FF} \times V_{y,t} \times I_{y,t} \\
\text{FF} = \frac{V_{MPP} \times I_{MPP}}{V_{oc} \times I_{sc}} \\
V_{y,t} = V_{oc} - K_V \times T_c^y \\
I_{y,t} = s_{y,t} \left[ I_{sc} + K_i \times (T_c^y - 25) \right] \\
T_c^y = \frac{N_{OT} - 20}{0.8}
\tag{80}
\]

where \(T_{c}^{y,t}\) is the cell temperature (\(^{o}C\)); \(T_A\) is the ambient temperature (\(^{o}C\)); \(K_V\) and \(K_i\) are voltage and current temperature coefficients (\(V/^{o}C\) and \(A/^{o}C\)), respectively; \(N_{OT}\) denotes nominal operating temperature of the cell in \(^{o}C\); \(\text{FF}\) is fill factor; \(I_{sc}\) and \(V_{oc}\) indicate short circuit current and open circuit voltage (in A and V), respectively; \(I_{MPP}\) and \(V_{MPP}\) are, respectively, the current and voltage at maximum power points, (A and V); \(P_{y,t}^{s}\) is the output power of the PV module; \(s_{y,t}\) solar irradiance; \(t\) and \(y\) are the indices of time periods and states.

- **Reducing the number of scenarios**

  A large number of scenarios may contribute to a more accurate model of the random variables. Nevertheless, this would increase the computational burden of the problem. Finally, a fast-forward scenario reduction method based on Kontorwisch distance [38] is used to reduce the number of scenarios, while providing a reasonable approximation of the random variables of the system.

The final scenarios used for PV and wind generation and the expected values are depicted in Fig. 10.

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