Finite-Time Nonlinear Observer Design for Uncertain DC Microgrids Feeding Constant Power Loads

Shekoufeh Neisarian, Mohammad Mehdi Arefi, Navid Vafamand
Shiraz University
Shiraz, Iran
{sh.neisarian, arefi, n.vafamand}@shirazu.ac.ir

Mohammad S. Javadi
INESC TEC
Porto, Portugal
msjavadi@gmail.com

João P. S. Catalão
FEUP and INESC TEC
Porto, Portugal
catalao@fe.up.pt

Abstract—Due to the salient features of direct current (DC) microgrids (MGs) in integrating renewable energy sources, this paper offers a robust finite-time nonlinear observer (FTNO) for DC MGs comprising linear resistive and nonlinear constant power loads (CPLs) and a buck converter. It is assumed that the capacitor voltage is only accessible and the power system is subject to unknown time-varying uncertainties. A novel nonlinear observer is designed to estimate the inductance current to prevent the ripples produced by current sensors and to eliminate the price of utilizing expensive sensors. The global finite-time stability analysis of the observer error dynamic is investigated via a Lyapunov function and an explicit finite convergence time (FCT) is derived. The convergence rate of the estimated current is tunable by adjusting the parameters in FCT. Eventually, simulations are carried out to confirm the superiority of the proposed observer performance in estimating unknown inductance current in a particular finite time.

Keywords—Uncertain DC microgrid, Buck converter, Constant power load, Nonlinear observer, Adjustable finite convergence time.

I. INTRODUCTION

Microgrids (MGs) have been presented to provide an impressive way of integrating different kinds of distributed renewable energy [1]. The MGs are categorized into AC and DC ones. In applications involving DC electronic loads and renewable DC sources like wind and photovoltaics, the DC MGs are more appropriate and affordable than conventional AC MGs [1]. However, there is an increasing share of loads that are tightly controlled by power converters in DC MGs. Such loads are nonlinear since they act as CPLs. From the small-signal point of view, they expose negative incremental resistance which makes the overall system unstable.

Recently, the stability issue of CPLs in the DC MGs has been extensively studied with several control methods proposed. Though, in those control approaches, it is presumed that all the system state variables are accessible and measurable [2], [3]. It is noteworthy that some papers like [4]–[7] considered disturbance observer or in [8], a finite-time disturbance observer is probed to compensate for the effects of disturbances and they are completely different from state observer.

A literature search reveals that there have been only a few works on the estimation of unknown and unmeasured variables of DC MGs with CPLs by utilizing observers. For instance, a fuzzy observer is presented in [9], which cannot theoretically assure the estimate convergence. To the best of the authors’ knowledge, there are no researches on the design of finite-time nonlinear observers (FTNOs) for uncertain DC MGs feeding CPLs. The finite-time approach to design nonlinear observers has superiority towards asymptotic estimation. It is capable of fast estimating and is robust against uncertainties.

This paper discusses the problem of global FTNO design for DC MGs with linear loads and CPLs subjected to unknown time-varying bounded matched disturbances and a finite convergence time (FCT) of estimation is extracted to give the freedom of adjusting the rate of convergence of estimation error. A complete mathematical proof of universal finite-time stability of the observer error dynamic is performed by properly introducing a Lyapunov function candidate. Simulation results display and confirm the effectiveness of the offered FTNO.

The advantages and innovations of the proposed scheme are listed as below:

- The inductance current is estimated in a specific finite estimation time and not only the ripples caused by deploying physical sensors are alleviated but also the expenses emanating from installing sensors are omitted.
- An explicit finite convergence time of the observer error dynamic is extracted that gives the choice of adjusting the rate of convergence of the estimation process.
- The estimation process is done as fast as possible.
- The stability analysis of the observer error dynamic is held globally.
- The suggested approach is robust against uncertainties comprising external disturbances, perturbations, and unmodeled dynamics, which is not the case in the existing results.

J.P.S. Catalão acknowledges the support by FEDER funds through COMPETE 2020 and by Portuguese funds through FCT, under POCI-01-0145-FEDER-029803 (02/SAICT/2017).
This paper is organized as follows: Section II introduces a typical class of nonlinear DC MGs. In Section III, the recommended FTNLO is presented and the finite-time stability analysis for the observer error dynamic is done. In Section IV, simulation results are given. Section V ends this paper by evoking some concluding remarks.

II. NONLINEAR MODEL OF DC MGs WITH CPLs

A conventional DC MG possesses some power generators, power supply, and loads. These loads can be resistive or constant power, as indicated in Fig. 1. The difference between the resistive loads and CPLs is that the latter requires power electronic load converters while the prior does not. The CPLs are commonly integrated into DC MGs at the input point of the load converter by assuming the converters ideal or consume constant power.

This section represents the nonlinear state-space model of an uncertain DC MG feeding CPL, linear resistive loads, a buck converter, and DC input voltage source by considering that the inductance current is not measurable and inaccessible. The simplified electronic schematic of the DC MG is demonstrated in Fig. 2.

The system is subjected to unknown time-varying bounded matched uncertainties. The mathematical model of an uncertain nonlinear DC MG with CPL is brought in (1).

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{LC} \left( v_c u - x_1 + \frac{P L x_2}{x_2^2} \right) - \frac{x_2}{RC} + d(t,x)
\end{align*}
\]  

(1)

where \( x_1 = v_c \), \( x_2 = \dot{v}_c \), \( R = \frac{1}{\sum_{i=1}^{I} R_i} \), \( P = \sum_{i=1}^{K} P_{cPL_i} \), and \( d(t,x) \) denotes matched uncertainties, which appear as the same channel as the control input, comprising of unmodeled dynamics, perturbations, and external disturbances. \( u \) is a known control input signal that stabilizes the system or leads to reference tracking.

III. FINITE-TIME NONLINEAR OBSERVER DESIGN

The purpose of this paper is to design a robust FTNLO to estimate the unmeasured inductance current of the DC MG with CPL subjected to the unknown time-varying bounded uncertainties (i.e., \( \|d(t,x)\| \leq \gamma \)). A block diagram of the proposed scheme is depicted in Fig. 3.

This section includes two subsections. First, some basic mathematical preliminaries are given. Then, a nonlinear observer is designed and a Lyapunov function is constructed to prove the finite-time convergence of the observer estimation errors by applying a finite-time lemma. Finally, an explicit tunable FCT is obtained.

A. Preliminaries

The definition of finite-time stability and several useful lemmas, which are used in the FTNLO design and stability analysis of the observer error dynamics are presented in the following.

**Definition 1** [7], [10]. Consider time-invariant nonlinear system (2) where \( f: \Gamma \rightarrow \mathbb{R}^n \) is a continuous vector function and \( \Gamma \subseteq \mathbb{R}^n \) is an open neighborhood around the equilibrium point \( x = 0 \).

\[
\dot{x} = f(x) \text{ with } f(0) = 0, \quad x \in \Gamma \subseteq \mathbb{R}^n
\]  

(2)

Assume that the system has a unique solution \( x(t,x_0) \), for any arbitrary initial condition \( x(0) = x_0 \). Then, the zero equilibrium point of the system is locally finite-time stable (LFTS) if both below constraints are fulfilled:

a) It is locally asymptotically stable in the region \( \bar{\Gamma} \), where \( \bar{\Gamma} \subseteq \Gamma \) is an open neighborhood around the equilibrium point.

![Fig. 2. An electronic circuit diagram of the DC MG with \( \mathcal{K} \) CPLs, \( \mathcal{J} \) resistive loads, and a buck converter.](image1)

![Fig. 3. A block diagram of a finite-time nonlinear observer for an uncertain DC MG with CPL.](image2)
b) For any initial condition \(x_0\), there exists a finite convergence time \(T(x_0): \Gamma \setminus \{0\} \to [0,\infty)\) such that \(t \to T(x_0)\) and \(x(t,x_0) \to 0\) and \(x(t,x_0) = 0\) for \(t \geq T(x_0)\).

System (2) is globally finite-time stable (GFTS), if \(\Gamma = \mathbb{R}^n\).

**Lemma 1** [7], [11]. Consider the nonlinear system (2) owning the equilibrium point \(x = 0\) and the initial condition \(x_0\). Its equilibrium point is LFTS, if there exists a continuously differentiable function \(\psi_2\) such that the following constraint holds for any unique solution \(x(t,x_0)\) of the system:

\[
\dot{V}(x) \leq -\rho_1 V^{\rho_2}(x)
\]

where, \(\rho_1 > 0, 0 < \rho_2 < 1\) are arbitrary real coefficients. The finite convergence time \(T(x_0)\) is calculated as follows:

\[
T(x_0) \leq (\rho_1(1 - \rho_2))^{-1} V^{1-\rho_2}(x_0)
\]

**Lemma 2** [7]. Suppose \(0 < \kappa < 1, a_j, j = 1,\ldots,n\) are real scalars. The following inequalities hold:

\[
\sum_{i=1}^{n} |a_i|^{1+\kappa} \geq \left(\sum_{i=1}^{n} |a_i|^2\right)^{\frac{1+\kappa}{2}}
\]

\[
\sum_{i=1}^{n} |a_i|^{0.5} \geq \left(\sum_{i=1}^{n} |a_i|^{0.5}\right)
\]

**B. FTNO Design and Stability Analysis**

The nonlinear structure of the proposed observer for system (1) is as follows:

\[
\begin{cases}
\dot{\tilde{x}}_1 = \tilde{x}_2 + (a_1 + func(x_1, \tilde{x}_2))sgn(x_1 - \tilde{x}_1) \\
\dot{\tilde{x}}_2 = -\frac{\tilde{x}_2}{RC} + \frac{P_{\tilde{x}_2}}{C_{\tilde{x}_2}^2} u + a_2 sgn(x_1 - \tilde{x}_1) \\
&+ a_4 (x_1 - \tilde{x}_1) \\
&+ func(x_1, \tilde{x}_2) \\
&+ \left[(-1 + \frac{P}{RC} + a_3 + a_4 + 1) (|\tilde{x}_2 + k|)\right]
\end{cases}
\]

where \(a_j, j = 1,\ldots,5\) are optional positive numbers that fulfill the conditions \(a_3 \geq a_2 \geq a_1 \geq \tilde{x}_1, \tilde{x}_2\) are the estimated states produced by the observer (2), \(0 < \beta < 1\) is a positive constant number and \(|\tilde{y}| \leq k\). By estimating \(x_2\), the estimation of inductance current can be easily obtained. \(\gamma\) is the upper bound of matched uncertainty \(d(t,x)\).

It is noted that \(func(x_1, \tilde{x}_2)\) is a positive scalar function and this feature will be used in the stability analysis of the observer error dynamic. By ascertaining \(\tilde{x}_1 = x_1 - \hat{x}_1\) and \(\tilde{x}_2 = x_2 - \hat{x}_2\) as observer estimation errors, the observer error dynamics are obtained as:

\[
\begin{cases}
\dot{\hat{x}}_1 = \hat{x}_2 - a_3 sgn(\hat{x}_1) - a_2 \hat{x}_1 \\
&- func(x_1, \hat{x}_2)) sgn(\hat{x}_1) \\
\dot{\hat{x}}_2 = -\frac{\hat{x}_2}{RC} + \frac{P_{\hat{x}_2}}{C_{\hat{x}_2}^2} - a_2 sgn(\hat{x}_1) \\
&- a_4 \hat{x}_1 + d(t,x)
\end{cases}
\]

Theorem 1 states and proves that the above observer error dynamic is finite-time stable and it gives an FCT.

**Remark 1:** For any arbitrary vectors \(h, r \in \mathbb{R}^n\), the subsequent inequality holds which is called the Cauchy-Schwarz inequality.

\[
h^T r \leq |h|^2 \leq |h||r|
\]

**Theorem 1:** Consider observer error dynamic system (7) for the uncertain DC MG (1). The dynamic is finite-time stable and the estimated states will reach their real values in an FCT.

**Proof:** Choose a radially unbounded Lyapunov function candidate as \(V = |\tilde{x}_1| + |\tilde{x}_2|\). Its time derivative is calculated based on:

\[
\dot{V} = \frac{\tilde{x}_1 \dot{\tilde{x}}_1}{|\tilde{x}_1|} + \frac{\tilde{x}_2 \dot{\tilde{x}}_2}{|\tilde{x}_2|}
\]

By substituting \(\dot{\tilde{x}}_1\) and \(\dot{\tilde{x}}_2\) into \(\dot{V}\) from observer error dynamics (7), the pursuant equation is resulted.

\[
\dot{V} = |\tilde{x}_1|^{-1} \tilde{x}_1 (\tilde{x}_2 - a_3 sgn(\tilde{x}_1) - a_2 \tilde{x}_1 \\
&- func(x_1, \tilde{x}_2)) sgn(\tilde{x}_1)) \\
&+ |\tilde{x}_2|^{-1} \tilde{x}_2 \left(-\frac{\hat{x}_2}{RC} \\
&+ \frac{P_{\hat{x}_2}}{C_{\hat{x}_2}^2} - a_2 sgn(\tilde{x}_1) - a_4 \tilde{x}_1 \\
&+ d(t,x))
\]

Employing the Cauchy-Schwarz inequality based on Remark 1 leads to the following relation.

\[
|\tilde{x}_1|^{-1} \tilde{x}_1 \tilde{x}_2 \leq |\tilde{x}_1|^{-1}|\tilde{x}_1| |\tilde{x}_2| \leq |\tilde{x}_1|^{-1}|\tilde{x}_1||\tilde{x}_2| = |\tilde{x}_2|
\]

Term \(-a_3 |\tilde{x}_1|^{-1} \tilde{x}_1 sgn(\tilde{x}_1)\) is simplified to:

\[
-a_3 |\tilde{x}_1|^{-1} \tilde{x}_1 sgn(\tilde{x}_1) = -a_3 |\tilde{x}_1|^{-1} |\tilde{x}_1| = -a_3
\]

The next term is detracted to:

\[
-a_3 |\tilde{x}_1|^{-1} \tilde{x}_1 \tilde{x}_1 = -a_3 |\tilde{x}_1|^{-1} |\tilde{x}_1|^2 = -a_3 |\tilde{x}_1|
\]

For \(-func(x_1, \tilde{x}_2)|\tilde{x}_1|^{-1} \tilde{x}_1 sgn(\tilde{x}_1)\) one obtains:

\[
-func(x_1, \tilde{x}_2)|\tilde{x}_1|^{-1} \tilde{x}_1 sgn(\tilde{x}_1) = -func(x_1, \tilde{x}_2)
\]

It should be pointed out that \(func(x_1, \tilde{x}_2)\) is a positive scalar function.
Term $|\vec{x}_2|^{-1}\vec{x}_2 \left(-\frac{\vec{x}_2}{RC} + \frac{P \vec{x}_2}{CX_1^2}\right)$ turns into:

$$|\vec{x}_2|^{-1}\vec{x}_2 \left(-\frac{\vec{x}_2}{RC} + \frac{P \vec{x}_2}{CX_1^2}\right) = |\vec{x}_2|^{-1} \left(-\frac{1}{RC} + \frac{P}{CX_1^2}\right) \vec{x}_2$$

(15)

Applying Remark 1 for $-\alpha_2|\vec{x}_2|^{-1}\vec{x}_2 sgn(\vec{x}_1)$ results in:

$$-\alpha_2|\vec{x}_2|^{-1}\vec{x}_2 sgn(\vec{x}_1) \leq -\alpha_2|\vec{x}_2|^{-1}|\vec{x}_2| sgn(\vec{x}_1) = -\alpha_2 sgn(\vec{x}_1) \leq \alpha_2$$

(16)

The last term in $V$ that is $|\vec{x}_1|^{-1}\vec{x}_1 d(t,x)$ is converted to:

$$|\vec{x}_1|^{-1}\vec{x}_1 d(t,x) \leq |d(t,x)| \leq \gamma$$

(18)

According to (11)-(18), (10) is transformed into the subsequent inequality.

$$\dot{V} \leq |\vec{x}_2| - \alpha_1 - f unc(x_1, \vec{x}_2) - \alpha_3|\vec{x}_1| + \left[\frac{1}{RC} + \frac{P}{CX_1^2}\right]|\vec{x}_2| + \gamma + \alpha_2$$

(19)

Inserting $f unc(x_1, \vec{x}_2)$ from (6) into (19) simplifies (19) to (20).

$$\dot{V} \leq (\alpha_4 - \alpha_3)|\vec{x}_1| + \left[\frac{1}{RC} + \frac{P}{CX_1^2}\right]|\vec{x}_2| + |\vec{x}_2| - \frac{1}{RC} + \frac{P}{CX_1^2} + 1 + \alpha_3$$

(20)

Applying the triangular inequality to $|\vec{x}_2| = |x_2 - \vec{x}_2|$ and remarking that $|\dot{x}_1| \leq k$ holds, the following inequality is yielded:

$$-(k + |\vec{x}_2|) \leq -|\vec{x}_2|$$

(21)

Substituting (21) into (20) gives:

$$\dot{V} \leq (\alpha_4 - \alpha_3)|\vec{x}_1| + |\vec{x}_2| - \alpha_5|\vec{x}_1|^\beta - \alpha_5|\vec{x}_1|^\beta$$

(22)

Referring to (21), $|\vec{x}_2| \leq k + |\vec{x}_2|$ holds and the pursuant inequality is resulted:

$$-(k + |\vec{x}_2|) \leq -|\vec{x}_2|^\beta$$

(23)

Combining (22) and (23) leads to:

$$\dot{V} \leq (\alpha_4 - \alpha_3)(|\vec{x}_1| + |\vec{x}_2|) - \alpha_5|\vec{x}_1|^\beta + |\vec{x}_2|^\beta$$

(24)

According to Lemma 2, (25) is extracted.

$$(|\vec{x}_1| + |\vec{x}_2|)^\beta \leq |\vec{x}_1|^\beta + |\vec{x}_2|^\beta$$

(25)

Substituting (25) into (24) leads to (26).

$$\dot{V} \leq (\alpha_4 - \alpha_3)(|\vec{x}_1| + |\vec{x}_2|) = \alpha_5(\vec{x}_1 + |\vec{x}_2|)^\beta$$

(26)

Since the condition $\alpha_3 \geq \alpha_4$ is fulfilled, (26) is converted into (27).

$$\dot{V} \leq -\alpha_5(\vec{x}_1 + |\vec{x}_2|)^\beta$$

(27)

By determining the parameters $\rho_1 = \alpha_5$ and $\rho_2 = \beta$ and due to the definition of the Lyapunov function candidate, the prior relationship is transformed into (28).

$$\dot{V} \leq -\rho_1 V^{\rho_2}$$

(28)

Resorting to Lemma 1, the observer error dynamic is finite-time stable with the following adjustable FCT:

$$T_{obs} \leq \frac{1}{\alpha_5(1-\beta)} \times (|\vec{x}_1(0)| + |\vec{x}_2(0)|)^{1-\beta}$$

(29)

Thereby, the finite-time stability of the observer error dynamic is satisfied which means that $\vec{x}_1$ and $\vec{x}_2$ reach to zero in a specific finite time (i.e., $T_{obs}$). In other words, the estimated states will reach their real values after $T_{obs}$. Moreover, by appropriately selecting the initial conditions, the rate of change of reaching is tunable. Therefore, the proof is completed.

IV. SIMULATION RESULTS

To display and verify the effectiveness, the preciseness, and the particular FCT of the suggested observer from the standpoint of rapid estimation, simulations are employed. The parameters of the uncertain DC MG with CPL and the parameters of the designed FTNO are given in Table I and Table II.

Besides, the matched uncertainties entered into the system is considered as $d(t,x) = 0.1 \sin(t)$ which its absolute value is upper bounded by $\gamma = 0.167$.

The estimations and real values of the DC MG’s inductance current and capacitor voltage are represented in Fig. 4 and Fig. 5, respectively.

It is apparent from Fig. 4 and Fig. 5 that the estimated values of inductance current and capacitor voltage reach their real values in the finite time.

| Table I. Parameter Values of the DC MG with CPL |
|---------------------------------|-----------------|-----------------|
| Parameters | Values | Parameters | Values |
| $L$ | 0.003 H | $R$ | 30 $\Omega$ |
| $C$ | 0.0005 F | $P$ | 300 W |
| $V_e$ | 250 V |
Table II. Parameter values of the FTNO.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.1</td>
<td>$\alpha_5$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>1.0</td>
<td>$k$</td>
<td>10</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>50</td>
<td>$\beta$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>49</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. The inductance current estimation and its actual value of the DC MG with CPL.

Fig. 5. The capacitor voltage estimation and its actual value of the DC MG with CPL.

V. CONCLUSIONS

Due to the key role of DC MGs in renewable energy resources, a novel nonlinear observer is designed in this paper for uncertain DC MGs feeding linear resistive loads, CPLs, a buck converter, and an input DC voltage source to estimate unknown and unmeasurable inductance current in a specific adjustable finite estimation time. This leads to the elimination of ripples produced by physically measuring the inductance current, the deletion of sensor costs, and the reduction of noise effects. The proposed method is robust against external disturbances, unmodeled dynamics, and perturbations entered into the systems. The finite-time stability of the observer error dynamic is ensured by a radially unbounded Lyapunov function candidate and a tunable FCT is derived to give the ability to increase the rate of change of estimation time by properly adjusting the parameters existing in this time. Numerical simulations demonstrate the effectiveness of the suggested FTNO. It is shown that the estimated value of the inductance current reaches its real value in some seconds.

REFERENCES