Optimal hydro scheduling and offering strategies considering price uncertainty and risk management

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Abstract

Hydro energy represents a priority in the energy policy of Portugal, with the aim of decreasing the dependence on fossil fuels. In this context, optimal hydro scheduling acquires added significance in moving towards a sustainable environment. A mixed-integer nonlinear programming approach is considered to enable optimal hydro scheduling for the short-term time horizon, including the effect of head on power production, start-up costs related to the units, multiple regions of operation, and constraints on discharge variation. As new contributions to the field, market uncertainty is introduced in the model via price scenarios and risk management is included using Conditional Value-at-Risk to limit profit volatility. Moreover, plant scheduling and pool offering by the hydro power producer are simultaneously considered to solve a realistic cascaded hydro system.

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Keywords: Hydro scheduling; Pricing; Uncertainty; Risk

1. Introduction

The share of renewable energies in power production has been augmented significantly in countries all over the world [1,2], especially in Europe, namely Denmark [3–5], Ireland [6], Spain [7], and Portugal [8], with the aim of building a low carbon society [9].

The Portuguese targets for renewable energies are rather impressive: the goal is for renewables to contribute 60% of total power production by 2020. The renewable technology with the greatest share in electricity generation in Portugal today stems from hydropower. The total installed capacity in 2009 attained 16738 MW, of which 4578 MW (27%) corresponded to hydro plants. Hydro energy is cost-competitive and can be employed as a storage system to diminish the effect of the stochasticity of wind power. Besides, an advantage of a hydro plant is that it can be brought into operation very quickly. This makes such plants suitable for peak load operation [10].
Hence, hydro energy represents a pivotal priority for the near future: forecasted investments will allow Portugal to reach an installed capacity of 8600 MW by 2020, dramatically increasing the country's hydropower potential [11]. Thus, the optimal management of hydro energy systems is important, as is the case in Norway for example [12].

Several hydro systems in cascaded configuration including reservoirs of relatively small size are typical in Portugal, as is the case in the Douro River, which accounts for approximately two thirds of the nation’s hydro production. The hydro plants in which the storage capacity is considered small are identified as being of run-of-river type, characterized by an efficiency of operation that is highly dependent on the head [13]. Hence, it is essential to include the effect of head dependency on short-term hydro scheduling (STHS).

The final goal of a hydro power producer operating in a market environment is to maximize profits [14]. The water in the reservoirs should be handled in an optimal way by using a self-schedule, which is essential for the survival of a hydro power producer in the competitive framework.

One of the first methods applied to solve the STHS problem was dynamic programming (DP) [15]. DP can deal with the nonlinear features of the hydro model. However, it is impractical to apply DP directly to cascaded hydro systems because of the problem of dimensionality.

Techniques based on artificial intelligence have also been tested to solve the STHS problem [16,17]. Still, the computational requirements increase significantly for hydro systems in cascaded configuration, and the heuristics implemented may lead to sub-optimal solutions.

A classical way to model the STHS problem is to consider a network flow model, taking advantage of the structure implicit in the cascaded configuration [18]. This model is frequently simplified as a piecewise linear model, allowing direct application of a linear programming (LP) approach and commercially available software. Another frequently employed approach to solving the STHS problem is based on mixed-integer linear programming (MILP) [19–21], using binary variables to model the on–off behavior of the plants.

The simplification required by LP – linearizing hydropower generation and discarding head dependence – is simply not acceptable for the run-of-river hydro plants. In what concerns MILP, a significant increase in the computational burden has been reported in solving the STHS problem, associated with the discretization of the nonlinear relation between power production, the discharge of water, and the head effect.
Besides, linearization methods based on successive iterations depend on operator know-how in the tuning of the parameters. For example, the way the under-relaxation factor is selected in [22] is empiric and depends on the specific case study considered, posing some ambiguities.

A nonlinear model possesses some advantages relative to a linear model, being able to express the hydro production features more precisely, incorporating head dependency into the STHS problem. A 4% profit increase, on average, has been reported [23] for a nonlinear model over a linear one while maintaining an acceptable CPU time. The nonlinear model is unable to avoid discharging at forbidden zones of operation, so other approaches based on mixed-integer nonlinear programming (MINLP) have been recently presented [8,24] to solve the STHS problem. Still, the problem was considered to be deterministic, ignoring uncertainties, which may not be a realistic assumption nowadays.

Day-ahead energy market prices are quite volatile, hard to predict, and subject to data uncertainty caused by unanticipated market conditions. Price volatility throughout the day can have a remarkable influence on the profits of the hydro power producer [22]. Since water inflows for the next 24 hours can be forecasted with rather good precision in many systems [25], uncertainty is restricted to energy market prices. Moreover, most power producers are averse to risk [21]. So, in order to manage risk along with generation scheduling and to achieve a distribution of profit among scenarios with enhanced uniformity, an appropriate risk measure should be taken into account.

A structured approach to risk management encourages decision-makers to examine their business processes in order to identify the various risks that can affect them. Therefore, it has become a core interest for generation companies to develop optimal bidding strategies to maximize the profits and minimize the risk in a competitive market [26].

There are some challenging issues about the development of offering strategies with risk management. For example, an earlier model to build generation bids neglects the nonlinear head effect [25]. In [27] an integrated bidding and scheduling algorithm with risk management is presented for a hydrothermal power system using Lagrangian relaxation and stochastic dynamic programming. The bidding risks are managed using the mean-variance model, where the objective function has a risk penalty term related to the price variances.
In [28], a mixed-integer quadratic programming approach is used to model the risk of a self-scheduling problem by taking into account the variance of market-clearing prices. In [29], a stochastic mid-term risk-constrained hydrothermal scheduling algorithm is presented considering the financial risks associated with uncertainties by applying expected downside risks.

A step forward in the previous study is described in [30], where the aim is to compare the downside risk with the absolute deviation risk in order to solve the stochastic price-based unit commitment problem. The difference between these two risk measures is that the downside risk measures the profit shortfall whereas the absolute deviation risk measures the deviation from the payoff target.

In [31] the Value-at-Risk (VaR) approach has been applied to risk assessment in electricity markets. However, VaR suffers from being unstable and difficult to optimize except when losses assume a normal distribution [32]. In contrast, Conditional Value-at-Risk (CVaR) can be applied using linear programming, which is a computationally robust method to deal with large-scale calculations that could be out of reach otherwise.

To conclude the literature review, note that earlier MINLP models assumed that the problem was deterministic, thus ignoring uncertainties. Also, other models used to construct hydro generation bids neglected the nonlinear head effect.

As new contributions to earlier MINLP models [8,24], market uncertainty is introduced in the model via price scenarios and risk management is included using CVaR to limit profit volatility. CVaR is incorporated in the objective function to deter extremely unfavorable situations. Our approach also includes the efficient frontier curve, providing the tradeoff of maximum expected profit versus minimum risk. Moreover, plant scheduling and pool offering by the hydro power producer are simultaneously considered to solve a realistic cascaded hydro system. Hence, the optimal offers are now presented for several levels of risks, which is important to define the bid strategies in the day-ahead market.

The structure of the paper is presented next. Section 2 provides the model and formulation of the STHS problem. Section 3 presents the approach based on MINLP for solving the STHS problem. Section 4 provides the results of applying the proposed approach to a Portuguese hydro system in cascaded configuration. Section 5 provides the analysis of errors. Finally, Section 6 delineates some conclusions.
<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$I, i$</td>
<td>set and index of reservoirs</td>
</tr>
<tr>
<td>$K, k$</td>
<td>set and index of periods</td>
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<tr>
<td>$N$</td>
<td>total number of scenarios</td>
</tr>
<tr>
<td>$B_n$</td>
<td>benefit in scenario $n$</td>
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<tr>
<td>$\zeta$</td>
<td>value-at-risk</td>
</tr>
<tr>
<td>$\delta$</td>
<td>per unit confidence level</td>
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<tr>
<td>$\rho_n$</td>
<td>probability of occurrence of scenario $n$</td>
</tr>
<tr>
<td>$\eta_n$</td>
<td>auxiliary variable employed to calculate CVaR</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>positive weighting factor to attain a suitable tradeoff between profit and risk</td>
</tr>
<tr>
<td>$\lambda_{kn}$</td>
<td>energy price for scenario $n$ at period $k$</td>
</tr>
<tr>
<td>$SU_i$</td>
<td>start-up cost of plant $i$</td>
</tr>
<tr>
<td>$y_{ik}$</td>
<td>binary variable assuming a value of 1 if plant $i$ is starting up at period $k$</td>
</tr>
<tr>
<td>$z_{ik}$</td>
<td>binary variable assuming a value of 1 if plant $i$ is shutting down at period $k$</td>
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<tr>
<td>$p_{ik}$</td>
<td>power generated by plant $i$ during period $k$</td>
</tr>
<tr>
<td>$v_{ik}$</td>
<td>reservoir $i$ storage at end of period $k$</td>
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<td>$a_{ik}$</td>
<td>reservoir $i$ inflow during period $k$</td>
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<td>$q_{ik}$</td>
<td>plant $i$ discharge during period $k$</td>
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<td>$s_{ik}$</td>
<td>water spillage by reservoir $i$ during period $k$</td>
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<td>plant $i$ efficiency during period $k$</td>
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<td>$\eta_{i}^{\text{min}}$</td>
<td>minimum efficiency of plant $i$</td>
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<td>$h_{ik}$</td>
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<td>maximum head of plant $i$</td>
</tr>
<tr>
<td>$h_{i}^{\text{min}}$</td>
<td>minimum head of plant $i$</td>
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</table>
2. Problem formulation

2.1 Risk management

CVaR represents an appropriate approach to address risk management for a hydro power producer. Previous MINLP approaches [8,24] did not consider risk management.

Value-at-risk requires the use of binary variables for its modeling, which represents a significant disadvantage. Instead, CVaR does not need to use binary variables, and can be modeled using linear constraints.

The concept of CVaR is illustrated in Fig. 1.

"Insert Fig. 1 here".

Risk is managed in our approach by imposing a lower limit and confidence level on CVaR. For a given time horizon $K$ and confidence level $\delta$, $\text{CVaR}(1-\delta)$ is the conditional expectation of the profit above $\text{VaR}(1-\delta)$. CVaR is the expected profit not surpassing a measure $\zeta$, named Value-at-Risk:

$$\text{CVaR} = E(B \mid B < \zeta) \quad (1)$$

Value-at-Risk is given by:

$$\text{VaR} = \max(x \mid p\{B \leq x\} \leq 1-\delta) \quad (2)$$
The value of $\delta$ is commonly set between 0.90 and 0.99 \[3\]. In this paper, $\delta$ is considered equal to 0.95. Mathematically, CVaR can be defined as:

$$\max \; \zeta - \frac{1}{1-\delta} \sum_{n} \rho_{n} \eta_{n}$$ \hspace{1cm} (3)

subject to

$$-B_{n} + \zeta - \eta_{n} \leq 0$$ \hspace{1cm} (4)

$$\eta_{n} \geq 0$$ \hspace{1cm} (5)

Constraints (4) and (5) impose conditions regarding the risk term. In (4), $\eta_{n}$ is zero if scenario $n$ gives a greater profit than $\zeta$. For all other scenarios, $\eta_{n}$ is given by the difference between $\zeta$ and the corresponding profit.

2.2 Objective function

The objective function considers all the price scenarios at once, weighted by their probability of occurrence.

The STHS problem is formulated as maximizing:

$$F = \sum_{n} \rho_{n} B_{n} + \alpha \left( \zeta - \frac{1}{1-\delta} \sum_{n} \rho_{n} \eta_{n} \right)$$ \hspace{1cm} (6)

The objective function \(6\) is defined as the total profit of the hydro power producer in addition to a risk measure on profit. The CVaR approach is included in the formulation, providing a tradeoff between maximum profit and profit volatility. A risk-averse producer will tend to minimize the risk by selecting a large value of $\alpha$ to increase the influence of the risk measure in \(6\). Otherwise, a risk-neutral producer tends to maximize the risk by selecting a small value of $\alpha$ to obtain a higher profit. $B_{n}$ is the benefit for each price scenario, taking into account start-up costs, and is given by:

$$B_{n} = \sum_{k=1}^{K} \lambda_{nk} \sum_{l=1}^{L} p_{lk} = \sum_{k=1}^{K} \sum_{l=1}^{L} S U_{ik} \gamma_{kl}$$ \hspace{1cm} (7)

2.3 Hydro constraints

The following constraints are considered:
1) Water Balance Equation:

\[ v_{ik} = v_{i,k-1} + u_{ik} - q_{ik} - s_{ik} + q_{i+1,k} + s_{i+1,k} \]  

(8)

2) Power Generation Equation:

\[ p_{ik} = P_{ik} (q_{ik}, \eta_{ik}) \]  

(9)

3) Head Equation:

\[ h_{ik} = H_{ik} (l_{ik}, l_{i+1,k}) \]  

(10)

4) Water Storage Constraints:

\[ v_{i}^{\text{min}} \leq v_{ik} \leq v_{i}^{\text{max}} \]  

(11)

5) Water Discharge Constraints:

\[ u_{ik} q_{i}^{\text{min}} \leq q_{ik} \leq u_{ik} q_{i}^{\text{max}} \]  

(12)

6) Discharge Ramping:

\[ q_{ik} - R_i \leq q_{i,k-1} \leq q_{ik} + R_i \]  

(13)

7) Commitment Status:

\[ y_{ik} - z_{ik} = u_{ik} - u_{i,k-1} \]  

(14)

8) Water Spillage Constraints:

\[ s_{ik} \geq 0 \]  

(15)

Equation (8) corresponds to the water balance equation for the reservoirs. In (9), the hydro power generation is considered a function of water discharge and efficiency, which in turn depends on the head. The operating points are restricted by the maximum and minimum values of the discharged water. In (10), the head is considered a function of the water levels in the upstream and downstream reservoirs, depending on the reservoir’s storage. In (11), lower and upper bounds on water storage are set. In (12), the same occurs with water discharge. Also, discharge ramping constraints are considered in (13), which may be compulsory due to environmental or navigation requirements. Equation (14) models the start-up and shut-down status of the plants. Equation (15) imposes a non-negative value for the spillage of water.

The hydro power producer analyzed in this paper is considered to be a price-taker; that is, it does not have market power, as in [8,24].
3. Proposed approach

The general formulation for a MINLP problem can be stated as:

\[
\begin{align*}
\text{Max } & F(x) \\
\text{subject to } & b_{\text{min}} \leq Ax \leq b_{\text{max}} \\
& x_{\text{min}} \leq x \leq x_{\text{max}} \\
& x_j \text{ integer}
\end{align*}
\]  

(16) (17) (18) (19)

Note that (6) is substituted into (16). The water conservation equation (8) is substituted into (17), as well as the lower and upper bounds for water discharge given in (12), the discharge ramping constraints given in (13), and the logical status of commitment given in (14). Equation (18) corresponds to the inequality constraints in (11) and (15).

The effect of head on power production is considered by a single function (20) of water storage and water discharge, as in [8], which can be implemented in a clear-cut way:

\[
p_{ij} = q_{ij}\left(\alpha_i\beta_jv_{ij} + \alpha_i\beta_1v_{1ij} - \alpha_i\beta_1v_{1ij} - \alpha_i\beta_1v_{1ij} + \eta_{ij}\right)
\]  

(20)

MINLP is still a hot research topic among specialists in optimization. If a solution is found, there is no guarantee that it is the global optimal. Instead, a local optimal is usually obtained. Therefore, an MILP approach is used to find a starting point for the MINLP approach. Afterwards, we check for an enhanced objective function value using the proposed MINLP approach. Note that the initial values do not affect the optimization results. Convergence towards a superior solution is always achieved in our case study, guaranteeing also a reasonable computation time.

The model presented in this paper is especially indicated for systems in which the daily policy of water discharges places a significant weight on hourly heads [22], that is, when it is truly important to take into account head variations to obtain optimal or near-optimal practical schedules, as occurs for instance in Portugal and Spain.

As a new contribution building on earlier studies [8,24], market uncertainty is introduced in the MINLP model via price scenarios and risk management is included using CVaR to limit profit volatility. Therefore, the maximum profit versus minimum risk tradeoff is now duly tackled. The hydro generation scheduling is then used to develop appropriate offering strategies for the pool-based system.
A mid-term model can offer water level targets to be reached by each reservoir at the end of the day. It is possible to obtain the water value associated with each reservoir by imposing a selected final water level. The water value curves will directly affect the bid curves’ shape. The supply functions, considered on an hourly basis, should be monotonically increasing functions. These supply functions can be built by solving autonomous problems with different water level targets at each reservoir [25].

4. Case study

The application of the proposed MINLP approach to a realistic case study is described hereafter, considering not only the effect of head on power production, start-up costs related to the units, multiple regions of operation, and constraints on discharge variation, but also price scenarios and risk management. A hydro energy system in Portugal in cascade configuration has been chosen. The modeling and simulations were carried out in a MATLAB/CPLEX environment using a 600 MHz computer with 256 MB of memory.

In [8,24] energy prices were regarded as deterministic data. In this paper, several price scenarios are considered using the neural network approach proposed in [36]. The hydro power producer is considered a price-taker, and thus does not have the ability to change market prices.

The price scenarios over the 24-hour time horizon are shown in Fig. 2 (where $ represents an economic quantity). The number of price scenarios generated is \( N = 100 \), and each scenario is equally probable.

"Insert Fig. 2 here".

The hydro energy system considered is presented in Fig. 3.

"Insert Fig. 3 here".

The only reservoir with inflow is the first reservoir. This inflow is presented in Fig. 4.

"Insert Fig. 4 here".

Water storage at the end of the time horizon is chosen to be equal to the initial value. The storage targets are defined by medium-term strategies, as in [37].

The expected profit versus standard deviation of profit is presented in Fig. 5, considering six values of \( \alpha \). This figure provides the maximum achievable expected profit for each risk level or, alternatively, the minimum achievable risk level for each expected profit.

"Insert Fig. 5 here".
An analysis of Fig. 5, known as the efficient frontier or Markowitz frontier, reveals that a risk-neutral producer \( \alpha = 0 \) expects to achieve a profit of $206,129 with a standard deviation of $5,967. On the other hand, a risk-averse producer \( \alpha = 1 \) expects to achieve a profit of $202,923 but with a lower standard deviation of $5,272.

Table 1 establishes a numerical comparison of the increase in profit for several risk levels. The maximum profit, corresponding to a risk level \( \alpha = 0 \), represents an increase of 1.58% in comparison with \( \alpha = 1 \). Different hydro power producers may choose different behaviors towards risk.

"Insert Table 1 here".

A comprehensive comparison of the optimal scheduling for the two extreme risk levels is presented hereafter.

The optimal storages of the reservoirs are provided in Fig. 6 and the optimal discharges of the plants are provided in Fig. 7. The results obtained using a risk level \( \alpha = 0 \) are represented by a solid line, while the results obtained using a risk level \( \alpha = 1 \) are represented by a dashed line.

"Insert Fig. 6 here".

"Insert Fig. 7 here".

Risk makes possible a different behavior, especially for the first reservoir, implying that for a risk-neutral producer the effect of head on power production is more relevant. The results in Fig. 7 are consistent with those in Fig. 6. The risk-neutral producer aims at discharging mostly during peak hours.

Figs. 8 and 9 present the histograms of the expected profits for \( \alpha = 0 \) and \( \alpha = 1 \), respectively.

"Insert Fig. 8 here".

"Insert Fig. 9 here".

Analyzing Figs. 8 and 9, it can be verified that the risk level corresponding to \( \alpha = 0 \) may imply a higher expected profit than that corresponding to \( \alpha = 1 \). However, \( \alpha = 0 \) is riskier than \( \alpha = 1 \), because economic loss can happen under some scenarios. Also, for \( \alpha = 1 \) the profit outcomes are tightened, as shown in Fig. 9. Thus, a risk-averse investor would prefer \( \alpha = 1 \) because it exhibits lower financial risk. Hence, our model provides the decision maker with different possible solutions according to the preferred risk level.
Table 2 shows the hourly scheduling results in the first hydro plant of this case study for $\alpha = 0$ and $\alpha = 1$.

"Insert Table 2 here".

The results listed in Table 2 demonstrate that different risk levels lead to different hourly schedules. In the hours in which the hydro plant is online, the production is usually slightly higher for $\alpha = 0$ than for $\alpha = 1$.

Figure 10 presents the hourly bids (quantity-price pairs) for the hydro system considered in this case study. The monotonically increasing hourly supply functions were built by solving autonomous problems with different water level targets at each reservoir. In our case study, 11 water level targets were taken into account, with water values ranging from $5.5/\text{MWh}$ to $60.5/\text{MWh}$. Hence, the curves are represented by piecewise linear approximations formed by 10 segments. Considering higher water levels at the end of the day implies that the power produced will be lower, since less water can be discharged.

"Insert Fig. 10 here".

The optimal solution requires about 3 seconds of CPU time. Hence, the proposed approach provides accurate results with a low computational burden.

5. Error analysis

The volatility of the expected profit is analyzed by means of dispersion. Accordingly, the dispersion of profit for the 100 scenarios is shown in Fig. 11 for three levels of risk. Also, Table 3 presents the 95% confidence intervals regarding the expected profit.

"Insert Fig. 11 here".

"Insert Table 3 here".

Although the expected profit is higher for $\alpha = 0$, the dispersion of profit is also greater when compared with the other risk levels. Instead, the lowest dispersion of profit is attainable for $\alpha = 1$. Hence, a risk-averse producer would expect a lower variability of the expected profit.
6. Conclusions

An MINLP approach is proposed in this paper to solve the STHS problem, considering the effect of head on power production, start-up cost related to the units, multiple regions of operation, and constraints on discharge variation. As new contributions building on earlier studies, price scenarios and risk management are also taken into account. The objective is to maximize the total profit of a hydro plant operating in a day-ahead electricity market, including an appropriate risk measure, the CVaR. The optimal self-schedule is used to derive appropriate strategies for making offerings to the pool. The hydro power producer is considered a price-taker, so market prices are exogenous variables modeled through scenarios. The proposed approach includes the efficient frontier curve, providing the tradeoff of maximum expected profit versus minimum risk. The efficient frontier curve can be used by decision-makers to make informed decisions on the pool. The presented results on a realistic cascaded hydro system validate the proficiency of the proposed approach, enabling the selection of the best solution according to the desired risk exposure level and simultaneously guaranteeing a satisfactory computation time.

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References


Figures

Fig. 1. CVaR concept.

Fig. 2. Price scenarios over the 24-hour time horizon.
Fig. 3. Hydro energy system with three reservoirs in cascaded configuration.

Fig. 4. Inflow into the first reservoir.
Fig. 5. Expected profit versus standard deviation of profit.

The results obtained using a risk level $\alpha = 0$ are represented by a solid line, while the results obtained using a risk level $\alpha = 1$ are represented by a dashed line.

Fig. 6. Optimal storages of the reservoirs.
Fig. 7. Optimal discharges of the plants. The results obtained using a risk level $\alpha = 0$ are represented by a solid line, while the results obtained using a risk level $\alpha = 1$ are represented by a dashed line.
Fig. 8. Histogram of the expected profits corresponding to $\alpha = 0$.

Fig. 9. Histogram of the expected profits corresponding to $\alpha = 1$. 
Fig. 10. Hourly supply functions generated for the risk levels corresponding to $\alpha = 0$ (○) and $\alpha = 1$ (Δ).
Fig. 11. Dispersion of profit for three levels of risk.
### Tables

**Table 1**
Comparison of the increase in profit for several levels of risk

<table>
<thead>
<tr>
<th>Level of risk</th>
<th>Profit standard deviation ($)</th>
<th>Expected profit ($)</th>
<th>% Increase</th>
<th>CPU time (s)</th>
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<tbody>
<tr>
<td>1.0</td>
<td>5,272</td>
<td>202,923</td>
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<td>0.8</td>
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<td>206,129</td>
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**Table 2**
Hourly scheduling results for two levels of risk – Plant 1

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<tr>
<th>Hour</th>
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<th>Power generation per water discharge [MWh/hm³]</th>
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<tr>
<td></td>
<td>$\alpha = 0$</td>
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Table 3
Confidence intervals

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<th>Risk level</th>
<th>95% confidence intervals regarding the expected profit</th>
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<td>[204,933; 207,144]</td>
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<td>[204,077; 206,254]</td>
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