Chapter 12

LINEAR PROGRAMMING APPLIED FOR THE OPTIMIZATION OF HYDRO AND WIND ENERGY RESOURCES

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ABSTRACT

In this book chapter, two important applications of linear programming are presented from the electric power industry, namely short-term hydro scheduling and the development of offering strategies for wind power producers. The linear programming approach is proposed to solve the problems related with generation companies whose main goal is to maximize profits. On the one hand, the main concern of hydroelectric companies is to find the optimal scheduling of hydroelectric power plants, for a short-term period in which the electricity prices are forecasted. The actual size of hydro systems, the continuous reservoir dynamics and constraints, still pose a real challenge to the modelers. On the other hand, wind power producers are entities owning generation resources and participating in the electricity market. The challenges for wind power producers are related to two kinds of uncertainties: wind power and electricity prices. It can be concluded that linear programming represents a robust approach for these two problems.

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1. INTRODUCTION

Nowadays, renewable energy sources play an increasingly important role in electricity production [1], since they produce clean energy, respecting the compromise established by the Kyoto protocol. These renewable energy sources can partly replace carbon emitting fossil-based electricity generation, and thereby reduce CO₂ emissions [2]. Hence, the use of renewable energy has been increasing in the last decade worldwide, particularly in European countries such as Denmark [3] and Ireland [4].

Hydro energy is currently one of the priorities in the Portuguese energy policy. Under this energy policy, the optimal management of hydro energy systems is of crucial importance, as occurs for instance in Norway [5].

In this book chapter, the short-term hydro scheduling (STHS) problem of a cascaded hydro energy system is considered. In the STHS problem a time horizon of one day to one week is considered, usually divided into hourly intervals. Hence, the STHS problem is treated as a deterministic one. Where the problem includes stochastic quantities, such as inflows to reservoirs or electricity prices, the corresponding forecasts are used.

In a deregulated environment, a hydro generating company (H-GENCO) is usually an entity owning generation resources and participating in the electricity market with the ultimate goal of maximizing profits, without concern of the system, unless there is an incentive for it. A day-ahead electricity market based on a pool is considered in this book chapter.

The optimal management of the water available in the reservoirs for power generation, regarding future operation use, delivers a self-schedule and represents a major advantage for the H-GENCO to face competitiveness given the economic stakes involved. Based on the self-schedule, the H-GENCO is able to submit bids with rational support to the electricity market. Thus, for deregulation applications, STHS solution is important as a decision support for developing bidding strategies in the market [6], guided by the forecasted electricity prices, and a more realistic modeling is crucial for surviving nowadays competitive framework.

Dynamic programming (DP) is among the earliest methods applied to the STHS problem [7]. Although DP can handle the nonconvex, nonlinear characteristics present in the hydro model, direct application of DP methods for cascaded hydro energy systems is impractical due to the well-known DP curse of dimensionality.

Artificial intelligence techniques have also been applied to the STHS problem [8–10]. However, due to the heuristics used in the search process, only sub-optimal solutions can be reached.

A natural approach to STHS is to model the system as a network flow model, because of the underlying network structure subjacent in cascaded hydro energy systems [11]. For cascaded hydro energy systems, as there are water linkage and electric connections among plants, the advantages of the network flow technique are salient. Hydroelectric power generation characteristics are often assumed as linear or piecewise linear in hydro scheduling models. Accordingly, the solution procedures can be based on linear programming (LP).
Hence, LP is proposed in this book chapter for solving the STHS problem in the day-ahead electricity market.

Wind, as a renewable energy source, has been generally applied as a means to reach emission reduction goals as a result of increasing concern regarding environmental protection [12]. Actually, wind power is the world’s fastest growing renewable energy source [13].

In Portugal, the wind power goal foreseen for 2010 was established by the government as 3750 MW, representing about 25% of the total installed capacity in 2010 [14]. This value has recently been raised to 5100 MW, by the most recent governmental goals for the wind sector.

In deregulated markets, wind power producers are entities owning generation resources and participating in the market with the ultimate goal of maximizing profits [15]. The challenges for wind power producers are related to two kinds of uncertainty: wind power and electricity prices. A large variability of wind power or electricity prices means a large variability in profit [16]. Thus, the decision makers have to consider these two kinds of uncertainty, as well as the several technical constraints associated to the operation of wind farms.

The offer decisions to submit for the electricity market have to be done in each hour, without knowing exactly what will be the value of power generation. The differences between the produced energy and supplied energy constitute the energy imbalances. The imbalances should be penalized by the market balance [17,18]. A wind power producer needs to know how much to produce in order to make realistic bids, because in case of excessive or moderate bids, other producers must reduce or increase production to fill the so-called deviation, causing economic losses. These economic losses are reflected in so-called penalties for deviation. To take into account these uncertain measures, multiple scenarios can be build using wind power forecasting [19–21] and electricity price forecasting [22,23] tools. A scenario tree represents the different stages that can take the random parameters, i.e., different realizations of uncertainty. The tree is a natural and explicit way of representing nonanticipativity decisions.

The stochastic nature of the uncertain measures can be modeled through a two-stage stochastic programming approach [24–27]. In this approach, the set of decisions inherent to the problem can be divided into two distinct stages: first-stage decisions, which must be taken before resolving the uncertainty; second-stage decisions, which are made after the uncertainty occurs and are influenced by the decisions taken in first stage. The first-stage decisions correspond to the hourly bids to be submitted to the day-ahead market, while the second-stage decisions correspond to the power output of the wind farm in each hour for a given scenario.

Figure 1 shows the scenario tree that will be used to represent the decisions to be taken in the two stages previously mentioned.

In this book chapter, a stochastic programming approach is proposed to generate the optimal offers that should be submitted to the day-ahead market by a wind power producer, in order to maximize its expected profit.

The book chapter is organized as follows. In Section 2, the mathematical formulation of both problems is provided. Section 3 presents the proposed LP approach. In Section 4, the proposed LP approach is applied on two case studies, to demonstrate its effectiveness. Finally, concluding remarks are given in Section 5.
NOMENCLATURE

$I$, $i$ set and index of reservoirs
$K$, $k$ set and index of hours in the time horizon
$S$, $s$ set and index of scenarios
$\rho$, probability of occurrence of scenario $s$
$\lambda$, forecasted electricity price in hour $k$
$\lambda_{s,k}$, expected electricity price in scenario $s$ in hour $k$
$p_{i,k}$, power generation of plant $i$ in hour $k$
$\Psi_i$, future value of the water stored in reservoir $i$
$a_{i,k}$, inflow to reservoir $i$ in hour $k$
$M$, set of upstream reservoirs to reservoir $i$
$v_{i,k}$, water storage of reservoir $i$ at end of hour $k$
$q_{i,k}$, water discharge by reservoir $i$ in hour $k$
$s_{i,k}$, water spillage by reservoir $i$ in hour $k$
$h_{i,k}$, head of plant $i$ in hour $k$
$\nu$, penalty factor over the electricity price for energy imbalances.
$x_s$, energy offered by the wind power producer in the day-ahead market for hour $k$
$\omega_{s,k}$, cost of penalization for deviation in scenario $s$ in hour $k$
$dev_{s,k}$, deviation for wind production in scenario $s$ in hour $k$
2. Problem Formulation

In this section, we present the formulation for each problem discussed previously. The short-term hydro scheduling problem is formulated first, and afterwards the problem of developing offering strategies for wind power producers is formulated.

2.1. Short-Term Hydro Scheduling

The STHS problem can be stated as to find out the water discharges, the water storages, and the water spillages, for each reservoir $i$ at all scheduling time periods $k$ that maximizes (or minimizes) a performance criterion subject to all hydraulic constraints.

1. Objective Function

In this problem, the objective function to be maximized is expressed as:

$$ F = \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_{ik} p_{i,k} + \sum_{i=1}^{I} \Psi_{i} (v_{i,K}) $$
In (1), the first term is related to the revenues of each plant \( i \) in the hydro energy system during the short-term time horizon, whereas the last term expresses the water value, \( \Psi_i \), for the future use of the water stored in the reservoirs at the last period, \( v_{iK} \).

The future value of the water stored in the reservoirs is not considered in (1) if the water storage in the reservoirs in the last period is fixed. An appropriate representation when this term is explicitly taken into account can be seen for instance in [28]. The storage targets for the short-term time horizon can be established by medium-term planning studies.

2. Hydro Constraints

The hydro constraints are of two kinds: equality constraints and inequality constraints or simple bounds on the decision variables.

The water balance equation for each reservoir is formulated as:

\[
v_{ik} = v_{i,k-1} + a_{ik} + \sum_{m \in M_i} (q_{im} + s_{im}) - q_{ik} - s_{ik} \quad \forall \ i \in I, \ \forall \ k \in K
\]

(2)

assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period.

The head of a hydro plant \( i \) measures the difference between the forebay elevation and the tailrace elevation. Forebay elevation is dependent upon reservoir contents and also on the flows through the reservoirs. Tailrace elevation depends upon discharges and for some plants on the elevation of water in the immediate downstream reservoir. Therefore, it can be expressed as a function of its reservoir storage \( v_{iK} \), and the immediate downstream reservoir storage \( v_{\text{f0}k} \) [29]:

\[
h_{ik} = h_{ik}(v_{iK}, v_{\text{f0}k}) \quad \forall \ i \in I, \ \forall \ k \in K
\]

(3)

If the tailrace elevation is considered constant, this relationship can be simplified [29]:

\[
h_{ik} = h_{ik}(v_{\text{f0}k}) \quad \forall \ i \in I, \ \forall \ k \in K
\]

(4)

so that the head depends only on the storage of the upstream reservoir.

Power generation is considered a function of water discharge:

\[
p_{ik} = P(q_{ik}) \quad \forall \ i \in I, \ \forall \ k \in K
\]

(5)

Water storage has lower and upper bounds, given by:
Linear Programming Applied for the Optimization ...

\[ v_i \leq v_{ik} \leq v_i \quad \forall i \in I, \quad \forall k \in K \]  

(6)

Water discharge has lower and upper bounds, given by:

\[ q_i \leq q_{ik} \leq \bar{q}_i \quad \forall i \in I, \quad \forall k \in K \]  

(7)

Each hydro plant has a mechanism to spill a given quantity of water, if required. A null lower bound for water spillage is considered, given by:

\[ s_{ik} \geq 0 \quad \forall i \in I, \quad \forall k \in K \]  

(8)

thus, water spillage can occur when without it the water storage exceeds its upper bound, so spilling is only necessary due to safety considerations. The spillage effects were considered in [30].

The initial water storages and inflows to reservoirs are assumed known. The H-GENCO analyzed in this book chapter is considered to be a price-taker, i.e., it does not have market power. Therefore, electricity prices \( \lambda_k \) in (1) are also assumed known, as in [31,32].

The size of the LP problem (1)–(8), expressed as the number of continuous variables and constraints, is provided in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Problem size – Short-term hydro scheduling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous variables</td>
</tr>
<tr>
<td>Equality constraints</td>
</tr>
<tr>
<td>Inequality constraints</td>
</tr>
</tbody>
</table>

2.2. Development of Offering Strategies for Wind Power Producers

The mathematical formulation of the optimization model related to development of offering strategies for wind power producers is presented thereafter. This formulation uses an absolute value function, since it can be expressed in the context of LP by adding some auxiliary variables for positive and negative deviations.

1. Objective Function

The objective function to be maximized can be expressed as:

\[ F = \sum_{i=1}^{s} \sum_{k=1}^{K} \rho_i \left[ \hat{\lambda}_k \rho \alpha - v \hat{\lambda}_k \text{dev}_\alpha \right] \]  

(9)
The objective function represents the total profit on the sale of wind energy in each scenario \( s \), taking into account the probability of occurrence \( \rho_s \), less a penalty for deviations from the bids.

The penalties for deviations are associated with short-term variability (e.g. hour to hour variation) and the lack of predictability of wind power. Hence, the deviations are measured in absolute value, and can be generated by excess or deficit of energy:

\[
\text{dev}_a = \begin{cases} 
  p_a - x_s, & p_a - x_s \geq 0 \\
  -p_a + x_s, & p_a - x_s < 0 
\end{cases}
\] (10)

The deviation cost is set as a percentage of the daily market price:

\[
\omega_a = v \lambda_a 
\] (11)

The penalty for the deviation corresponds to product of the cost for the shifted power in absolute value:

\[
P_{\text{dev}_a} = \omega_a \text{dev}_a
\] (12)

The revenue is given by the product of the expected electricity price by the power output of the wind farm:

\[
L_a = \hat{\lambda}_a p_a
\] (13)

The profit of the operation is calculated as the difference between the revenue of the wind farm and penalization for deviation:

\[
F = L_a - P_{\text{dev}_a}
\] (14)

The objective function is obtained by substituting (12) and (13) into (14), resulting in the following equation:

\[
F = \sum_{i=1}^{k} \sum_{s=1}^{K} \left( \hat{\lambda}_a p_a - v \hat{\lambda}_a |p_a - x_s| \right)
\] (15)

2. Constraints

In order to make the offers to the market, it is required to satisfy the technical limitations of the wind farm. So, the optimal value of the objective function is determined subject to inequality constraints or simple bounds on the variables. The constraints are indicated as follows:

\[
0 \leq p_a \leq W_a
\] (16)
In inequality (16), wind power is limited superiorly by the value of wind generation forecast, $W_s$, in scenario $s$ in hour $k$. The value of wind generation forecast is not always attainable due to the intermittent wind.

$$0 \leq x_k \leq \bar{P}$$  \hspace{1cm} (17)

In inequality (17), the offers are limited by the maximum power installed in the wind farm $\bar{P}$.

3. Linearization of the Objective Function

The objective function, presented in the previous subsection, is characterized by a nonlinearity due to the existence of the absolute value. So, it is required to use a mathematical process that allows reformulating into a linear problem.

In this subsection, the problem involving absolute value terms is transformed into an LP formulation. Initially, it is considered:

$$\text{Max } F = c^T x - |x|$$  \hspace{1cm} (18)

subject to

$$\underline{x} \leq x \leq \bar{x}$$  \hspace{1cm} (19)

$$x \in \mathbb{R}^n$$  \hspace{1cm} (20)

In (18), the function $F(\cdot)$ is an objective function of decision variables, where $c$ is the vector of coefficients for the linear term.

In (19), $\underline{x}$ and $\bar{x}$ are the lower and upper bound vectors on variables. The variable $x$ is a set of decisions variables. Subsequently, absolute-valued variables are replaced with two strictly positive variables:

$$|x| = x^+ + x^-$$  \hspace{1cm} (21)

In addition, each variable is substituted by the difference of the same two positive variables, as:

$$x = x^+ - x^-$$  \hspace{1cm} (22)
The equivalent LP problem is given by:

\[
\begin{align*}
\text{Max } F &= c^T x - (x^+ + x^-) \\
\text{subject to} & \\
-x & \leq x \leq x \\
x &= x^+ - x^- \\
x^+ & \geq 0, \quad x^- \geq 0
\end{align*}
\] (23)

The size of the LP problem (9)–(17), expressed as the number of continuous variables and constraints, is provided in Table 2.

<table>
<thead>
<tr>
<th>Continuous variables</th>
<th>(3 \times S \times K + K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equality constraints</td>
<td>(S \times K)</td>
</tr>
<tr>
<td>Inequality constraints</td>
<td>(3 \times S \times K + K)</td>
</tr>
</tbody>
</table>

### 3. PROPOSED APPROACH

LP is an optimization procedure that minimizes (or maximizes) a linear objective function with variables that are also subject to linear constraints. A solution that satisfied all conditions of the problem and the given objective is called optimal solution. LP characterizes itself by its simple mathematical structure but powerful in its adaptability to a wide range of applications. These algorithms provide extremely robust and efficient codes.

### 3.1. LP Applied for Short-Term Hydro Scheduling

The LP approach is used for solving the STHS problem. The LP problem can be stated as to maximize:

\[
F(x)
\] (27)

subject to:
Linear Programming Applied for the Optimization ...

\[ \begin{align*}
  b & \leq A x \leq \bar{b} \\
  \underline{x} & \leq x \leq \bar{x}
\end{align*} \tag{28} \tag{29} \]

In (27), the function \( F (\cdot) \) is a linear function of the vector \( x \) of decision variables. Equality constraints are defined by setting the lower bound equal to the upper bound, i.e. \( b = \bar{b} \). Equation (29) corresponds to the inequality constraints or simple bounds on the variables in (6), (7) and (8).

### 3.2. LP Applied for the Development of Offering Strategies for Wind Power Producers

#### 1. Two-stage Stochastic Programming

The two-stage stochastic programming model can be formulated as:

\[
\begin{align*}
  \text{Max} & \quad c^T x + E \{ \max_{\omega} q^T \omega y^\omega \} \\
\text{subject to} & \quad b \leq A x \leq \bar{b} \\
& \quad \underline{h}_{x\omega} \leq T_{x\omega} x + W_{x\omega} y_{x\omega} \leq \bar{h}_{x\omega} \\
& \quad x \geq 0, \quad y_{x\omega} \geq 0.
\end{align*} \tag{30} \tag{31} \tag{32} \tag{33}
\]

In the first-stage, the “here-and-now” decisions should be taken, before the uncertainties represented by \( x \) are known. In the second-stage, where the information \( x \) is already available, the decision is made about the value of the vector \( y \). The first-stage decision of \( x \) depends only on the information available until that time; this principle is called nonanticipativity constraint. The problem of two stages means that the decision \( x \) is independent of the achievements of the second-stage, and thus the vector \( x \) is the same for all possible events that may occur in the second-stage of the problem.

#### 2. Deterministic Equivalent Problem

The stochastic model is usually a difficult computational problem, so it is common to choose the deterministic model solution using the average of random variables or solving a deterministic problem for each scenario.

The deterministic equivalent problem is given by:

\[
\begin{align*}
  \text{Max}_{x, y} & \quad c^T x + \sum_{s=1}^{S} \rho_s q^T_s y_s \\
\end{align*} \tag{34}
\]
subject to
\[ \underline{b} \leq Ax \leq \bar{b} \]  
\[ \underline{b}_s \leq T_s x + W_s y_s \leq \bar{b}_s \]  for \( s = 1, \ldots, S \)  
\[ x \geq 0, \quad y_s \geq 0. \]  for \( s = 1, \ldots, S \)  

The matrix composed by (35) and (36), for large-scale linear problems, can be generally represented according with Figure 2.

![Diagram of constraints]

Figure 2. Layout of the constraints associated with two stages.

### 4. Case Studies

The proposed LP approach has been developed and implemented in MATLAB and solved using the optimization solver package CPLEX. The numerical simulation has been performed on a 2-GHz based processor with 2GB of RAM.

#### Table 3. Hydro data

<table>
<thead>
<tr>
<th>#</th>
<th>( \bar{v}_i ) (hm³)</th>
<th>( \tilde{v}_i ) (hm³)</th>
<th>( v_{i0} ) (hm³)</th>
<th>( p_i ) (MW)</th>
<th>( \tilde{p}_i ) (MW)</th>
<th>( q_i ) (m³/s)</th>
<th>( \tilde{q}_i ) (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.18</td>
<td>12.94</td>
<td>10.35</td>
<td>28.00</td>
<td>188.08</td>
<td>168.13</td>
<td>1144.50</td>
</tr>
<tr>
<td>2</td>
<td>5.32</td>
<td>13.30</td>
<td>10.64</td>
<td>29.99</td>
<td>237.14</td>
<td>104.70</td>
<td>1080.00</td>
</tr>
<tr>
<td>3</td>
<td>39.00</td>
<td>97.50</td>
<td>78.00</td>
<td>10.67</td>
<td>60.00</td>
<td>3.00</td>
<td>16.40</td>
</tr>
<tr>
<td>4</td>
<td>4.80</td>
<td>12.00</td>
<td>9.60</td>
<td>24.99</td>
<td>185.99</td>
<td>104.67</td>
<td>900.00</td>
</tr>
<tr>
<td>5</td>
<td>4.40</td>
<td>11.00</td>
<td>8.80</td>
<td>29.99</td>
<td>201.02</td>
<td>93.23</td>
<td>881.31</td>
</tr>
<tr>
<td>6</td>
<td>36.89</td>
<td>58.38</td>
<td>46.70</td>
<td>39.99</td>
<td>134.02</td>
<td>94.99</td>
<td>326.34</td>
</tr>
<tr>
<td>7</td>
<td>8.60</td>
<td>21.50</td>
<td>17.20</td>
<td>19.99</td>
<td>117.01</td>
<td>182.83</td>
<td>1356.51</td>
</tr>
</tbody>
</table>
Figure 3. Hydro energy system with seven cascaded reservoirs, where $a$ represents inflow, $v$ represents water storage, $q$ represents water discharge and $s$ represents water spillage.

Figure 4. Electricity price profile.
The two case studies are defined by:

- Case A. Short-term hydro scheduling.
- Case B. Development of offering strategies for wind power producers.

4.1. Case A

The proposed LP approach has been applied on one of the main Portuguese cascaded hydro energy systems. The realistically-sized hydro energy system has seven cascaded reservoirs and is shown in Figure 3. Table 3 shows the data of these plants.

The hydro plants numbered in Figure 3 as 1, 2, 4, 5 and 7 are run-of-the-river hydro plants. The hydro plants numbered as 3 and 6 are storage hydro plants. Inflow is considered only on reservoirs 1 to 6. The final water storage in the reservoirs is constrained to be equal to the initial water storage.

The time horizon is one day divided into 24 hourly intervals. The electricity price profile considered over the short-term time horizon is shown in Figure 4 ($ is a symbolic economic quantity). The electricity price values are based on real market operation.

The competitive environment coming from the deregulation of the electricity markets brings energy prices uncertainty, placing higher requirements on forecasting. A good price forecasting tool reduces the risk of under/over estimating the profit of the H-GENCO and provides better risk management. In the short-term, a generating company needs to forecast energy prices to derive its bidding strategy in the market and to optimally schedule its energy resources [33].

Price forecasting has become in recent years an important research area in electrical engineering, and several techniques have been tried out in this task. In general, hard and soft computing techniques could be used to predict energy prices. The hard computing techniques include auto regressive integrated moving average (ARIMA) [34] and wavelet-ARIMA [35] models. The soft computing techniques include neural networks [22] and hybrid approaches [36,37]. These energy prices are considered as deterministic input data for our STHS problem.

The storage trajectories of the reservoirs are shown in Figure 5. The discharge profiles for the reservoirs are shown in Figure 6. The main numerical results are summarized in Table 4.

The optimal solution requires only 1.20 seconds of CPU time, on a 2-GHz based processor with 2GB of RAM, using CPLEX. Hence, the LP approach provides a solution for this problem with a negligible CPU time requirement.

### Table 4. LP results

<table>
<thead>
<tr>
<th>Method</th>
<th>Average Discharge</th>
<th>Average Storage (%)</th>
<th>Total Profit ($ \times 10^3$)</th>
<th>CPU (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>25.00</td>
<td>83.08</td>
<td>714.57</td>
<td>1.20</td>
</tr>
</tbody>
</table>

7
Figure 5. Storage trajectories of the reservoirs considering the proposed LP approach.
Figure 6. Discharge profiles for the reservoirs considering the proposed LP approach.
4.2. Case B

The proposed LP approach has also been applied on a case study based on Portuguese wind farm. The total installed capacity of the plant is 265 MW. The deviation cost has been fixed at 30% of the daily market price, $v = 0.3$. Therefore, it is liable to a penalty other than by the price and according to the magnitude of the deviation in terms of power.

The time horizon chosen is one day divided into 24 hourly periods. This case study is composed of six electricity price scenarios computed by the approach proposed in [22], Figure 7, and six wind power scenarios computed by the approach proposed in [21], Figure 8, over the time horizon.

The number of scenarios generated for the day-ahead market in the optimization problem is \( S = 36 \). The probability of each generated scenario will be \( 1/S \).

![Figure 7. Electricity price scenarios considered in the case study.](image)

![Figure 8. Wind power scenarios considered in the case study.](image)
Table 5. Scenarios considered, the number and probability

<table>
<thead>
<tr>
<th>Number of scenarios</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price scenarios</td>
<td>6</td>
</tr>
<tr>
<td>Wind scenarios</td>
<td>6</td>
</tr>
<tr>
<td>Total scenarios</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 5 summarizes the data of the scenarios that compose the probability tree. The optimal bids shown in Figure 9 are common to the 36 scenarios that provide the probability tree.

Figure 9. Optimal hourly bids.

Choosing one scenario of the problem, it can be verified in Figure 10 that the wind farm adjusts its production to minimize deviations. Nevertheless, in almost every hour there are small differences between the offers and the power output of the wind farm.

The deviations from generated power for this scenario are shown in Figure 11.

The expected value of profit is 276685 €. The dispersion of profit for the 36 scenarios is show in Figure 12.

Table 6 provides the confidence interval for the profit.

The optimal solution requires only 1.59 seconds of CPU time, on a 2-GHz based processor with 2GB of RAM, using CPLEX. Hence, the LP approach provides a solution for this problem with a negligible CPU time requirement.

Table 6. Confidence interval 95% of the expected profit

<table>
<thead>
<tr>
<th>Wind farm</th>
<th>Confidence interval 95% of the expected profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind farm</td>
<td>[271008; 283075]</td>
</tr>
</tbody>
</table>
Figure 10. Optimal offers to be submitted to the day-ahead market and power produced.

Figure 11. Deviations resulting from the difference between the offers and the power produced.

Figure 12. Dispersion of profit.
5. CONCLUSIONS

An LP approach is proposed to solve the STHS problem in the day-ahead electricity market. The goal in the STHS problem is to maximize the value of total hydroelectric generation throughout the time horizon, while satisfying all hydraulic constraints, aiming the most efficient and profitable use of the water. The results obtained by the proposed LP approach are feasible, assuring simultaneously a negligible computation time. The proposed LP approach has also been applied to allow wind power producers to achieve better offering strategies in the market. The goal here is to maximize the profit of the wind power producer, reducing deviations, and taking into account the uncertainty associated with wind energy production and electricity prices. It is seen that the wind farm adjusts its production to minimize deviations. Hence, the LP approach is also proficient in the development of offering strategies for wind power producers.

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