Abstract—Electricity has been an integral part of public transport systems and it has become a strategic priority due to the versatile potential of electric vehicles (EVs) as distributed energy resources (DERs). Despite the logistic ease of transitioning conventional public transport systems to fully-electric ones, the latter must be properly designed to adhere to technical constraints and minimize their total ownership cost (TOC). Most published work addresses this issue with a narrow scope, such as modeling a very specific case study, leading to non-generic models which are difficult to apply for a universal case. In this study, a mixed-integer linear programming (MILP) model is formulated to model a generic public transport network, comprising the routes, electric bus models, and charging infrastructures. The objective function minimizes the TOC of a generic network, which is applied to any universal case study. The constructed model is tested by simulating a system with different routes, bus models, and types of chargers to obtain the optimal configuration of the charging infrastructure.

Keywords— Battery Sizing, Electric buses, Mixed-Integer Linear Programming, Optimal Charging Infrastructure.

I. INTRODUCTION

A. Background and Motivation

Globally, the transport sector is simultaneously a major energy consumer and greenhouse gas emitter. With the increasing popularity of electric vehicles (EVs) as a highly versatile distributed energy resource (DER), the sector becomes a strategic priority in energy systems research and development. There have been numerous research studies aiming at harnessing the benefits of consumer-owned EVs for modern smart grids (SGs) [1]. A lesser-investigated element to consider is electric buses (EBs), which can be used to bring about techno-economic benefits in SG operation if optimized.

In the context of public transport systems, the transition to a fully electric fleet is quite easy to carry out for three main reasons. First, due to heavy usage, public transport buses are frequently replaced and thus EBs can gradually replace conventional buses in the fleet without causing any interruption. Second, public transportation schedules are largely fixed (on the short-to-medium term), and thus individual upgrades to EBs can be seamlessly performed. Third, investment stability in public transportation facilitates the acquisition of new EB technologies. In addition to the aforementioned facts, EB fleets have been shown to have a lower total ownership cost (TOC) compared to their conventional counterparts. With all this being said, the main challenge hindering the transition there to is the complexity involving designing an optimal charging infrastructure which meets the needs of the transport system and adheres to techno-economic constraints while maintaining the minimal TOC of the system.

With this being the primary motivation behind this work, a survey or recent scientific literature has been performed to identify the state-of-the-art progress on this topic.

B. State-of-the-Art

In a recent paper [2], the design of an EB transport system was optimized in terms of the fleet size and mix (with specifications of different bus types), and the charging infrastructure. The study identified that range limitation is indeed a main hurdle in electrification of public transport systems and that optimal design thereof is of paramount importance. By modeling the transport network of two European cities, a genetic algorithm (GA) was used to obtain the optimal mix of EB models and the required number of each. The objective function of the GA was formulated as the TOC.

Another study [3] utilized a GA as an optimization approach for EB-based public transport systems. In this work, the objective was to determine. A real-world transit network in China was modeled, and the objective was to determine the optimal EB scheduling and charging infrastructure in order to meet the (constraint) scheduled routes with minimal charging costs. A sensitivity analysis was used to assess the economic viability of the charging power and discharging depth (direct functions of charging infrastructure and EB schedules, respectively).

While the main focus of some studies was optimizing the EB schedules, others were concerned with optimizing the charging infrastructure, given a specified EB fleet. The previous studies [2], [3], like many others, considered only the presence of a charger at the EB depot, meaning they to return to the original depot in order to recharge. Other studies tackled this problem by considering other locations for energy storage systems (ESSs) and/or fast chargers throughout the network which can be used to charge the EBs without having to make a full trip back.

In [4], mixed-integer programming (MIP) was used to minimize the TOC of a real world transportation network of a town in the United States. The optimal deployment of fast charging stations and ESS throughout the network was achieved. Similarly, another study [5] utilized MIP to find optimal charging station planning for a transport network of a town in the United States. The optimal deployment of fast charging stations and ESS throughout the network was achieved. Similarly, another study [5] utilized MIP to find optimal charging station planning for a transport network of a town in the United States. The optimal deployment of fast charging stations and ESS throughout the network was achieved.
C. Contributions

From the conducted review of literature it was observed that the majority of previous works have either modeled a limited system (e.g. disregarding commercially available charging infrastructure options) or a particular one (e.g. constructing the mathematical model for a specific case study). In both cases, this limits or eliminates the possibility of generalizing the model for a universal transport system. The contributions of this work are summarized as follows:

- Construct a universal mathematical model for fully-electric public transportation networks.
- Formulate a mixed-integer linear programming (MILP) model to minimize the TOC of a public transport system.
- Guarantee a universal nature of the model, such that the any set of routes, buses, and charging infrastructures can be considered as input for the model.
- In this sense, the model is versatile and can be used to optimize already existing systems or design new ones due to its generic formulation.

D. Paper Organization

This manuscript is organized as follows: Section I introduced the background and motivation behind this work. In Section II, the modeling of a public transport model is performed, first by introducing all the components of the system, and then by constructing the MILP formulation to minimize the total TOC. In Section III, different routes, bus models, and types of chargers are modeled as a generic case study to obtain the optimal configuration of the charging infrastructure for each route. In Section IV, the results of the simulation for different routes are presented along with a discussion of the. Finally, the conclusions and suggestions for future work are made in Section V.

II. SYSTEM MODEL

A. Electric Bus System

In Fig. 1, a public transport system is illustrated along with its components. A generic system is comprised of the following components:

- **Depot**: The depot is where the buses are dispatched from, and is where they park and charge while they are not in service.
- **Electric Bus**: The electric buses (EBs) are the backbone of the network, traversing the routes with passengers on board. EBs have onboard batteries which are recharged at designated charging locations in the network.
- **Routes**: The routes are the paths which EBs must traverse to transport passengers. Routes are made up of bus stops and are scheduled. The scheduling can be based on a specific time at which the EB must arrive/depart from/to each spot, or a frequency for the EBs to traverse the route (e.g. 1 bus to pass by a stop every X minutes).
- **Terminals**: Terminals are usually bus stops at which several routes intersect and therefore have an allocated area and infrastructure for use by the EBs.
- **Charging Infrastructure**: The charging infrastructure provides the energy needs of the system. The chargers where buses can recharge their batteries can be off-route (e.g. at depots) or on-route (e.g. at terminals).

More details regarding the charging infrastructures, EBs, and the routes must be discussed and elaborated before the MILP optimization model is formulated for the TOC of the system.

1) **Charging Infrastructure**

As illustrated in Fig 1, three main types of chargers exist. The first type is the depot charger (DC), which is typically used to charge the buses during the time when they are out of service and parked at the depot (off-route). DCs typically have rated powers ranging from 50 kW to 100 kW, intended for slow charging of the batteries overnight or while they are out of service. The second type of chargers is the terminal charger (TC). As the name suggests, a TC is typically installed for on-route charging at terminals, with a rated power ranging from 500 kW to 600 kW. The third type is the flash charger (FC), which is used for on-route fast charging at any stop with a rated power ranging from 400 kW to 500 kW.
Typically, on-route chargers are associated with much higher (an order of magnitude) capital costs than depot chargers. The investment is justified by their fast charging rates, which allow EBs to charge on-route, decreasing the parking time at the depot, and thereby minimizing the number of idle buses in the network and total investment in batteries. This is one of the trade-offs which upholds the need for an optimization model for designing the charging infrastructure.

Accordingly, all three types of chargers (DC, TC, and FC) and their aforementioned technical specifications are to be considered in the current model.

2) Electric Bus Specifications

Accurate modeling of EBs is a complex task due to the complexity of the electro-mechanical physical system and the wide variety of commercially available EB models. Deriving the exact energy consumption profiles of each EB model is field of study of its own, and therefore this work builds on the conclusions of studies which have performed a thorough analysis of most widely available commercial EB models [2], [6]-[8]. These conclude that commercially available EBs can be classified into three main categories based on the average energy consumption per distance driven as shown in Table I.

Note that EBs belonging to the same category may be fitted with different battery capacities, emphasizing the importance of optimal charging infrastructure design. Due to the high initial cost of the batteries, it is crucial to minimize their cost. In order to do so, for each route, the optimal battery size must be chosen taking into consideration the remaining system costs. Most commercially available EBs are fitted with batteries with capacities ranging from 80 kWh to 320 kWh [7], [8]. As such, in the current model the battery capacity of EBs assigned to each route are modeled as a design variable for the optimization problem.

3) Bus Routes

Defining generic routes is crucial to establish an adequate framework for the optimization model. Routes can be categorized based on two key parameters [2], [9]:

- **Average distance between stops:** This parameter is an indicator of the route location. Routes within large cities or densely populated areas are associated with shorter average distances between stops compared to those in suburban areas. This is expressed as:

\[
d^s = \frac{L_r}{N_r^s - 1}
\]

Where \(d^s\) is the average distance between stops for route \(r\), \(L_r\) is the length of route \(r\), and \(N_r^s\) is the number of stops in route \(r\).

- **Average daily distance:** Considering normal operation in which an EB is assigned a specific route each day, this is expressed as:

\[
d^d = \frac{H_r}{T_r}, \quad L_r = N_r^d \cdot L_r
\]

Where \(d^d\) is the average daily distance on route \(r\), \(H_r\) is circulating hours of route \(r\) (difference between first and last bus of the day), \(T_r\) is the average duration of the route, and \(N_r^d\) is the daily number of trips in route \(r\).

Having defined the key parameters for generic routes, different scenarios can accordingly be constructed to test the optimization model.

### Table I. Classification of commercially available EBs according to average energy consumption [2], [7], [8].

<table>
<thead>
<tr>
<th>Bus Type</th>
<th>Average Consumption (kWh/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-meter</td>
<td>1.2</td>
</tr>
<tr>
<td>13-meter or articulated</td>
<td>1.8</td>
</tr>
<tr>
<td>24-meter or double articulated</td>
<td>2.2</td>
</tr>
</tbody>
</table>

B. Optimization Problem

As any optimization problem, the proposed MILP model consists of two main elements: the objective function and problem constraints, which are detailed subsequently.

1) Objective Function

The objective function to be minimized represents the TOC of the transport system and is shown in Eq. 3. Note that in the current formulation the TOC is calculated as an annual value, with charging infrastructure and battery costs being capital investments divided by the equipment lifetime and yearly operating costs:

\[
\text{annual cost} = \frac{\text{capital cost}}{\text{lifetime}} + \text{annual operating cost} \quad (3)
\]

The annual TOC is calculated as the summation of five cost terms, as shown in Eq. (4). The five cost terms, from left to right, correspond to: the annual running cost of the depot station(s), annual ownership costs of batteries for all buses in circulation, annual ownership cost of all the entire charging infrastructure, annual electricity cost for on-route charging (by TCs and FCs), and finally the annual electricity cost for off-route charging (by DCs). Each of the five terms is elaborated in Eq. (4)-(9).

The first term, \(C^\text{depot}\), is expressed in Eq. (5) as the multiplication of a binary variable \(d\) representing the existence of the depot multiplied by the annual ownership cost of running the depot. The second term, \(C^\text{batteries}\), is shown in Eq. (6) and is the summation of the battery costs of each bus \(i\), which is calculated as the capacity of each battery \(b_i\) multiplied by its annual ownership cost \(C_i^b\) per-kWh. The third term shown in Eq. (7) corresponds to the annual cost of the charging infrastructure, where \(x_{ih}\) and is a binary variable indicating the presence of a charger of type \(h\) at stop \(i\). and \(C^h\) is the annual ownership cost of a charger of type \(h\). Accordingly, \(C^\text{chargers}\) is calculated as the sum of the annual cost of all present charger types (decided by the binary variable) at each stop, and is summed for all stops.

The fourth and fifth terms in Eq. (8) and Eq. (9) correspond to the total cost of energy supplied to recharge the batteries through on-route and off-route chargers, respectively.

\[
\text{min TOC} = C^\text{depot} + C^\text{batteries} + C^\text{chargers} + C^\text{onroute} + C^\text{offroute} \quad (4)
\]

\[
C^\text{depot} = d \cdot C^d \quad (5)
\]

\[
C^\text{batteries} = \sum_{i} (b_i \cdot C_i^b) \quad (6)
\]

\[
C^\text{chargers} = \sum_{i} \sum_{h} C_i^h \cdot x_{ih} \quad (7)
\]

\[
C^\text{onroute} = \sum_{i} \sum_{j} (e_{ij} \cdot C_i^E_{ij}) \cdot d_{\text{year}} \cdot n_{\text{bus}} \quad (8)
\]

\[
C^\text{offroute} = e_{\text{last}} \cdot C_{\text{end,end}}^E \cdot d_{\text{year}} \cdot n_{\text{bus}} \quad (9)
\]
In Eq. (8), \( C^b_{ij} \) and \( e_{ij} \) are the energy charged at stop \( i \) during trip \( j \), and the corresponding cost per unit of electricity, respectively. \( d_{\text{year}} \) is the number of days in a year, set as 365, and \( n^{\text{bus}} \) is the total number of buses traversing the route. This last value can be calculated based on the two parameters of each route which were introduced in Eq. (1) and Eq. (2), as shown below:

\[
n^{\text{bus}} = \frac{H_k}{P^b_{i} \cdot N_{k}}
\]

Where \( P^b_{i} \) is the frequency of buses is route \( i \) and the other variables have been defined in Section II.A.2. In Eq. (9), the final term of the TOC objective function is shown: the cost of electricity charged off-route (while the EBs parked or are not in service). In this equation, \( C^b_{\text{end-end}} \) and \( e_{\text{last}} \) correspond to the energy charged at the end of the route (i.e., off-route), respectively. It is important to note that in this formulation, the last stop in a bus schedule corresponds to the depot. However, this does not dictate the presence of a charger at the depot (DC), which is a decision variable dependent on the binary variable \( d \).

2) Constraints

The constraints of the optimization problem can be divided into four groups:

- **Infrastructure Constraints:**

  The first constraint is associated with the charging infrastructure, and guarantees that at each stop there is only one type of charger installed (based on the binary decision variable \( h \) which was previously introduced), as represented in Eq. (11).

\[
\sum_{h \in H} x_{ih} \leq 1, \quad \forall i \in I
\]

- **Battery Constraints:**

  The second set of constraints are associated with the batteries onboard the EBs, and are represented by Eq. (12)-(14). To protect the health of the batteries, for each bus \( k \), the battery State-of-Charge (SoC) must be within the upper and lower bounds \( \bar{b} \) and \( \underline{b} \), as set by Eq. (12) and Eq. (13), respectively. Eq. (14) sets the SoC boundary conditions to be at the maximum value (i.e., the EB starts from the depot with full charge).

\[
\begin{align*}
E_k &\leq \bar{b} \cdot b, \quad \forall k \in K \\
E_k &\geq \underline{b} \cdot b, \quad \forall k \in K \\
E_1 &\equiv E_{\text{end}} = \bar{b} \cdot b
\end{align*}
\]

- **Charged Energy Constraints:**

  The third set of constraints in Eq. (15)-(20) are related to the energy exchange between the EBs and the charging infrastructure. Eq. (15) ensures that energy can only be injected from the electrical grid to the EBs through the chargers and not vice-versa. This constraint can easily be modified or removed in case bi-directional energy flow with the power grid is possible and to be considered. Eq. (16) dictates that if there is no charger installed at a stop \( x_{ih} = 0 \), then the energy exchanged at that stop must be equal to zero \( (e_{ij} = 0) \). Eq. (17) and Eq. (18) set the charging power according to the charger type installed at a stop \( x_{i1r}, x_{i2r}, \text{etc.} \). As previously explained, the two on-route charger types correspond to TC and FC.

\[
\begin{align*}
\sum_{h \in H} x_{ih} = 0 &\Rightarrow e_{ij} = 0, \quad \forall i \in I, \quad \forall j \in J \\
x_{i1} = 1 &\Rightarrow e_{ij} \leq E_{ij}, \quad \forall i \in I, \quad \forall j \in J \\
x_{i2} = 1 &\Rightarrow e_{ij} \leq E_{ij}, \quad \forall i \in I, \quad \forall j \in J \\
d = 1 &\Rightarrow e_{\text{last}} \leq \bar{b}
\end{align*}
\]

\[d = 0 \Rightarrow e_{\text{last}} \leq E_{i1}\]

Finally, Eq. (19) and Eq. (20) define the maximum energy charged at the last stop, depending on the existence or not of a depot charger (binary decision variable \( d \)). As it can be seen, in case there is no DC, the maximum energy charged is limited by the rated power of the first terminal charger, hence the condition \( e_{\text{last}} \leq F_{i1} \).

- **Energy Balance Constraints:**

  The final constraint in Eq. (21) is associated with the total energy balance of the system, such that the total SoC consumed by all buses is equal to the total SoC charged. The equation is applied for each bus in the network, such that \( E_k \) (the SoC of the bus at stop \( k \)) is equal to the SoC at the previous stop \( E_{k-1} \) minus the energy charged at the current stop \( (e^{\text{bus}}_k) \), plus the energy charged at the final stop.

\[
E_k = E_{k-1} - e^{\text{bus}}_k + [e_{ij} - e_{\text{last}}], \quad \forall k \in K, k \neq 0
\]

C. Computational Implementation

The proposed MILP formulation was modeled on MATLAB using the YALMIP package, and the solution was obtained using the Gurobi solver.

III. CASE STUDY

In order to test the proposed mathematical formulation, three generic routes are used, as detailed in Table II. Based on length of the route, different bus sizes are needed for each route, whose specifications are in accordance with Table I. The techno-economic specifications of the available chargers to choose from and the batteries are provided in Tables III and IV, respectively. The latter are constrained between 80 kWh and 320 kWh with 20 kWh increments. In this study, \( n^{\text{bus}} \) is set to unity corresponding to one EB dispatched to each route.

<table>
<thead>
<tr>
<th>Specifications of Routes Used for the Case Study.</th>
<th>Route A</th>
<th>Route B</th>
<th>Route C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Trips (per day)</td>
<td>5</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Number of Stops (per trip)</td>
<td>75</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>Total Number of Stops (per day)</td>
<td>375</td>
<td>900</td>
<td>1200</td>
</tr>
<tr>
<td>Trip Length (km)</td>
<td>20</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Bus Size (m)</td>
<td>18</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>Average Consumption (kWh/km)</td>
<td>1.8</td>
<td>1.8</td>
<td>2.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specifications of Techno-Economic Specifications of Chargers.</th>
<th>DC</th>
<th>TC</th>
<th>FC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charger Classification</td>
<td>Depot</td>
<td>On-Route</td>
<td>On-Route</td>
</tr>
<tr>
<td>Speed Model</td>
<td>Standard</td>
<td>Slow</td>
<td>Fast</td>
</tr>
<tr>
<td>Rated Power (kW)</td>
<td>50</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Capital Cost (EUR)</td>
<td>273k</td>
<td>278k</td>
<td>278k</td>
</tr>
<tr>
<td>Operating Cost (EUR/year)</td>
<td>100</td>
<td>2k</td>
<td>2k</td>
</tr>
<tr>
<td>Lifetime (years)</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

| Specifications of Techno-Economic Specifications of Batteries. | |
|---|---|---|---|
| Capital Cost (EUR/kWh) | 250 | |
| Operating Cost (EUR/year) | - | |
| Battery Lifetime (years) | 5 | |
| State-of-Charge Upper Boundary (%) | 90 | |
| State-of-Charge Lower Boundary (%) | 10 | |
IV. RESULTS

A. Optimal Charger Deployment and Battery Sizing

The result for the optimal charger deployment in Route A is shown in Fig. 2. As can be seen, only the depot charger with a 50 kW power rating is sufficient to sustain the energy demand of the EB throughout its 5 cycles of the route per day. The result of the optimal battery capacity was 260 kWh. In Fig. 3, one can see that a full charge at the depot can sustain the full daily cycle of the route by the EB before reaching the minimum bound of 10% SoC.

With Route B being significantly longer (threefold the distance of Route A), investment in a higher charging power was necessary. In Fig. 4, the optimal deployment is shown to be that of one 600 kW TC to sustain the route. With this, only an 80 kWh battery is needed. As such, the optimal solution as here as opposed to Route A consisted of investing in a more powerful charger while saving the costs by using smaller batteries on the deployed EB. The optimal charging schedule is shown in Fig. 5, where it can be seen that the EB occasionally stops at charges at the TC to recharge its battery throughout the day, guaranteeing a full SoC at the end of the route for its next deployment.

For Route C (the longest of the three), the optimal charger configuration consisted of both a 600 kW TC and a 50 kW DC (as shown in Fig. 6), with a medium-sized 200 kWh battery capacity for deployed EBs. The SoC variation throughout the day shows that the EB stops to recharge its battery every cycle of the route, gradually decreasing the SoC at the end of every cycle. Finally, at the end of the day, the bus is recharged at the depot to reach a full SoC for its next deployment.
TABLE V. OPTIMAL DESIGN AND COST FOR THE CASE STUDY ROUTES.

<table>
<thead>
<tr>
<th>Route</th>
<th>Optimal Battery Size (kWh)</th>
<th>Number of DC (50 kW)</th>
<th>Number of TC (500 kW)</th>
<th>Number of TC (600 kW)</th>
<th>Number of FC (400 kW)</th>
<th>Number of FC (500 kW)</th>
<th>Cost of Chargers (EUR/year)</th>
<th>Cost of Batteries (EUR/year)</th>
<th>Cost of Electricity (EUR/year)</th>
<th>Annual TOC (EUR/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route A</td>
<td>260</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>5000</td>
<td>13000</td>
<td>3918</td>
<td>21918</td>
</tr>
<tr>
<td>Route B</td>
<td>80</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>16000</td>
<td>4000</td>
<td>11181</td>
<td>31181</td>
</tr>
<tr>
<td>Route C</td>
<td>200</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>21000</td>
<td>10000</td>
<td>18614</td>
<td>49614</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

In this study, a mathematical model for fully-electric public transportation networks was formulated, and MILP optimization was implemented to minimize the TOC of a public transport system. The generic nature of the model was guaranteed by allowing the consideration of any set of routes, different EB models, battery capacities, and different charging technologies as input for the model. In this sense, the model is versatile and can be used to optimize already existing systems or design new ones due to its generic formulation. The mathematical formulation was tested by simulating three routes of different lengths and frequencies in to determine the optimal design of charging infrastructure and battery sizing of each. The results show the significant effect of the route properties on the design choices which minimize the TOC. For future work, it is recommended to employ and test this model on intersecting routes to account for possible congestions at stops, which was not considered in this case study.

REFERENCES