New Multi-Stage and Stochastic Mathematical Model for Maximizing RES Hosting Capacity—Part I: Problem Formulation

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Abstract—This two-part work presents a new multi-stage and stochastic mathematical model, developed to support the decision-making process of planning distribution network systems (DNSs) for integrating large-scale “clean” energy sources. Part I is devoted to the theoretical aspects and mathematical formulations in a comprehensive manner. The proposed model, formulated from the system operator’s viewpoint, determines the optimal sizing, timing and placement of distributed energy technologies (particularly, renewables) in coordination with energy storage systems and reactive power sources. The ultimate goal of this optimization work is to maximize the size of renewable power absorbed by the system while maintaining the required/standard levels of power quality and system stability at a minimum possible cost. From the methodological perspective, the entire problem is formulated as a mixed integer linear programming optimization, allowing one to obtain an exact solution within a finite simulation time. Moreover, it employs a linearized AC network model which captures the inherent characteristics of electric networks, and balances well accuracy with computational burden. The IEEE 41-bus radial DNS is used to test validity and efficiency of the proposed model, and carry out the required analysis from the standpoint of the objectives set. Numerical results are presented and discussed in Part II of this paper to unequivocally demonstrate the merits of the model.

Index Terms—Distributed generation, distribution network systems, energy storage systems, integrated planning, stochastic programming, variability and uncertainty.

I. NOMENCLATURE

a) Sets/Indices

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Ω</td>
<td>Index/set of substations</td>
</tr>
<tr>
<td>C</td>
<td>Cost of branch i-j (€)</td>
</tr>
<tr>
<td>e</td>
<td>Energy storage limits (MWh)</td>
</tr>
<tr>
<td>emax</td>
<td>Emission rates of new and existing DGs (€/MWh)</td>
</tr>
<tr>
<td>g</td>
<td>Investment cost of DG, line, energy storage system and capacitor banks, respectively</td>
</tr>
<tr>
<td>L</td>
<td>Lifetimes of DG, line, energy storage system and capacitor banks, respectively (years)</td>
</tr>
<tr>
<td>MC</td>
<td>Maintenance cost of capacitor bank and energy storage system per year (€)</td>
</tr>
<tr>
<td>MGN</td>
<td>Maintenance costs of new and existing DGs per year (€)</td>
</tr>
<tr>
<td>MK</td>
<td>Maintenance costs of new and existing branch k per year (€)</td>
</tr>
<tr>
<td>N</td>
<td>Number of buses and substations, respectively</td>
</tr>
<tr>
<td>OC</td>
<td>Operation cost of unit energy production by new and existing DGs (€/MWh)</td>
</tr>
<tr>
<td>pchmax</td>
<td>Charging and discharging power limits of a storage system (MW)</td>
</tr>
<tr>
<td>psoilh</td>
<td>Hourly solar PV output (MW)</td>
</tr>
<tr>
<td>Pr</td>
<td>Rated power of a DG unit (MW)</td>
</tr>
<tr>
<td>Pwindh</td>
<td>Hourly wind power output (MW)</td>
</tr>
<tr>
<td>Q</td>
<td>Rating of minimum capacitor bank</td>
</tr>
<tr>
<td>R</td>
<td>A certain radiation point (often taken to be 150 W/m²)</td>
</tr>
<tr>
<td>Rh</td>
<td>Hourly solar radiation (W/m²)</td>
</tr>
<tr>
<td>Tk</td>
<td>Resistance and reactance of branch k, respectively</td>
</tr>
<tr>
<td>B</td>
<td>Solar radiation in standard condition (usually set to 1000 W/m²)</td>
</tr>
<tr>
<td>V</td>
<td>Cut-in wind speed (m/s)</td>
</tr>
<tr>
<td>Vc</td>
<td>Cut-out wind speed (m/s)</td>
</tr>
<tr>
<td>Vnom</td>
<td>Observed/sampled hourly wind speed (m/s)</td>
</tr>
<tr>
<td>Vt</td>
<td>Nominal voltage (kV)</td>
</tr>
<tr>
<td>Z</td>
<td>Impedance of branch i-j (Ω)</td>
</tr>
<tr>
<td>α,β</td>
<td>Slopes of linear segments</td>
</tr>
</tbody>
</table>

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Motivation and Aims

NOWADAYS, the issue of integrating distributed generations (DGs) (renewable DGs, in particular) is globally gaining momentum because of several techno-economic and environmental factors. Since recent years, the size of DGs integrated into distribution systems has been increasing. This trend is more likely to continue in the years to come because it is now widely accepted that DGs bring wide-range benefits to the system. However, given the current set-up of distribution networks (which are generally passive), large-scale DG integration is not technically possible because this brings about tremendous challenges to the system operation, especially in undermining the power system quality and stability.

Such challenges’ limitations are expected to be alleviated when distribution networks undergo the anticipated evolutionary process from passive to active networks or smart grids. This transition is expected to result in a system that is adequately equipped with appropriate technologies, state-of-the-art solutions and a new operational philosophy that is totally different from the current ‘fit and forget’ approach. This is expected to offer sufficient flexibility and control mechanism in the system. Nevertheless, the process is not straightforward as it demands exceptionally huge investments in smart-grid technologies and concepts to fully automate the system, and this should be accompanied by a new operational philosophy. Therefore, the whole transformation process (i.e., the transformation of current distribution systems to full-scale smart-grids) might be very slow, and its realization might take several decades.

However, given the techno-economic factors and global concerns about environmental issues, the integration of renewable energy sources (RESs) cannot be postponed. It is likely that the integration of DGs in distribution systems will go ahead along with smart-grid enabling technologies that have the capability to alleviate the negative consequences of large-scale integration of DGs. In other words, in order to facilitate (speed up) the much-needed transformation of conventional (passive) DNSs and support large-scale RES integration, different smart-grid enabling technologies such as reactive power sources, advanced switching and storage devices are expected to be massively deployed in the near term.

To this end, developing strategies, methods and tools to maximize the penetration level of DGs (particularly, RESs) has become very crucial to guide such a complex decision-making process.

In this respect, this work focuses on the development of multi-stage mathematical models to determine the optimal sizing, timing and placement of energy storage systems and reactive power sources as well as that of RESs in distribution networks. The ultimate goal of this optimization work is to maximize the RES power absorbed by the system at a minimum cost while maintaining the power quality and stability at the required/standard levels.

The problem is formulated from the system perspective (i.e. in centralized planning framework). In a deregulated environment and from the smart-grid context where the current regulatory and technical challenges are expected to be fully resolved, planning will most likely involve distributed decision-making processes. One may rightfully argue that the investment decisions obtained from a coordinated planning model may not be implemented in reality. This is because distributed decisions often lead to sub-optimal solutions,
which may be different from the ones obtained by the centralized planning model. However, this does not mean that the outcomes of the coordinated planning model cannot be used. For instance, these outcomes can be regarded as the best investment targets. Given these targets, distributed decisions (solutions) can be systematically made to approach one or more of these targets (for example, via incentive or market-based mechanisms). From another perspective, in the absence of “attractive” market environment (seen from the private investors), distribution network systems may not see significant breakthrough when it comes to investments in DGs and energy storage systems (ESSs). In this case, DSOs may, instead, be given additional roles and responsibilities that include investing in DGs and ESSs in coordination with network investments. DSOs may also oversee investments in DGs and ESSs. In addition, DSOs may also be required to manage these assets in a coordinated manner to keep the system integrity, stability and power quality at the required/standard levels. Another issue which explains the versatility of the coordinated approach is related to the realization of smart-grids. Even if there is a general consensus on the smartification of power systems (distribution networks, in particular), and there are signs that some systems are evolving into smart-grids, the whole process is going to probably take very long time. Based on the aforementioned reasons, a centralized (coordinated) approach of the planning problem can provide vital solutions.

B) State-of-the-art Literature Review and Background

Reducing fossil fuel dependence and mitigating climate change have led to an increased pressure to change the current generation paradigm. The compounded effect of increasing demand for electricity, environmental and climate change concerns is triggering a policy shift all over the world, especially when it comes to energy production. Integration of DG, particularly, RESs, in electric distribution network systems is gaining momentum. In particular, the recent developments in a climate change conference held in Paris (COP21) are expected to accelerate renewable integration. It is highly expected that large-scale DG integration will be one of the solutions capable of mitigating the aforementioned problems and overcoming the challenges. Because of this, Governments of various nations have introduced targets to achieve large-scale integration of DGs. In particular, in the European Union, which strongly advocates the importance of integrating renewables, RESs are expected to cover 20% and 50% of the overall energy consumption by 2020 and 2050, respectively.

DGs offer a more environmentally friendly option through great opportunities with renewable-enabling technologies such as wind, photovoltaic, biomass, etc. RESs are abundant in nature, which, under a favorable RES integration policy and incentive mechanism, makes it attractive for the large-scale power generation sector. Nevertheless, there is no rule or partial rule on the DG unit’s connection; typically, these are traditionally connected at the end of radial feeder systems or nodes with greater load on the distribution system. This is realized often with several impeding restrictions put in place to alleviate the negative consequences of integrating DGs in the system.

The optimal and dynamic planning of the DG placement and sizing is becoming extremely important for energy producers, consumers and network operators in technical and economic terms. There are many studies in the literature on this topic, yet most of them only consider the optimal location of a single DG unit or do not consider simultaneously positioning and sizing RES units, mainly due to their high dispatch unpredictability. The increase in DG penetration increases the uncertainty and the fluctuations of power production. If the placement and proper sizing is not taken into account, the benefits of DG integration can be lost, leading to inefficient operation and increasing the electricity cost and energy losses.

Another major concern with the wide DG penetration is system reliability. The penetration of distributed systems can result in the degradation of power quality, particularly in cases of slightly meshed networks [1] or microgrids. In this paradigm, the use of ESS has been seen as one of the viable options to mitigate the aforementioned concerns.

The DG allocation and sizing subject have attracted special interest from researchers in recent years. An excellent review of previous works related to this subject area, published prior to the year 2013, is presented in [2].

In [2] and [3], an analysis of several techniques used on the DG impact assessment in the electrical system is presented. Most of these techniques analyze the distribution system to determine rules that can be used for DG integration [4]–[8]. Important issues related to the connection of DG units are the network topology, DG capacity and suitable location; because, each bus in the system has an optimal level of DG integration. If the value surpasses this level, system losses can increase [9], [10].

Recently, several methods have been proposed for planning and operation or in some cases for both location and sizing of DGs in the distribution system. In general, these methods can be classified as heuristic [11]–[26], numerical [27]–[34] and analytical [35], [36] based methods. Heuristic based methods apply advanced artificial intelligence algorithms, such as genetic algorithms (GA) [11]–[14], particle swarm optimization (PSO) [15]–[19], harmony search (HS) [20], [21] and big bang crunch (BBC) [22]–[24].

Numerical methods are algorithms that seek numerical results for different problems in particular to the problem in question. Some of the most recently works use nonlinear mixed integer programming (MINLP) [27]–[29], mixed integer linear programming (MILP) [30], [31], quadratic programming (QP) [32] and AC optimal power flow (OPF)-based [33].

The exhaustive search methods seek the optimal DG location for a given DG size under different load models. Therefore, these methods fail to represent accurately the behavior of the DG optimization problem involving two discrete variables, both for optimum DG size and optimal DG location. In [35], authors present a technique with a probabilistic basis for determining the capacity and optimal placement of wind DG units to minimize energy loss in the distribution system. A sensitivity analysis is presented in [36] for DG placement and sizing in the network.

Despite many studies in the literature on areas related to DG placement and sizing problem, most of them only consider the optimal location of a single DG unit, mostly of conventional DGs. The simultaneous consideration of placement, timing and sizing of DG units (especially RESs), along with the placement, timing and sizing of smart-grid enabling technologies, seems to be far from being addressed in the literature. The increase in RES-based DG penetration increases the uncertainty and the fluctuations of the system production. If the placement and proper sizing is not taken into account, the benefits of DG integration may not be exploited; instead, this may result in the degradation of system efficiency, increased cost of electricity and energy losses. Another major concern with the wide-range DG penetration is system reliability. However, the simultaneous investment planning of DGs, ESSs and
reactive power sources is expected to significantly alleviate these challenges and increase the penetration level of RES-based DGs.

C) Contributions
The novel contributions of this work are threefold:

- A new multi-stage and stochastic optimization model is proposed, which simultaneously considers the integration of smart-grid enabling technologies such as ESSs, reactive power sources, and network reinforcements to support large-scale RES-based DG integration. To the best of the authors’ knowledge, this is the first time such a joint optimization framework is proposed to such a problem.

- A MILP programming is proposed based on a linearized AC network model which captures well the inherent characteristics of electric networks. To the best of the authors’ knowledge, this network model has not been used in the literature related to the distribution network systems.

- The solution framework simultaneously finds the optimal sizing, time and placement of DGs, ESSs and reactive power sources in distribution networks, which is another innovative step forward.

D) Paper Organization

The rest of this paper is organized as follows. Section III presents a brief description of how the uncertainty and the variability pertaining to variable energy sources and electricity demand are handled in the planning process. The mathematical formulation and detailed description of the proposed model are presented in Section IV. The last section draws some conclusions.

III. UNCERTAINTY AND VARIABILITY MANAGEMENT

A. Description of Terminologies

The terminologies uncertainty and variability are sometimes incorrectly used interchangeably in the literature despite the fact that they are different. Variability, as defined in [37], refers to the natural variation in time of a specific uncertain parameter, whereas uncertainty refers to “the degree of precision with which the parameter is measured” or predicted. We follow these terminologies in our paper when referring to operational variability and uncertainty, which are introduced by model parameters. For example, wind power output is characterized by both phenomena; its hourly variation corresponds to the variability while its partial unpredictability (i.e. the error introduced in predicting the wind power output) is related to uncertainty. The schematic illustration in Fig. 1 clearly distinguishes both terminologies. As illustrated in this figure, the hourly differences in wind power outputs are due to the natural variability of primary energy source (wind speed); whereas, the likelihood of having different power outputs at a given hour is a result of uncertainty (partial unpredictability) in wind speed.

Other terminologies used in this paper are snapshot, scenario and time stage. A snapshot refers to an hourly operational situation. Alternatively, it can be understood as a demand—generation pattern at a given hour. A scenario, on the other hand, denotes the evolution of an uncertain parameter over a given time horizon (often yearly). For example, the hourly variations of wind power production and electricity consumption collectively form a group of snapshots; whereas, the annual demand growth (which is subject to uncertainty) and RES power output uncertainty are represented by a number of possible storylines (scenarios) [38]. Time stage (also referred to as decision stage) stands for the yearly decision stages throughout the planning horizon. The length of planning horizon in the present work three years, which is divided into yearly decision stages.

B. Uncertainty and Variability

There are various sources of uncertainty and variability in a distribution systems planning problem, particularly with intermittent renewable sources. These are related to the variability in time and the randomness of operational situations [39]. In addition, there are other uncertainties mostly related to the long-term electricity, carbon and fuel prices, rules, regulations and policies, etc. To account for uncertainty in demand, two demand scenarios are formed by assuming a ±5% prediction error margin from a long real-life demand profile (which is 8760 long). This gives a total of three demand profile scenarios, which are kept the same throughout the planning horizon. Due to the lack of sufficient historical data, first, synthetic hourly wind speed and solar radiation series (20 for each) are generated using the methods in [1] and [2], respectively considering the autocorrelations and diurnal patterns of each parameter. Then, the average wind speed and solar radiation profiles are determined from the generated series. The power outputs corresponding to each wind speed and solar radiation are then determined by plugging in these values in the respective power curves given by equations (1) and (2). Note that these power output samples (snapshots) cannot be used directly in the planning process because the resulting wind power output profiles may not respect the natural correlations that exist between them and the average electricity demand profile. These samples should be readjusted to reflect the temporal correlations that naturally exist among demand, solar radiation and wind speed series. To this end, the correlation between wind and solar sources is considered to be -0.3 while that of wind and demand is 0.28, which is in line with the results in [40]. A correlation of 0.5 is assumed between solar and demand, according to [41].

Given this desired correlation matrix, the wind and the solar power output profiles can be transformed into new ones, respecting the correlation among them. Such adjustments in the correlation of data series are performed using Cholesky factorization, a method used for generating correlated random variables. The method works as follows. Given a desired correlation matrix \( R \), and uncorrelated data \( D \), a new data \( Z \) whose correlation matrix is \( R \) can be generated by simply multiplying the Cholesky decomposition of \( R \) by \( D \).

Then, the hourly wind and solar power output are determined by plugging in these readjusted series into their corresponding power curves given by equations (1) and (2).

\[
P_{wind,n} = \begin{cases} 
0 & ; 0 \leq v_{n} \leq v_{cl} \\
\frac{P_{r} (A + B v_{n})}{P_{r}} & ; v_{cl} \leq v_{n} \leq v_{r} \\
0 & ; v_{r} \leq v_{n} \leq v_{co}
\end{cases} 
\]  

\[
P_{solar,n} = \begin{cases} 
0 & ; 0 \leq v_{n} \leq v_{co} \\
\frac{P_{r} (A + B v_{n})}{P_{r}} & ; v_{co} \leq v_{n} \leq v_{cl}
\end{cases} 
\]  

Fig. 1. Illustration of variability and uncertainty in wind power output.
In the above equation, A and B are parameters represented by the expressions in [42] and [43]. Similarly, the hourly solar power output $P_{\text{sol},h}$ is determined by plugging in the hourly solar radiation levels in the solar power output expression given in (2) [44].

$$
P_{\text{sol},h} = \begin{cases} 
\frac{R_h}{R_{\text{max}}+R_h} ; & 0 \leq R_h \leq R_e \\
\frac{R_h}{R_{\text{max}}} ; & R_e \leq R_h \leq R_{\text{std}} \\
P_r ; & R_h \geq R_{\text{std}} 
\end{cases}
$$

(2)

In the present work, uncertainty in wind speed and solar radiation is assumed to lead to a ±15% deviation on average from the corresponding average power output profiles. This approximately translates to ±5% forecasting error in wind speed or solar radiation. Note that such error or higher is induced even by the most advanced forecasting tools available today. Based on the assumptions, two hourly profiles of wind power output are generated by considering the ±15% margin (one above the average and one below the average profile). In total, this leads to 3 wind power output profiles (including the average profile). These are defined as wind scenarios. Similarly, to account for the uncertainty in solar power outputs, two profiles are generated for each by considering a ±15% uncertainty margin. The two generated solar power output profiles along with the average profile form the set of solar scenarios. An illustration of uncertainty characterization of wind and solar power outputs is shown in Fig. 2.

Note that each of these scenarios has 8760 snapshots of demand, wind and solar power outputs. These individual scenarios are combined to form a set of 27 scenarios (3x3x3), which are used in the analysis. These scenarios are assumed to be equally probable; hence, the probability of realization of each scenario $p_\ell$ is given by 1/27. To ensure problem tractability, the multi-dimensional input data (27x8760) is clustered into 27x200 groups via a standard clustering technique (k-means algorithm [45], which has been applied in investment planning problems as in [46]). Here, each cluster represents a group of similar operational situations. A representative snapshot, the medoid in this case, is then selected from each cluster. A weight is assigned to each representative snapshot, which is proportional to the number of operational situations in its group. Note that while clustering such a large number of operational situations in this manner is critically important to guarantee problem tractability, the chronological orders (and by implication the autocorrelations) of the considered data (time series) are not unfortunately preserved in the reduced number of operational situations. In other words, such information is lost during the clustering process. In the context of medium- to long-term planning problems, the impact of such information loss on the planning outcome may not be significant. However, if this is a concern, the chronological information can be somehow recovered by methods as in [47]. A comprehensive analysis on issues related to this will be addressed in future works by the authors.

IV. MATHEMATICAL FORMULATION OF THE PROBLEM

A. Brief Description of the Problem

As mentioned earlier, this work develops an integrated optimization model that simultaneously finds the optimal locations and sizes of installed DG power (particularly, focusing on wind and solar), energy storage systems and reactive power sources such as capacitor banks. The optimal deployment of the aforementioned enabling technologies should inherently meet the goal of maximizing the renewable power integration/absorbed into the system. The entire model is formulated as a stochastic mixed integer linear programming (SMILP) optimization. In addition, instead of the customary direct current (DC) network models, a linearized AC model is used here to better capture the inherent characteristics of the network system. An ideal representation of the network system would be to use the full AC network model. However, embedding this model in planning problems is computationally unaffordable. Because of their appealing computational performances, DC based optimization models are commonly used in distribution systems planning problems. However, the DC network model does not consider reactive power flows, which can have significant impact on the planning solutions, such as investment decisions related to capacitor banks. In addition, voltage magnitudes are often considered to be the same throughout the system. Voltage angles are also sometimes neglected in DC models. All these simplifying assumptions may result in sub-optimal solutions (often underinvestment) when embedded in planning models. The linearized AC model on the contrary acknowledges the presence of both active and reactive power flows, voltage magnitude and angle differences among nodes in the system. The model captures the physical characteristics of the network system in a better way when compared to DC network models, and very close to the ideal AC network model. Computationally speaking, the linearized AC model is more expensive than the DC network model because it involves a more detailed network representation than the DC model. However, embedding the linearized AC model in planning problems yields far better solutions than when using the DC one. Hence, the linearized AC model can be generally regarded as a bridge between the DC and the AC network models. Planning models based on this linearized AC model are tractable enough, and the results are accurate enough.

The schematic representation in Fig. 3 illustrates the multi-stage and multi-scenario modeling framework and the expansion solution structure (i.e. $X^\ell$, where $X^\ell$ represents the solution vectors of several investment variables). At each stage of the planning horizon, we obtain a single investment solution which is good enough for all scenarios [38], [48]. Note that while operational variables depend on each scenario and snapshot, the investment decision variables only depend on the time stage index. This means that the investment solution obtained should satisfy all conditions in every scenario, making the solution robust against any realization of the considered scenarios. It should be noted here that the robustness of the solution is directly related with the level of details of uncertainty and
variability characterization. Generally, the higher the numbers of snapshots and scenarios considered are, the more robust the solution is. However, there is always a threshold beyond which adding more snapshots and scenarios does not significantly change the solution but increases unnecessary computational burden. If the scenarios considered in the planning are carefully selected to be representative enough of all possible uncertainty realizations, then, the robustness and reliability of the solution can be more guaranteed.

The length of the planning horizon in the present work is assumed to be three years, which is then divided into yearly decision stages. In each stage, investment decisions related to DGs, ESSs, capacitor banks and lines are made. These decisions can be regarded as here-and-now because such decisions are independent of any scenario or snapshot. However, operation variables (such as actual power productions, storage level, power flows, etc.) depend on scenarios and snapshots, as well as decision stages.

B. Objective Function

As mentioned earlier, the objective of this work is to maximize RES integration in DNS from the system perspective (or, from the Distribution System Operators’ point of view) by optimally deploying different smart-grid enabling technologies at a minimum cost. Here, it is assumed that the DSO owns some generation sources and ESSs.

The resulting problem is formulated as a multi-stage stochastic MILP an overall cost minimization as an objective (3). The objective function in (3) is composed of Net Present Value (NPV) of five cost terms each multiplied by a certain relevance factor \( \rho_j; \forall j \in \{1,2,\ldots,5\} \). Note that, in this work, all cost terms are assumed to be equally important; hence, these factors are set equal. However, depending on the relative importance of the considered costs, different coefficients (relevance factors) can be adopted in the objective function. Note that all cost terms have the same units (Euros). In reality, the objective function is one: the total cost in the system which is the sum of various cost components (operation, maintenance, emission, investment, etc.). However, a decision-maker may not be interested in some of these costs, for instance, because their values (and/their expected influences on the solution) are negligible compared with others. Such cost terms would then have their relevance factors set to zero. It is for this sole purpose that the relevance factors (which should not be confused with weights like in the Pareto-type optimization) are included in the formulation.

In the present work, a perpetual planning horizon [49] is assumed when formulating the integrated planning problem, as in [50]. This is purposely done to balance different cost terms within and outside the actual planning horizon. To further clarify this, consider the illustrative example in Fig. 4. It is understood that investments are made in a specific year within the planning horizon (the second year in this case) and the investment costs are prorated throughout its lifetime i.e. distributed into equal payments among the years within the life span of the asset. However, the maintenance and operation (O&M) related costs are incurred every year within and after the planning horizon. To balance these cost terms, a perpetual planning horizon, i.e. an endless payment of fixed payments is assumed. Based on the finance theory [49], the present value of perpetuity, which is the sum of the net worth of infinite annual fixed payments, is determined by dividing the fixed payment at a given period by the interest rate \( r \). Based on this, the O&M costs include the associated annual costs within (part I) and outside the planning horizon (part II). The latter (part II) are determined by the perpetuity of the costs in the last planning stage updated by net present value factor in this case \((1+r)^{-3}\). Note that after the lifetime of a given asset elapses, investments will be made in the same asset with the same cost, leading to a seemingly perpetual planning horizon.

The first term in (3), \( TInvC \), represents the total investment costs under the assumption of perpetual planning horizon [49]. In other words, “the investment cost is amortized in annual installments throughout the lifetime of the installed component”, as is done in [50]. Here, the total investment cost is the sum of investment costs of new and existing DGs, feeders, energy storage system and capacitor banks, as in (4). The second term, \( TMC \), in (3) denotes the total maintenance costs, which is given by the sum of individual maintenance costs of new and existing DGs as well as that of feeders, energy storage system and capacitor banks in the system at each stage and the corresponding costs incurred after the last planning stage, as in (5). Note that the latter costs depend on the maintenance costs of the last planning stage. Here, a perpetual planning horizon is assumed. The third term \( TEC \) in (3) refers to the total cost of energy in the system, which is the sum of the cost of power produced by new and existing DGs, purchased from upstream and supplied by energy storage system at each stage as in (6). Eq. (6) also includes the total energy costs incurred after the last planning stage under a perpetual planning horizon. These depend on the energy costs of the last planning stage. The fourth term \( TENS \) represents the total cost of
unserved power in the system and is calculated as in (7). The last term $TEmiC$ gathers the total emission costs in the system, given by the sum of purchased power from the existing and new DGs as well that of power purchased from the grid at the substation.

$$\text{Minimize } TC = a_1 * TInvC + a_2 * TMC + a_3 * TEC + a_4 * TENS + a_5 * TImiC$$

$$TInvC = \sum_{g \in G}(1+\gamma)^{-t}(EInv_{DG}^C + EInv_{LN}^C + EInv_{ES}^C + EInv_{AP}^C)$$

$$TMC = \sum_{i \in I}(1+\gamma)^{-t}(\text{MntC}_{DG}^I + \text{MntC}_{LN}^I + \text{MntC}_{ES}^I + \text{MntC}_{Cap}^I)$$

$$TEC = \sum_{j \in J}(1+\gamma)^{-t}(EOp_{DG}^I + EOp_{ES}^I + EOp_{Cap}^I)$$

$$EInvC = \sum_{g \in G}(1+\gamma)^{-1} \text{ENS}_{g}^C + \frac{\text{NPV of maintenance costs}}{1+\gamma} \text{ENS}_{g}^C$$

$$TEmiC = \sum_{i \in I}(1+\gamma)^{-1} \text{Emi}_{g}^C + \frac{\text{NPV of emission costs}}{1+\gamma} \text{Emi}_{g}^C$$

The individual component costs in (4)–(8) are computed by the following expressions. Eqs. (9)–(12) represent the investment costs of DGs, feeders, energy storage system and capacitor banks, respectively. Notice that all investment costs are weighted by the capital recovery factor, $\frac{1}{(1+\gamma)^{t}}$. The formulations in (9)–(12) ensure that the investment cost of each asset added to the system is considered only once in the summation. In this regard, there are two issues that need to be taken care of in the formulation. On one hand, it is required that investment decisions already made at a given stage cannot be reversed back (divested) in the subsequent stages. This condition is met by the set of logical constraints described in the following subsection in the model formulation, for example, $x_{g,i} \leq x_{g,i-1}$. Such a logical constraint states that the investment decision at a planning stage $t$ should be at least equal to the investment decision in the preceding stage $t-1$. In other words, $x_{g,i}$ should be equal to the investments made in the preceding stages plus the additional investment in stage $t$. On the other hand, only the investment costs for the marginal (additional) investment made at each stage should be considered in the investment cost summations in (9)–(12). In the example, the additional (marginal) investments made at each stage are given by: $(x_{g,i} - x_{g,i-1})$. This is why the investment cost function in (9) contains this expression.

Now, suppose the decision variable on DG investments $x_{g,i}$ is defined as a binary one. This means that only one DG of type $g$ can be installed at node $i$ in either of the planning stages. Suppose it becomes most economical to install it in the second year i.e. $x_{g,0} = 0$. The logical constraint in (70) leads to $x_{g,0} \geq x_{g,1}$ i.e. $x_{g,0} \geq 0$. For this particular example, the binary variable for each stage i.e. $x_{g,0,1,i}x_{g,2,i}$ is equal to $[0; 1; 1]$, respectively. Recall that the investment cost of this DG should be considered only once (in the second year) in the summation, and this is taken care of by the expression $(x_{g,1,i} - x_{g,0,i})$ in Eq. (9).

All the differences for this particular example are $(x_{g,0,i} - x_{g,1,i}) = 0$, $(x_{g,1,i} - x_{g,0,i}) = 1$, and $(x_{g,2,i} - x_{g,1,i}) = 0$, which indicates that the investment cost is considered only once at the second stage. Instead of defining the variable $x_{g,i}$ as a binary variable, one may allow it to have any integer value as far as it is deemed optimal. In this case, for the above example, suppose the optimal solution is to install one DG in the second stage and one more DG in the third stage which means $(x_{g,1,i}x_{g,2,i}; x_{g,3,i})$ is equal to $(0; 0; 1)$. Note that $x_{g,3,i}$ should be equal to 2 because the investment decision made in the preceding stages should be also available in the third stage. For this example, $(x_{g,1,i} - x_{g,0,i}) = 0$, $(x_{g,2,i} - x_{g,1,i}) = 1$, and $(x_{g,3,i} - x_{g,2,i}) = 1$, showing that the investment cost each DG is considered only once in the summation.

In general, the formulation remains valid regardless of how the investment variables are defined. Note that investment variables refer to the decision variables corresponding to investments in DGs, energy storage systems, capacitor banks and distribution lines in each of the decision stages along the 3-year planning horizon.

Eq. (13) stands for the maintenance costs of new and existing DGs at each time stage. The maintenance cost of a new/existing feeder is included only when its corresponding investment/ utilization variable is different from zero. Similarly, the maintenance costs of new and existing feeders at each stage are given by Eq. (14). Eqs. (15) and (16) are related to the maintenance costs at each stage of energy storage and capacitor banks, respectively.

$$Invc_{DG} = \sum_{g \in G} \sum_{i \in I}(1+\gamma)^{-t} \text{IC}_{g,i} (x_{g,i} - x_{g,i-1})$$

where $x_{g,0} = 0$ (9)

$$Invc_{LN} = \sum_{k \in K} \sum_{t \in T}(1+\gamma)^{-t} \text{IC}_{k,t} (x_{k,t} - x_{k,t-1})$$

where $x_{k,0} = 0$ (10)

$$Invc_{ES} = \sum_{e \in E} \sum_{t \in T}(1+\gamma)^{-t} \text{IC}_{e,t} (x_{e,i,t} - x_{e,i,t-1})$$

where $x_{e,0} = 0$ (11)

$$Mntc_{DG} = \sum_{g \in G} \sum_{i \in I} \text{MC}_{DG}^e x_{g,i} + \sum_{g \in G} \sum_{i \in I} \text{MC}_{DG}^f x_{g,i}$$

(13)

$$Mntc_{LN} = \sum_{k \in K} \text{MC}_{LN}^e x_{k,t} + \sum_{k \in K} \text{MC}_{LN}^f x_{k,t}$$

(14)

$$Mntc_{ES} = \sum_{e \in E} \sum_{t \in T} \text{MC}_{ES}^e x_{e,i,t} + \sum_{e \in E} \sum_{t \in T} \text{MC}_{ES}^f x_{e,i,t}$$

(15)

$$Mntc_{Cap} = \sum_{e \in E} \sum_{t \in T} \text{MC}_{Cap}^e x_{e,i,t}$$

(16)

The total cost of power produced by new and existing DGs is given by Eq. (17). Note that these costs depend on the amount of power generated at each scenario, snapshot and stage. Therefore, these costs represent the expected costs of operation. Similarly, Eqs. (18) and (19) respectively account for the expected costs of energy supplied by the energy storage system, and that purchased from upstream (i.e., transmission grid). The penalty for the unserved power, given by (20), is also dependent on the scenarios, and time stages. Eq. (20) therefore gives the expected cost of unserved energy.
in the system. The expected emission costs of power generated by new and existing DGs are given by (21)—(23), and that of energy purchased from the grid is calculated using (24). Note that, for the sake of simplicity, a linear emission cost function is assumed here. In reality, the emission cost function is highly nonlinear and nonconvex, as in [44].

\[
E_C^{DG} = \sum_{s \in S^D} \sum_{w \in W^D} \pi_w \sum_{g \in G^D} \sum_{d \in D} (\delta C_{g,w,s,D}^{N} + \delta C_{g,s,D}^{N} + \delta C_{g,s,w,D}^{N})
\]

(17)

\[
E_C^{ES} = \sum_{s \in S^E} \sum_{w \in W^E} \sum_{d \in D^E} \pi_w \sum_{g \in G^E} \sum_{d \in D^E} \delta C_{g,w,s,D}^{E} + \delta C_{g,s,D}^{E} + \delta C_{g,s,w,D}^{E}
\]

(18)

\[
E_C^{SS} = \sum_{s \in S^S} \sum_{w \in W^S} \sum_{d \in D^S} \pi_w \sum_{g \in G^S} \sum_{d \in D^S} \delta C_{g,w,s,D}^{S} + \delta C_{g,s,D}^{S} + \delta C_{g,s,w,D}^{S}
\]

(19)

\[
ENSC_F = \sum_{s \in S^F} \sum_{w \in W^F} \sum_{d \in D^F} \pi_w \sum_{g \in G^F} \sum_{d \in D^F} \delta C_{g,w,s,D}^{F} + \delta C_{g,s,D}^{F} + \delta C_{g,s,w,D}^{F}
\]

(20)

\[
Emic^{DG}_{t} = Emic^{CN}_{t} + Emic^{E}_{t}
\]

(21)

\[
Emic^{CN}_{t} = \sum_{s \in S^C} \sum_{w \in W^C} \sum_{d \in D^C} \pi_w \sum_{g \in G^C} \sum_{d \in D^C} \delta C_{g,w,s,D}^{C} + \delta C_{g,s,D}^{C} + \delta C_{g,s,w,D}^{C}
\]

(22)

\[
Emic^{E}_{t} = \sum_{s \in S^E} \sum_{w \in W^E} \sum_{d \in D^E} \pi_w \sum_{g \in G^E} \sum_{d \in D^E} \delta C_{g,w,s,D}^{E} + \delta C_{g,s,D}^{E} + \delta C_{g,s,w,D}^{E}
\]

(23)

\[
Emic^{F}_{t} = \sum_{s \in S^F} \sum_{w \in W^F} \sum_{d \in D^F} \pi_w \sum_{g \in G^F} \sum_{d \in D^F} \delta C_{g,w,s,D}^{F} + \delta C_{g,s,D}^{F} + \delta C_{g,s,w,D}^{F}
\]

(24)

C. Constraints

1) Kirchhoff’s Voltage Law

The customary AC power flow equations, given by (25) and (26), are highly non-linear and non-convex. Understandably, using these flow equations in power system planning applications is increasingly difficult. Because of this, Eqs. (25) and (26) are often linearized by considering two practical assumptions. The first assumption is concerning the bus voltage magnitudes, which in distribution systems are expected to be close to the nominal value \(V_{nom}\). The second assumption is in relation to the voltage angle difference \(\theta_k\) across a line which is practically small, leading to the trigonometric approximations \(\sin \theta_k \approx \theta_k\) and \(\cos \theta_k \approx 1\). Note that this assumption is valid in distribution systems, where the active power flow dominates the total apparent power in lines. Furthermore, the voltage magnitude at bus \(i\) can be expressed as the sum of the nominal voltage and a small deviation \(\Delta V_i\), as in (27).

\[
P_k = V_i^2 g_k - V_i V_j (g_k \cos \theta_k + b_k \sin \theta_k)
\]

(25)

\[
Q_k = -V_i^2 b_k + V_i V_j (g_k \cos \theta_k - b_k \sin \theta_k)
\]

(26)

\[
V_i = V_{nom} + \Delta V_i, \text{ where } \Delta V_{min} \leq \Delta V_i \leq \Delta V_{max}
\]

(27)

Note that the voltage deviations at each node \(\Delta V_i\) are expected to be very small. Substituting (27) in (25) and (26) and neglecting higher order terms, we get:

\[
P_k \approx (V_{nom}^2 + 2V_{nom}\Delta V_i) g_k - (V_{nom}^2 + V_{nom}\Delta V_i + V_{nom}\Delta V_j)(g_k + b_k \theta_k)
\]

(28)

\[
Q_k \approx -(V_{nom}^2 + 2V_{nom}\Delta V_j) b_k + (V_{nom}^2 + V_{nom}\Delta V_i + V_{nom}\Delta V_j)(b_k - b_k \theta_k)
\]

(29)

Note that Eqs. (28) and (29) still contain nonlinearities because of the products of two continuous variables—voltage deviations and angle differences. However, since these variables \(\Delta V_i, \Delta V_j\) and \(\theta_k\) are very small, their products can be neglected. Hence, the above flow equations become:

\[
P_k = V_{nom}(\Delta V_i - \Delta V_j) g_k - V_{nom}^2 b_k \theta_k
\]

(30)

\[
Q_k = -V_{nom}(\Delta V_i - \Delta V_j) b_k - V_{nom}^2 g_k \theta_k
\]

(31)

The linear planning model proposed here is based on the above linearized flow equations. This linearization approach was first introduced in [52] in the context of transmission expansion planning problem. When the investment planning problem includes network switching, reinforcement, replacement and expansion of feeders, Eqs. (30) and (31) must be multiplied by the corresponding binary variables as in (32)–(35). This is to make sure the flow through an existing/new feeder is zero when its switching/investment variable is zero; otherwise, the flow in that feeder should obey the Kirchhoff’s law.

\[
P_k \approx u_{k,t}(V_{nom}(\Delta V_i - \Delta V_j) g_k - V_{nom}^2 b_k \theta_k)
\]

(32)

\[
Q_k \approx u_{k,t}(-V_{nom}(\Delta V_i - \Delta V_j) b_k - V_{nom}^2 g_k \theta_k)
\]

(33)

\[
P_k = x_{k,t}(V_{nom}(\Delta V_i - \Delta V_j) g_k - V_{nom}^2 b_k \theta_k)
\]

(34)

\[
Q_k = x_{k,t}(-V_{nom}(\Delta V_i - \Delta V_j) b_k - V_{nom}^2 g_k \theta_k)
\]

(35)

The bilinear products, involving binary with voltage deviation and angle difference variables, introduces undesirable nonlinearity to the problem. This nonlinearity can be avoided using the big-M formulation i.e. by reformulating the above equations into their respective disjunctive equivalents as in (36)–(39). As a rule-of-thumb, the big-M parameter is often set to the maximum transfer capacity in the system.

\[
[P_{k,s,w,t} - (V_{nom}(\Delta V_{i,s,w,t} - \Delta V_{j,s,w,t}) g_k - V_{nom}^2 b_k \theta_k)] \leq MP_k (1 - u_{k,t})
\]

(36)

\[
\left[Q_{k,s,w,t} - (-V_{nom}(\Delta V_{i,s,w,t} - \Delta V_{j,s,w,t}) b_k - V_{nom}^2 g_k \theta_k) \right] \leq MQ_k (1 - u_{k,t})
\]

(37)

\[
[P_{k,s,w,t} - (V_{nom}(\Delta V_{i,s,w,t} - \Delta V_{j,s,w,t}) g_k - V_{nom}^2 b_k \theta_k)] \leq MP_k (1 - x_{k,t})
\]

(38)

\[
\left[Q_{k,s,w,t} - (-V_{nom}(\Delta V_{i,s,w,t} - \Delta V_{j,s,w,t}) b_k - V_{nom}^2 g_k \theta_k) \right] \leq MQ_k (1 - x_{k,t})
\]

(39)

2) Flow Limits

The apparent power flow through a line \(S_k\) is given by \(\sqrt{P_k^2 + Q_k^2}\) and this has to be less than or equal to the rated value which is denoted as:

\[
P_k^2 + Q_k^2 \leq (S_k^{max})^2
\]

(40)

Considering line switching/investment, Eq. (38) can be rewritten as:

\[
P_{k,s,w,t}^2 + Q_{k,s,w,t}^2 \leq u_{k,t}(S_k^{max})^2
\]

(41)

\[
P_{k,s,w,t}^2 + Q_{k,s,w,t}^2 \leq x_{k,t}(S_k^{max})^2
\]

(42)

The quadratic expressions of active and reactive power flows in (41) through (42) can be easily linearized using piecwise linearization, considering a sufficiently large number of linear segments, \(L\). There are a number of ways of linearizing such functions such as incremental, multiple choice, convex combination and other approaches in the literature [53]. Here, the first approach
(which is based on first-order approximation of the nonlinear curve) is used because of its relatively simple formulation. To this end, two non-negative auxiliary variables are introduced for each of the flows \( P_k \) and \( Q_k \) such that \( P_k = P_k^+ - P_k^- \) and \( Q_k = Q_k^+ - Q_k^- \).

Note that these auxiliary variables (i.e., \( P_k^+ \), \( P_k^- \), \( Q_k^+ \) and \( Q_k^- \)) represent the positive and negative flows of \( P_k \) and \( Q_k \), respectively. This helps one to consider only the positive or the quadratic angle differences, resulting in a significant reduction in the mathematical complexity, and by implication the computational burden. In this case, the associated linear constraints are:

\[
P_{k,x,w,t}^+ = \sum_{i=1}^i a_{i,k} p_{k,x,w,t,l} \tag{43}
\]

\[
Q_{k,x,w,t}^+ = \sum_{i=1}^i b_{i,k} q_{k,x,w,t,l} \tag{44}
\]

\[
P_{k,x,w,t}^+ + P_{k,x,w,t}^- = \sum_{i=1}^i p_{k,x,w,t,l} \tag{45}
\]

\[
Q_{k,x,w,t}^+ + Q_{k,x,w,t}^- = \sum_{i=1}^i q_{k,x,w,t,l} \tag{46}
\]

where \( p_{k,x,w,t,l} \leq P_{k,x,w,t}^{\text{max}} / L \) and \( q_{k,x,w,t,l} \leq Q_{k,x,w,t}^{\text{max}} / L \).

3) Line Losses

The active and reactive power losses in line \( k \) can be approximated as follows:

\[
P_L = P_{k,i} + P_{k,j} \approx 2V_i^2\text{nom}b_k(1 - \cos \theta_k) \approx V_i^2\text{nom}b_k^2 \tag{47}
\]

\[
Q_L = Q_{k,i} + Q_{k,j} \approx -2V_i^2\text{nom}b_k(1 - \cos \theta_k) \approx -b_kV_i^2\text{nom}^2 \tag{48}
\]

Clearly, Eqs. (47) and (48) are nonlinear and nonconvex functions, making the problem more complex to solve. This can be overcome by having the quadratic angle differences piecewise-linearized, as it is done for the quadratic flows in the above. However, instead of doing this, the expressions in (47) and (48) can be expressed in terms of the active and the reactive power flows as in (49) and (50). Note that Eq. (49) can be easily obtained by multiplying the squared expressions of both sides of the equations in (30) and (31) by the resistance of the branch, combining the resulting equations, neglecting higher order terms and reordering both sides of the resulting equation. Eq. (50) is also obtained in a similar fashion but by multiplying the squared expressions by reactance. For the sake of completeness, details concerning the derivations (49) and (50) are presented in Appendix A.

\[
P_L = P_{k,x,w,t} \approx V_i^2\text{nom}b_k^2 \tag{49}
\]

\[
Q_L = Q_{k,x,w,t} \approx b_kV_i^2\text{nom}^2 \tag{50}
\]

Note that expressing the losses as a function of flows has two advantages. First, doing so reduces the number of nonlinear terms that has to be linearized, which in turn results in a model with a reduced number of equations and variables. For example, if Eqs. (47) and (48) are used instead, in addition to the quadratic power flow terms \( P_k^2 \) and \( Q_k^2 \), the quadratic angle differences \( \theta_k^2 \) should also be linearized to make the problem linear and convex. On the contrary, when Eqs. (49) and (50) are used, one is only required to linearize \( P_k^2 \) and \( Q_k^2 \). Second, it avoids unnecessary constraints on the angle differences when a line between two nodes is not connected or remains not selected for investment. This is often avoided by introducing binary variables and using a so-called big-M formulation [52]. However, this adds extra complexity to the problem.

4) Kirchhoff’s Current Law (Active and Reactive Load Balances)

All the time, load balance should be respected at each node i.e. the sum of all injections should be equal to the sum of all withdrawals at each node. This is enforced by adding the following two constraints:

\[
\sum_{s \in \text{gen}}(P_{g_s,x,w,t}^+ + P_{g_s,x,w,t}^-) + \sum_{s \in \text{con}}(P_{c_s,x,w,t}^+ - P_{c_s,x,w,t}^-) + P_{s_s,x,w,t}^+ + \sum_{s \in \text{out}}(P_{s_s,x,w,t}^+ - P_{s_s,x,w,t}^-) = D_{x,w,t} + P_{L,c,x,w,t} \tag{51}
\]

\[
\sum_{s \in \text{gen}}(Q_{g_s,x,w,t}^+ + Q_{g_s,x,w,t}^-) + \sum_{s \in \text{con}}(Q_{c_s,x,w,t}^+ - Q_{c_s,x,w,t}^-) + Q_{s_s,x,w,t}^+ + \sum_{s \in \text{out}}(Q_{s_s,x,w,t}^+ - Q_{s_s,x,w,t}^-) = Q_{x,w,t} + Q_{L,c,x,w,t} \tag{52}
\]

Eqs. (51) and (52) stand for the active and the reactive power balances at each node, respectively.

5) Bulk Energy Storage Model Constraints

The generic bulk ESS is modeled by constraints (53) — (59).

\[
\begin{align*}
0 & \leq P_{\text{ess},x,w,t}^{\text{ch}} \leq P_{\text{ess},x,w,t}^{\text{max}} \tag{53} \\
0 & \leq P_{\text{ess},x,w,t}^{\text{disch}} \leq P_{\text{ess},x,w,t}^{\text{max}} \tag{54} \\
\min & \leq P_{\text{ess},x,w,t}^{\text{ch}} \leq P_{\text{ess},x,w,t}^{\text{max}} \tag{55} \\
\max & \leq P_{\text{ess},x,w,t}^{\text{disch}} \leq P_{\text{ess},x,w,t}^{\text{max}} \tag{56} \\
E_{\text{ess},x,w,t} & = E_{\text{ess},x,w,t-1} + \min \beta_{\text{ess}} P_{\text{ess},x,w,t}^{\text{ch}} - \max \beta_{\text{ess}} P_{\text{ess},x,w,t}^{\text{disch}} \tag{57} \\
E_{\text{ess},x,w,t} & = E_{\text{ess},x,w,t} \leq E_{\text{ess},x,w,t}^{\text{max}} \tag{58} \\
E_{\text{ess},x,w,t+1} & = E_{\text{ess},x,w,t} + P_{\text{ess},x,w,t}^{\text{max}} \tag{59}
\end{align*}
\]

The limits on the capacity of ESS while being charged and discharged are considered in Eqs. (53) and (54), respectively. Inequality (55) prevents simultaneous charging and discharging operation of ESS at the same operational time \( w \). The amount of stored energy within the reservoir of bulk ESS at the operational time \( w \) as a function of energy stored until \( w - 1 \) is given by (56). The maximum and minimum levels of storages in operational time \( w \) are also considered through inequality (57). Eq. (58) shows the initial level of stored energy in the bulk ESS as a function of its maximum reservoir capacity. In a multi-stage planning approach, Eq. (59) ensures that the initial level of energy in the bulk ESS at a given year is equal to the final level of energy in the ESS in the preceding year and the reservoir level at the end of the planning horizon should be equal to the initial level. The latter constraint guarantees that the optimal solution returned by the solution algorithm is not because of the initial reservoir level. For the sake of simplicity, \( \eta_{\text{ess}}^{\text{ch}} \) is assumed to be equal to \( \eta_{\text{ess}}^{\text{disch}} \), as in [54], [55]. Both the charging \( \eta_{\text{ess}}^{\text{ch}} \) and discharging \( \eta_{\text{ess}}^{\text{disch}} \) efficiencies are expressed in terms of the energy at the nodes where the storage system is connected to. Because of this, a certain percentage of the energy fed to the storage system will be stored while the remaining will be lost in the form of losses (electrical, chemical, heat, etc.). This is related to the charging efficiency, which should then be less than 1. On the other hand, in order to withdraw a given amount of energy from the storage system, more energy is needed to cover the discharging losses. This is why we have \( 1/\eta_{\text{ess}}^{\text{disch}} \) in Eq. (56) associated with the
energy at the output side of the energy storage system.

Notice that inequalities (53) and (54) involve products of charging/discharging binary variables and investment variable. In order to linearize these, new continuous positive variables \( z_{dch}^{ch} \) and \( z_{dch}^{ch} \), which replaces the bilinear products in each constraint, is introduced such that the set of linear constraints in (60) and (61) hold. For instance, the product \( t_{dch}^{ch} \) is replaced by the positive variable \( z_{dch}^{ch} \). Then, the bilinear product is decoupled by introducing the set of constraints in (60) [56].

\[
z_{dch}^{ch} \leq x_{max}^{dch} t_{dch}^{ch}, \quad z_{dch}^{ch} \leq x_{c}^{ch}, \quad z_{dch}^{ch} \geq x_{dch} - (1 - t_{dch}^{ch}) x_{c}^{ch} \tag{60}
\]

Similarly, the product \( t_{dch}^{ch} \) is decoupled by including the following set of constraints:

\[
z_{dch}^{ch} \leq x_{max}^{dch} t_{dch}^{ch}, \quad z_{dch}^{ch} \leq x_{c}^{ch}, \quad z_{dch}^{ch} \geq x_{dch} - (1 - t_{dch}^{ch}) x_{c}^{ch} \tag{61}
\]

The large number of discrete variables in the storage model presented above can render significant computational burden. To overcome this, a relaxed ESS model can be formed without the charging and discharging indicator variables as:

\[
0 \leq p_{ch}^{ch} \leq x_{c} \leq p_{ch}^{max} \tag{53'}
\]

\[
0 \leq -p_{dch} \leq x_{dch} \leq p_{dch}^{max} \tag{54'}
\]

\[
E_{es,i,w,t} = E_{es,i,w,t-1} + \eta_{ch}^{ch} p_{ch}^{ch} t_{dch}^{ch} - \eta_{dch}^{ch} p_{dch}^{dch} t_{dch}^{dch} \tag{56'}
\]

Under normal conditions, the ESS model in (53'), (54') and (56') is exact because by the principle of optimality, at most one of the variables \( p_{ch}^{ch} \) and \( p_{dch}^{dch} \) can be greater than zero. In other words, it does not economic sense to have both variables to be greater than zero at the same time.

6) Active and Reactive Power Limits of DGs

The active and reactive power limits of existing generators are given by (62) and (63), respectively. In the case of new (candidate) generators, the corresponding constraints are (64) and (65). Note that the binary variables multiply the minimum and the maximum generation limits to make sure that the power generation variable is zero when the generator remains either unutilized or unselected for investment.

\[
p_{\min}^{p_{g,i,s,t}} \leq p_{g,i,s,t} \leq p_{\max}^{p_{g,i,s,t}} \tag{62}
\]

\[
q_{\min}^{q_{g,i,s,t}} \leq q_{g,i,s,t} \leq q_{\max}^{q_{g,i,s,t}} \tag{63}
\]

\[
p_{\min}^{p_{g,i,s,t}} \leq p_{g,i,s,t} \leq p_{\max}^{p_{g,i,s,t}} \tag{64}
\]

\[
q_{\min}^{q_{g,i,s,t}} \leq q_{g,i,s,t} \leq q_{\max}^{q_{g,i,s,t}} \tag{65}
\]

It should be noted that these constraints are applicable only for conventional DGs which have reactive power support capabilities. In the case of variable generation sources, slight modifications are required. For instance, for wind and solar PV generators, the upper bound \( p_{\max}^{p_{g,i,s,t}} \) should be equal to the actual production level at a specific hour, which in turn depends on the level of primary energy source (wind speed and solar radiation). The lower bound \( p_{\min}^{p_{g,i,s,t}} \) in this case is simply set to zero. In addition, conventional wind and solar PV sources do not often have the capability to provide reactive power support; hence, they are operated at a constant and lagging or unity power factor. Under such an operation, the following constraints should be used:

\[
q_{\min}^{q_{g,i,s,t}} = \begin{cases} \tan^{-1}(p_{g,i,s,t}) \cdot p_{\max}^{p_{g,i,s,t}} & 
\end{cases} \tag{66}
\]

\[
q_{\min}^{q_{g,i,s,t}} = \begin{cases} \tan^{-1}(p_{g,i,s,t}) \cdot p_{\max}^{p_{g,i,s,t}} & 
\end{cases} \tag{67}
\]

where \( p_{g,i,s,t} \) is the power factor of the wind or solar type generator.

Under normal cases, \( p_{\max}^{p_{g,i,s,t}} = p_{\max}^{q_{g,i,s,t}} \) and \( p_{\max}^{q_{g,i,s,t}} = p_{\max}^{q_{g,i,s,t}} \).

7) Reactive Power Limit of Capacitor Banks

Inequality (66) ensures that the reactive power produced by the reactive power sources (capacitor banks) is bounded between zero and the maximum possible capacity.

\[
0 \leq q_{\min}^{q_{g,i,s,t}} \leq q_{\max}^{q_{g,i,s,t}} \tag{66}
\]

8) Active and Reactive Power Limits of Power Purchased

For technical reasons, the power that can be purchased from the transmission grid could have minimum and maximum limits, which is enforced by (67) and (68). However, it is understood that, in reality, setting such limits is difficult. They are included here only for the sake of completeness.

\[
p_{\min}^{p_{c,s,w,t}} \leq p_{c,s,w,t} \leq p_{\max}^{p_{c,s,w,t}} \tag{67}
\]

\[
q_{\min}^{q_{c,s,w,t}} \leq q_{c,s,w,t} \leq q_{\max}^{q_{c,s,w,t}} \tag{68}
\]

For the analysis, the active power production limits are simply set to 1.5 times the minimum and the maximum levels of total load in the system.

Note that the multiplier is higher than one in the system because of the losses, which needs to be covered by generating extra power. The reactive power limits are determined by the power factor of the substation, which is assumed to be 0.9 throughout the analysis in this paper.

9) Logical Constraints

The following logical constraints ensure that an investment decision cannot be reversed i.e. an investment already made cannot be divested.
\[ x_{k,t} \geq x_{k,t-1} \quad (69) \]
\[ x_{g,t} \geq x_{g,t-1} \quad (70) \]
\[ x_{e,t} \geq x_{e,t-1} \quad (71) \]
\[ x_{c,t} \geq x_{c,t-1} \quad (72) \]

10) Radiality Constraints

Distribution networks are structurally meshed but predominantly operated in a radial manner because of technical reasons (particularly related to the protection). The presence of DGs and reactive power sources in the system nodes other than substation nodes (which is the subject of the present work) may lead to islanding i.e. some loads may be isolated from the mains (substations) and/or loops, breaking the radiality of the network. This is not desired mainly because of the aforementioned technical reasons. To ensure radiality, two conditions must be fulfilled. First, the solution must have \( N_I - N_C \) circuits. Second, the final topology should be connected. Eq. (73) represents the first necessary condition for maintaining the radial topology of DNS.

\[ \sum_{k \in E/D} OR(x_{k,t}, u_{k,t}) = N_I - N_C \quad \forall t \quad (73) \]

Note that the above equation assumes line investment is possible in all corridors. Hence, in a given corridor, we can have either an existing branch, a new one, or both connected in parallel, depending on the economic benefits of the final setup (solution) brings about to the system. The radiality constraint in (73) then has to accommodate this condition. One way to do this is using the Boolean logic operation, as in (73). Unfortunately, this introduces nonlinearity.

We show how this logic can be linearized using an additional auxiliary variable \( z_{k,t} \), and the binary variables associated to existing and new branches (i.e., \( u_{k,t} \) and \( x_{k,t} \)), respectively. Given 
\[ z_{k,t} = OR(x_{k,t}, u_{k,t}), \]
this Boolean operation can be expressed using the following set of linear constraints:

\[ z_{k,t} \leq x_{k,t} + u_{k,t}; \quad z_{k,t} \geq x_{k,t}; \quad z_{k,t} \geq u_{k,t}; \quad 0 \leq z_{k,t} \leq 1 \quad \forall t \quad (74) \]

Note that the auxiliary variable \( z_{k,t} \) is automatically constrained to be binary. Hence, it is not necessary to explicitly define \( z_{k,t} \) as a binary variable; instead, defining it as a continuous positive variable is sufficient. Alternatively, if \( z_{k,t} \) is defined to be binary variable from the outset, then, Eq. (73) can be converted into a single range constraint as:

\[ 0 \leq 2z_{k,t} - x_{k,t} - u_{k,t} \leq 1 \quad \forall t \quad (75) \]

Then, the radiality constraints in (73) can be reformulated using the \( z_{k,t} \) variables as:

\[ \sum_{k \in E/D} z_{k,t} = N_I - N_C \quad \forall t \quad (76) \]

When all loads in the DNS are only fed by power from substations, the final solution obtained automatically satisfies the two aforementioned conditions; hence, no additional constraints are required i.e. (74) or (75) along with (76) are sufficient to guarantee radiality. However, it should be noted that in the presence of DGs and reactive power sources, these constraints alone may not ensure the radiality of the distribution network, as pointed out in [57] and further discussed in [58]. This is however out of the scope of this work. If this is a critical issue, additional constraints need to be added to guarantee that all buses are linked, as proposed in [50], [53]–[55].

The optimization problem, developed here, encompasses Eqs. (3)–(24) and constraints (36)–(39), (41)–(46), (49)–(61), (62), (63), (64), (65), (66)–(72), (74) or (75) and (76) when all considered DGs are conventional generators with reactive power support capabilities. Otherwise, constraints (63') and (65') need to be added in case RES-based DGs which do not have such capabilities are present in the system or included in the planning. In addition, if there are RES-based DGs which are instead capable of operating as “generators” or “consumers” of reactive power depending on system operational situations, constraints (63'') and (65'') should be included and applied. In addition, if a relaxed ESS model is sought, constraints (53)–(61) can be replaced with the constraints in (53'), (54') and (56').

V. CONCLUSIONS

This work has developed a new multi-stage and stochastic model for jointly optimizing the integration of smart-grid enabling technologies such as ESS, reactive power sources, and network switching, reinforcement and/or expansion to support large-scale renewable integration. The novel integrated planning model, proposed here, simultaneously determines the optimal size, time and placement of ESSs and reactive power sources as well as that of conventional DGs and RESs in distribution networks, which can be considered as a major leap forward. The ultimate goal of this optimization work is to maximize the RES power absorbed by the system while maintaining the power quality and stability within the required/standard levels at a minimum possible cost. From a methodological perspective, the model has been formulated as a MILP optimization, employing a linearized AC network model which captures well the inherent characteristics of power network systems. The linearized AC model also strikes the right balance between accuracy and computational burden. The standard IEEE 41-bus distribution system is used to test the developed model and carry out the required thorough analysis from the standpoint of the objectives set in this work. Test results and discussions are presented in Part II of this paper.

APPENDIX A – FLOW-BASED LOSSES

The derivations related to the losses equations in (49) and (50) are provided here. Squaring both sides of the flow equations in (49) and (50) and dividing each by \( V_{\text{nom}}^2 \), we get:

\[ \frac{(P_k)^2}{V_{\text{nom}}^2} \approx \left( \frac{\Delta V_i - \Delta V_j}{t} \right) g_k^2 - 2 \cdot g_k V_{\text{nom}} b_k \theta_k \ast \left( \Delta V_i - \Delta V_j \right) \frac{1}{t} \quad (A.1) \]

\[ \frac{(Q_k)^2}{V_{\text{nom}}^2} \approx \left( \frac{\Delta V_i - \Delta V_j}{t} \right) b_k^2 + 2 \cdot b_k V_{\text{nom}} g_k \theta_k \ast \left( \Delta V_i - \Delta V_j \right) \frac{1}{t} \quad (A.2) \]

Since the variables \( \theta_k, \Delta V_i, \Delta V_j \) are very small, the second order terms (i.e. products of these variables) are close to zero. Hence, the first and the second terms in (A.1) and (A.2) can be neglected, leading to the following expressions, respectively:

\[ \frac{(P_k)^2}{V_{\text{nom}}^2} \approx (V_{\text{nom}} b_k \theta_k)^2 \quad (A.3) \]
Multiplying both sides of (A.3) and (A.4) by $r_k$ and summing gives:

$$r_k \left( \frac{P_k}{V_{nom}} \right) + r_k \frac{Q_k^2}{V_{nom}^2} \approx r_k (V_{nom} b_k \theta_k)^2 + r_k (V_{nom} g_k \theta_k)^2$$

(A.5)

After rearranging Eq. (A.5), we get:

$$r_k \left( P_k^2 + Q_k^2 \right)/V_{nom}^2 \approx g_k (V_{nom} \theta_k)^2 r_k \left( \frac{b_k}{g_k} \right)^2 + g_k$$

(A.6)

One can easily verify that $r_k \left( \frac{b_k}{g_k} \right)^2 + g_k = 1$, reducing Eq. (A.5) to:

$$r_k \left( P_k^2 + Q_k^2 \right)/V_{nom}^2 = g_k (V_{nom} \theta_k)^2$$

(A.7)

Recall that the right hand side of (A.7) corresponds to the active power losses expression in (49), which proves the derivation. The flow-based reactive power losses in (50) are derived in a similar way. Multiplying both sides of (A.3) and (A.4) by $x_q$ instead of $r_k$, adding both and rearranging the resulting equation leads to:

$$r_k \left( P_k^2 + Q_k^2 \right)/V_{nom}^2 \approx -b_q V_{nom} \theta_k x_q \left[ -b_k + (g_k)^2 / (-b_q) \right]$$

(A.8)

Note that, in Eq. (A.8), $x_q \left[ -b_k + (g_k)^2 / (-b_q) \right] = 1$. Hence, the equation reduces to:

$$r_k \left( P_k^2 + Q_k^2 \right)/V_{nom}^2 \approx -b_q V_{nom}^2 \theta_k$$

(A.9)

Notice that the right hand side of Eq. (A.8) is equal to the reactive losses expression in (50).

REFERENCES


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