Abstract—A coordinated strategy between wind and reversible hydro units for the midterm planning that reduces the imbalance of wind power and improves system efficiency is proposed. A stochastic mixed integer linear model is used, which maximizes the joint profit of wind and hydro units, where conditional value at risk (CVaR) is used for model risk. The offering strategies studied are 1) separate wind and hydro pumping offer, where the units work separately without a physical connection and 2) a single wind and hydro pumping offer with a physical connection between them to store wind energy for future use. The effects of a coordinated wind–hydro strategy for midterm planning are analyzed, considering CVaR and the future water value. The future water value in the reservoirs is analyzed hourly for a period of 1 week and 2 months, in two realistic case studies.

Index Terms—Midterm planning, offering strategies, reversible hydro unit, wind power.

NOMENCLATURE

Indexes

\( i \) Index referring to a hydro unit.

\( l \) Index referring to each block resulting from the linearization of the production curve of a hydro turbine.

\( t \) Index referring to a period (h).

\( w \) Index referring to a scenario.

Parameters

\( \alpha \) Per unit confidence level.

\( b_{t,w}^+ \) Upper limit of the wind farm power offer in period \( t \) and scenario \( w \) (MW).

\( b_{t,w}^- \) Lower limit of the wind farm power offer in period \( t \) and scenario \( w \) (MW).

\( B_0 \) Hydro unit \( i \) power capacity (MW).

\( c_i \) Start-up cost of hydro unit \( i \) (€).

\( c_i^H \) Generating cost of hydro unit \( i \) (€/MWh).

\( c_i^P \) Pumping cost of hydro unit \( i \) (€/MWh).

\( cv \) Conversion factor \( \left( \frac{\text{Hm}^3}{\text{s}} \right) \).

\( \beta \) Risk aversion of the producer, \( \beta \in (0, 1) \).

\( d_{t,w} \) Day-ahead market price in period \( t \) and scenario \( w \) (€/MWh).

\( fwp1_w \) Future water value price in the first evaluation period (€/Hm³).

\( fwp2_w \) Future water value price in the second evaluation period (€/Hm³).

\( gw_{t,w} \) Power produced by the wind farm using a Weibull distribution in period \( t \) and scenario \( w \) (MW).

\( if_{t,i,w} \) Incoming flow associated with hydro unit \( i \) in period \( t \) and scenario \( w \) (Hm³/h).

\( \lambda_{t,w}^+ \) Positive imbalance market price in period \( t \) and scenario \( w \) (€/MWh).

\( \lambda_{t,w}^- \) Negative imbalance market price in period \( t \) and scenario \( w \) (€/MWh).

\( eff \) Hydro pumping efficiency.

\( P_{\text{max}} \) Maximum installed power of the wind farm (MW).

\( \text{porho}_{i} \) Minimum power of hydro unit \( i \) for the upper curve (MW).

\( \text{porho}_{i} \) Minimum power of hydro unit \( i \) for the lower curve (MW).

\( \text{porhom}_{i} \) Minimum power of hydro unit \( i \) for the intermediate curve (MW).

\( \text{ppm}_{i} \) Pumping upper limit of hydro unit \( i \) (MW).

\( \rho_{w} \) Probability of occurrence of scenario \( w \).

\( \text{rho}_{i} \) Slope of block \( l \) of hydro unit \( i \) for the upper curve (MW/\( \frac{\text{Hm}^3}{\text{s}} \)).

\( \text{rho}_{i} \) Slope of block \( l \) of hydro unit \( i \) for the lower curve (MW/\( \frac{\text{Hm}^3}{\text{s}} \)).

\( \text{rhom}_{i} \) Slope of block \( l \) of hydro unit \( i \) for the intermediate curve (MW/\( \frac{\text{Hm}^3}{\text{s}} \)).

\( \text{rhop}_{i} \) Conversion factor from total hydro unit \( i \) capacity in MWh to \( \frac{\text{MWh}}{\text{s}} \).

\( \text{tp} \) Total number of periods of study (h).

\( \text{umin}_{i} \) Minimum water discharge by hydro unit \( i \) (\( \frac{\text{m}^3}{\text{s}} \)).
\( u_{\text{max},i} \) Maximum water discharge of block \( l \) of hydro unit \( i \) (\( m^3/s \)).

\( \text{vc1}_i \) Lower level of the reservoir associated with hydro unit \( i \) used in the discretization of the hydro production curves (\( \text{Hm}^3 \)).

\( \text{vc2}_i \) Upper level of the reservoir associated with hydro unit \( i \) used in the discretization of the hydro production curves (\( \text{Hm}^3 \)).

\( r_{0,i,w} \) Initial reservoir volume of hydro unit \( i \) and scenario \( w \) (\( \text{Hm}^3 \)).

\( V_{\text{max},i} \) Maximum volume of the reservoir of hydro unit \( i \) (\( \text{Hm}^3 \)).

\( V_{\text{min},i} \) Minimum volume of the reservoir of hydro unit \( i \) (\( \text{Hm}^3 \)).

**Continuous Variables**

\( b_{t,i} \) Joint power offer in the day-ahead market associated to the wind farm and hydro unit \( i \) in period \( t \) (MW).

\( \text{CVaR} \) Conditional value at risk (\( \text{€} \)).

\( \Delta_{t,i,w} \) Imbalance between the actual joint production and the joint power offer in period \( t \) and scenario \( w \) (MW).

\( \Delta_{t,i,w}^+ \) Positive imbalance between the actual joint production and the joint power offer in period \( t \) and scenario \( w \) (MW).

\( n_{p,t,i,w} \) Power produced by hydro unit \( i \) in period \( t \) and scenario \( w \) to eliminate the negative imbalance (MW).

\( p_{t,i,w} \) Power produced by hydro unit \( i \) in period \( t \) and scenario \( w \) (MW).

\( p_{\text{n},t,i,w} \) Net pumping of hydro unit \( i \) in period \( t \) and scenario \( w \) (MW).

\( p_{r,t,i,w} \) Auxiliary variable associated with \( p_{p,t,i,w} \) (MW).

\( PF \) Sum of all profits of the wind and hydro units (\( \text{€} \)).

\( \text{PFE}_{t,i,w} \) Profits from energy sales in the electricity market at each period \( t \) for the wind farm and the hydro units in scenario \( w \) (\( \text{€} \)).

\( p_{t,i,w} \) Total pumping of hydro unit \( i \) in period \( t \) and scenario \( w \) (MW).

\( p_{\text{pp},t,i,w} \) Power purchased by hydro unit \( i \) that is used for pumping in period \( t \) and scenario \( w \) (MW).

\( p_{\text{pw},t,i,w} \) Power produced by the wind farm that is used for pumping by hydro unit \( i \) in period \( t \) and scenario \( w \) (MW).

\( p_{\text{ppw},t,i,w}^+ \) Auxiliary variable associated with \( p_{\text{pw},t,i,w} \) (MW).

\( p_{\text{ppw},t,i,w}^- \) Excess wind power that is used for pumping by hydro unit \( i \) when there is a joint offer to sell power in period \( t \) and scenario \( w \) (MW).

\( p_{\text{ppw},t,i,w}^+ \) Excess wind power that can be used for pumping by hydro unit \( i \) in period \( t \) and scenario \( w \) (MW).

\( r_{t,i,w} \) Reserve of hydro unit \( i \) in period \( t \) and scenario \( w \) (\( \text{Hm}^3 \)).

\( \eta_{w} \) Auxiliary variable in scenario \( w \) used to compute CVaR (\( \text{€} \)).

\( s_{t,i,w} \) Spillage of hydro unit \( i \) in period \( t \) and scenario \( w \) (\( m^3/s \)).

\( u_{t,i,w} \) Water discharge of hydro unit \( i \) in period \( t \) and scenario \( w \) (\( m^3/s \)).

\( u_{l,t,i,w} \) Water discharge of block \( l \) of hydro unit \( i \) in period \( t \) and scenario \( w \) (\( m^3/s \)).

\( \text{VaR} \) Value at risk (\( \text{€} \)).

\( v_{t,i,w} \) Water flow pumped by hydro unit \( i \) in period \( t \) and scenario \( w \) (\( m^3/s \)).

**Binary Variables**

\( a_{t,i,w} \) 0/1 variable that is equal to 0 if hydro unit \( i \) pumps water in period \( t \) and scenario \( w \), and 1 otherwise.

\( b_{d,l,i} \) 0/1 variable that is equal to 1 if there is a joint sale in period \( t \) and 0 otherwise (purchase).

\( d_{t,i,w} \) 0/1 variable that is equal to 1 if there is a negative imbalance and 0 if there is a positive imbalance of wind and hydro unit \( i \) in period \( t \) and scenario \( w \).

\( d_{1,t,i,w} \) 0/1 variable used in the discretization of the hydro production curves of hydro unit \( I \) in period \( t \) and scenario \( w \).

\( d_{2,t,i,w} \) 0/1 variable used in the discretization of the hydro production curves of hydro unit \( i \) in period \( t \) and scenario \( w \).

\( v_{t,j,w} \) 0/1 variable that is equal to 1 if hydro unit \( i \) generates in period \( t \) and scenario \( w \) and 0 if the hydro unit is pumping.

\( w_{l,t,i,w} \) 0/1 variable that is equal to 1 if the water discharged by hydro unit \( i \) has exceeded block \( l \) in period \( t \) and scenario \( w \), and 0 otherwise.

\( y_{t,i,w} \) 0/1 variable that is equal to 1 if hydro unit \( i \) is started up in period \( t \) and scenario \( w \), and 0 otherwise.

\( z_{t,i,w} \) 0/1 variable that is equal to 1 if hydro unit \( i \) is shutdown in period \( t \) and scenario \( w \), and 0 otherwise.

I. INTRODUCTION

Investment in renewable technologies has plateaued in several countries, such as Portugal and Spain. In Spain, as a consequence of the economic crisis, premiums have been reduced [1], while they have been eliminated in other countries. In this vein, European countries such as the United Kingdom, France, Spain, and Portugal have delayed various onshore wind farm projects.

Nowadays, the importance of renewable energies in the reduction of CO\(_2\) emissions is vital, providing a competitive
advantage in the CO₂ market and reducing external energy dependence. Moreover, investment in renewable energies is the driving force behind the economy in many countries. Due to this, being more efficient in the reduction of CO₂ can be attained by combining renewable technologies to take advantage of their mutual synergies. An example for this is the combination of wind and hydro technologies.

Wind and reversible hydro power technologies may be more competitive against other renewable and conventional energies if they are coordinated through a joint offer in the day-ahead electricity market.

This paper analyzes both separate wind reversible hydro offering without a physical connection and a single wind reversible hydro offering with a physical connection in a midterm horizon. In this way, wind and reversible hydro units could be more efficient with fewer financial resources. A single offer with a physical connection may reduce wind imbalance through the use of a turbine/pumping hydro unit by selecting the best hours to sell/buy energy. Currently, there are some projects to study and analyze the joint coordination between wind and reversible hydro units, e.g., in the Canary Islands [2] and in Aegean Sea islands [3].

A. Literature Review

The main differences between nodal, uniform, and zonal pricing are shown in [4]. On the other hand, the integration of renewable energies in electricity markets is described in [5].

The impact of wind production in locational marginal prices is analyzed in [6]. Wind power trading is a well-known topic, where [7]–[9] evaluate the incorporation of wind energy into electricity markets. Another issue is the minimization of the imbalance cost of trading wind power in the market [10].

Hydro power scheduling models are well-known [11], some of them incorporating the future water value in the formulation as in [12].

Three optimization models for coordination between wind and reversible hydro technologies to offer energy in the day-ahead market are formulated in [13]. In this paper, the offering strategies in the day-ahead market are shown for the short-term, i.e., 168 h. These strategies are divided as 1) separate wind and reversible hydro offers without a physical connection between them; 2) separate wind and reversible hydro offers with a physical connection; and 3) single wind and reversible hydro offers with a physical connection.

Other authors study the combination of offering wind and hydro in an electricity market [14], but a single offer of the wind reversible hydro type has not been studied [15]. Risk-constrained coordination of cascaded hydro units with wind power through stochastic price-based unit commitment is presented in [16].

Reference [17] analyzes strategies for wind, thermal, and hydro power in a unit commitment model including the network. But, a network model is unnecessary when modeling energy offers only, as shown in [14]–[16].

Reference [18] considers 1 week as long-term, where they analyze a virtual power plant composed of dispatchable and nondispatchable technologies, adding a bilateral contract for a period of 168 h. In [19], a series of price-taker hydroelectric plants (H-GENCO) are modeled through mixed integer nonlinear programming for a period of 1 week, adding the future water value to the formulation.

B. Aims and Contributions

The combination of wind and reversible hydro power is studied because of the high uncertainty of wind generation, while the reversible hydro unit allows both energy generation and storage (by pumping), with a very quick response. Thus, reversible hydro power can reduce imbalances of wind generation on the condition that the profit must be equal or higher. Based on this, this paper extends the wind–hydro offering models 1) and 3) of [13] to a midterm horizon.

Consequently, the future water value (€) has two possible values. The first value is used for the first month and the second one is fixed at the end of the study period.

Hydro power is modeled as in [11] and the wind power model is based on [8], which models risk hedging using CVaR [19]. Our main aim is to compare the actual separate wind–hydro strategy against a coordinated joint strategy. To have enough data to determine the evolution in the midterm, this paper incorporates two future water values. Thus, the best strategy can be decided. The behavior of the profits offers and imbalances considering risk aversion is shown.

Accordingly, our contributions are threefold:

1) Simulation of an optimal wind reversible hydro unit offer for midterm planning by comparing a separate offer without a physical connection and a single offer with a physical connection.
2) Incorporation of two future water values for the midterm.
3) Use of a highly detailed model for the coordination of wind and reversible hydro units in the case of risk-hedging strategies using CVaR.

Compared to [13], there are several notable differences, such as changes in scenarios, changes in the study period (2 months now), and the incorporation of two future water values. Note that there are two kinds of profits, one related to the current profit and the other related to the future water value.

C. Paper Organization

This paper is organized as follows. Section II describes the two models, Section III shows the objective function subject to constraints, Section IV introduces the market and network interactions of the proposed algorithm, Section V presents the data of two realistic case studies, Section VI shows the results obtained, and Section VII presents the conclusion.

II. MODELS DESCRIPTION

This section describes the two kinds of strategies as follows.

A. Separate Wind Reversible Hydro Offering Without a Physical Connection

The scheme is depicted in Fig. 1. This strategy is the same as having wind power and reversible hydro power offering
Fig. 1. Separate wind reversible hydro offering without a physical connection (SO).

Fig. 2. Single wind reversible hydro offering with a physical connection (JOPC).

separately in a day-ahead electricity market. Wind power can only offer its production and the reversible hydro can offer/bid when using the turbine or pumping.

B. Single Wind Reversible Hydro Offering With a Physical Connection

A diagram of the single offer is shown in Fig. 2. A coordinated joint strategy can show that this strategy takes advantage of wind and reversible hydro synergy. Both generators, wind and reversible hydro, can only offer or purchase energy. Thus, when generators purchase energy for pumping, wind power is used to reduce the purchased energy through a physical connection.

In our case, it is impossible for a wind power unit to offer its energy to the market and for a reversible hydro unit to purchase energy at the same time, because the coordinated joint strategy is similar to having a single producer that can either offer or purchase energy.

This strategy makes it possible to reduce the two types of imbalances, positive and negative, where the imbalance can only be zero, positive, or negative.

III. MATHEMATICAL FORMULATION

The strategies are modeled through mathematical programming, where the objective function maximizes the profits of offering energy in the day-ahead electricity market. The problem is modeled using mixed integer linear programming (MILP).

The objective function and its constraints are presented for a joint offer with a physical connection (JOPC).

A. Objective Function

The objective function maximizes the profits of selling or purchasing energy where the measure of risk is CVaR. This function decides on the optimal offer (selling/purchase) per period $t$ for the wind farm and hydro unit $i$ for all scenarios, depending on water inflows, day-ahead market prices, positive and negative imbalance market prices, and wind production. These factors are parameters of the model.

The objective function is defined as

$$\max (1 - \beta) \cdot PF + \beta \cdot \text{CVaR}$$

where

$$PF = \sum_w \rho_w \left[ \sum_t (\lambda_{t,w} \cdot b_{t,i=1} + c_{i=1} \cdot (p_{t,i=1,w} + np_{t,i=1,w}) - c_{i=1} \cdot \eta_{t,i=1,w} + \lambda_{t,w}^+ \cdot \Delta_{t,i=1,w}^- - \lambda_{t,w}^- \cdot \Delta_{t,i=1,w}^+ - c_W \cdot gw_{t,w}) + \sum_t \left( r_{t=\frac{TP}{2},i,w} \cdot fwp1_w + r_{t=tp,i,w} \cdot fwp2_w \right) \right];$$

$$\text{CVaR} = \text{Var} - \frac{1}{1 - \alpha} \sum_w \rho_w \cdot \eta_w.$$

As can be seen in function $PF$ in (1), there is a single joint offer, $b_{t,i=1}$. $PF$ is the profit obtained from the offer remunerated at the day-ahead electricity market price. If there is an imbalance between the generation and the offer, the generator is sometimes penalized when the excess of production is considered as revenue equal to the imbalance power multiplied by the positive imbalance market price that is lower than the market price. Meanwhile, if the generation is less than the offer, this implies a cost equal to the imbalance power multiplied by the negative imbalance market price that is higher than the market price. Moreover, $PF$ has a production cost different from zero, including the start-up and shutdown hydro costs. The profit that comes from the future water value is evaluated for the reservoir in the middle of the time horizon $r_{t=\frac{TP}{2},i,w}$, and at the end of it $r_{t=tp,i,w}$.

CVaR defines the value at risk with a confidence level, usually 0.9.

$PF$ is composed of two terms: 1) the profit of selling energy in the electricity market and 2) the future value of water storage.

B. Constraints

The problem is subject to several constraints that are classified into seven blocks: 1) CVaR constraints; 2) hydro power curve linearization constraints; 3) hydro reserve constraints; 4) pumping constraints; 5) wind–hydro interconnection constraints; 6) offer constraints; and 7) imbalance constraints.
1) CVaR Constraints: These constraints consider all profits per period and scenario, fixing variable \( \eta_w \) per scenario and value of VaR

\[
PFE_{t,i,w} = \lambda_{t,w} \cdot b_{t,i,w} - c_{t,i} \cdot (p_{t,i,w} + np_{t,i,w}) - e^p \cdot \frac{1}{Pr_{t,i,w} - ci} \cdot h_{t,i,w} \\
+ \lambda^+_{t,w} \cdot \frac{\Delta h_{t,i,w}}{1} + \lambda^-_{t,w} \cdot \frac{\Delta h_{t,i,w}}{2} - e^w \cdot gw_{t,w}.
\]

Equation (3) incorporates new elements as compared to [13], i.e., the future water value for the midterm.

Equation (3) is composed of the profits from the sale/purchase of energy, future water value, VaR, and auxiliary variable \( \eta_w \).

Therefore, CVaR is evaluated with respect to the profit from the sale/purchase of energy and the future water value. Thus, risk aversion is related to profit through constraint (3) and the dependence between profit and risk is evaluated by means of the risk aversion of the producer \( \beta \)

\[
n_0 = \frac{\left( \sum_i \left( PFE_{t,i,w} + \left( \sum_i \left( r_{t,i,w} \cdot fwp1_{w} + r_{t,wp1_{i,w}} \cdot fwp2_{w} \right) \right) \right) \right) + VaR - \eta_w \leq 0.}
\]

2) Hydro Power Curve Linearity Constraints: The hydro power curve is linearized based on [11]. Equation (4) represents the maximum capacity in MW. Therefore, (4) is not really a constraint

\[
B_{0,i} = \text{porhol}_{i=1} + \sum_{i} \text{umax}_{i=1} \cdot \text{rhol}_{i=1}.
\]

Equation (5) is the discharge of hydro unit \( i \) depending on the linearization of the hydro power curve

\[
u_{t,i,w} = \sum_i (ul_{i,t,i,w} + umin_{i=1} \cdot v_{t,i,w}).
\]

The limits of each block of the linearized hydro power curve are presented in (6)–(9)

\[
ul_{i,t,i,w} \leq \text{umax}_{i=1} \cdot v_{t,i,w}
\]

\[
u_{t,i,w} \geq \text{umax}_{i=1} \cdot ul_{i,t} = 1, t_i = 1, w
\]

\[
u_{t,i,w} \leq \text{umax}_{i=1} \cdot ul_{i,t} = 1, t_i = 1, w
\]

\[
u_{t,i,w} \geq \text{umax}_{i=1} \cdot ul_{i,t} = 1, t_i = 1, w
\]

Equations (10)–(12) determine the power curves for higher, intermediate, or the lower one, depending on the reserve level. Also, the beginning and end of each hydro power curve are fixed

\[
d_{t,i,w}^1 \geq d_{t,i,w}^2
\]

\[
rt_{t,i,w} \geq vc_{1,i} \cdot \left( d_{t,i,w}^1 - d_{t,i,w}^2 \right) + vc_{2,i} \cdot d_{t,i,w}^2
\]

\[
rt_{t,i,w} \leq vc_{1,i} \cdot \left( 1 - d_{t,i,w}^1 \right) + vc_{2,i} \cdot \left( d_{t,i,w}^1 - d_{t,i,w}^2 \right)
\]

\[
V_{\text{max}}_i \cdot d_{t,i,w}^2
\]

Equations (13)–(18) determine the power production for the lower, intermediate, and higher power curves, respectively. These equations include variable \( np_{t,i,w} \), which represents the amount of power that a hydro unit could produce to reduce the negative wind imbalances

\[
p_{t,i,w} + np_{t,i,w} - \text{porhol}_{i=1} \cdot v_{t,i,w}
\]

\[
B_{0,i} + \text{porhol}_{i=1} \cdot \left( d_{t,i,w}^1 + d_{t,i,w}^2 \right) \leq 0
\]

\[
p_{t,i,w} + np_{t,i,w} - \text{porhol}_{i=1} \cdot v_{t,i,w}
\]

\[
B_{0,i} + \text{porhol}_{i=1} \cdot \left( d_{t,i,w}^1 + d_{t,i,w}^2 \right) \geq 0
\]

\[
p_{t,i,w} + np_{t,i,w} - \text{porhol}_{i=1} \cdot v_{t,i,w}
\]

\[
B_{0,i} + \text{porhol}_{i=1} \cdot \left( d_{t,i,w}^1 + d_{t,i,w}^2 \right) \geq 0
\]

Equation (19) shows the up/down hydro logic

\[
y_{t,1} = v_{t,1} - v_{t,1} = w_{t,1} - v_{t,1} = w_{1,1}.
\]

Equations (20) and (21) limit the production by its maximum capacity, previously calculated in (4). Also, binary variable \( a_{t,i,w} \) fixes the hydro production limit to range between zero and the maximum capacity.

Meanwhile, \( np_{t,i,w} \) variable is limited by the maximum hydro capacity minus the hydro production. In a separate offer without a physical connection strategy, \( np_{t,i,w} \) and the related equations would not exist

\[
p_{t,i,w} = B_{0,i} + \text{a}_{t,i,w}
\]

\[
B_{0,i} = a_{t,i,w} - p_{t,i,w}
\]

3) Hydro Reserve Constraints: Equations (22)–(24) model the balance of reserves, i.e., the minimum reserves of the hydro turbine and the initial and final conditions of the reserves

\[
r_{t,i,w} = r_{t-1,i,w} + \text{a}_{t,i,w} - cv \cdot u_{t,i,w} + cv \cdot u_{t-1,i,w}
\]

\[
r_{t,i,w} \geq \text{vc}_{1,i} \cdot \text{max}_{1,i} \cdot v_{t,i,w}
\]

\[
r_{t,i,w} \geq \text{vc}_{1,i} \cdot \text{max}_{1,i} \cdot v_{t,i,w}
\]

\[
r_{t,i,w} \geq \text{vc}_{1,i} \cdot \text{max}_{1,i} \cdot v_{t,i,w}
\]

\[
r_{t,i,w} \geq \text{vc}_{1,i} \cdot \text{max}_{1,i} \cdot v_{t,i,w}
\]

4) Pumping Constraints: Equations (25)–(27) represent the pumping efficiency and determine if there is a turbine/pumping
in the reversible hydro power unit through the binary variable 
\[ a_{t,i,w} \]

\[ \text{pr}_{t,i=1,w} \leq (1 - a_{t,i=1,w}) \cdot \text{ppm}_{i=1} \]
\[ \text{pm}_{t,i=1,w} = \text{pr}_{t,i=1,w} \cdot \text{eff} \]
\[ \text{wt}_{t,i=1,w} = \text{pm}_{t,i=1,w} / \text{rhopp}_{i=1} \].

5) Wind–Hydro Interconnection Constraints: The interconnection is represented by (28)–(38). The interconnection can be used to reduce the wind positive imbalance when there is an energy offer or to buy less energy when there is a purchase of energy. Equation (28) is used when there is an energy offer

\[ \text{if gw}_{t,w} > b_{t,w}^+ \]
\[ \text{ppwm}_{i=1,w} \leq \left( \text{gw}_{t,w} - b_{t,w}^+ \right) \cdot \text{bd}_{t,i=1} \]
\[ \text{else ppwm}_{i=1,w} = 0. \] (28)

Equations (29)–(37) calculate the energy that could be used for pumping coming from an excess of wind power when, in case of offering energy, a single offer with a physical connection is used, or to reduce the energy purchased in the other case (energy purchase)

\[ \text{ppw}_{t,i=1,w} = \text{gw}_{t,w} \cdot \left( 1 - \text{bd}_{t,i=1} \right) + \text{ppwm}_{i=1,w} \]
\[ \text{ppw}^-_{t,i=1,w} \leq \text{ppm}_{i=1,w} \cdot \left( 1 - \text{bd}_{t,i=1} \right) \]
\[ \text{ppw}^+_{t,i=1,w} \leq \text{ppm}_{i=1,w} \cdot \text{bd}_{t,i=1} \]
\[ \text{ppw}^\pm_{t,i=1,w} = \text{ppw}^+_{t,i=1,w} - \text{ppw}^-_{t,i=1,w} \]
\[ \text{ppbw}_{t,i=1,w} \leq \text{ppm}_{i=1,w} \cdot \left( 1 - \text{bd}_{t,i=1} \right) - \text{ppw}^-_{t,i=1,w} \]
\[ \text{ppw}_{t,i=1,w} = \text{ppw}^+_{t,i=1,w} + \text{ppw}^-_{t,i=1,w} + \text{ppbw}_{t,i=1,w} \]
\[ \text{ppw}_{t,i=1,w} \leq \text{ppm}_{i=1,w} \cdot \left( 1 - a_{t,i=1,w} \right) \]
\[ \text{pr}_{t,i=1,w} = \text{ppw}_{t,i=1,w} \].

When the imbalance is negative (38), the reversible hydro power unit could produce more energy, depending on its capacity limit.

\[ \text{if gw}_{t,w} < b_{t,w}^- \]
\[ \text{np}_{t,i=1,w} \leq \left( b_{t,w}^- - \text{gw}_{t,w} \right) \]
\[ \text{else np}_{t,i=1,w} = 0. \] (38)

6) Offer Constraints: Offers are limited by the capacity of each plant. Furthermore, the model needs to fix the constraints for offering or purchasing. Variable \( b_{t,i=1,w} \) depends on the period and this variable does not depend on the scenario

\[ b_{t,i=1} \leq P_{\text{max}} \cdot \text{bd}_{t,i=1} + p_{t,i=1,w} \]
\[ b_{t,i=1} \geq 0 \cdot \text{bd}_{t,i=1} + p_{t,i=1,w} - \text{ppm}_{i=1} \cdot \left( 1 - \text{bd}_{t,i=1} \right). \]

7) Imbalance Constraints: The limits of each type of imbalance are set in (41) and (42) for a negative and a positive imbalance, respectively

\[ \Delta_{t,i=1,w}^- \leq (B0_{i=1} + P_{\text{max}}) \cdot d_{t,i=1,w} \]
\[ \Delta_{t,i=1,w}^+ \leq (P_{\text{max}} + \text{ppm}_{i=1}) \cdot \left( 1 - d_{t,i=1,w} \right). \]

To quantify the excess or lack of energy, the imbalance is calculated in (43), and if the imbalance is either positive or negative, it is solved in (44)

\[ \Delta_{t,i=1,w} = gw_{t,w} \cdot \text{bd}_{t,i=1} + gw_{t,w} \cdot (1 - \text{bd}_{t,i=1}) + p_{t,i=1,w} - b_{t,i=1} + \text{np}_{t,i=1,w} - \text{ppbw}_{t,i=1,w} \]
\[ \Delta_{t,i=1,w}^- = \Delta_{t,i=1,w}^+ - \Delta_{t,i=1,w}^-. \]

IV. ELECTRICITY MARKET AND NETWORK INTERACTIONS

In this section, the features of a single wind and hydro offer with a physical connection are discussed in relation to the electricity market and the transmission network.

The generators’ economic surplus for different types of prices, such as nodal, zonal, and uniform price are studied in [4]. Note that most European markets have the same price for all the nodes (single-node systems) and the overall energy is integrated in the European Network of Transmission System Operators for Electricity (ENTSO-E) [20].

The uniform price is the marginal price obtained from the economic equilibrium between the demand and supply curves in the day-ahead market. After the day-ahead market is cleared, there are some mechanisms, such as the adjustment market and the balancing market, to find the energy balance between load and generation, representing an economic surplus for the generators. The algorithm proposed can be applied in electricity markets with a uniform price.

Hence, it is aimed at European markets, where the day-ahead market is used to sell the energy generated and the balancing market is used to penalize imbalances between the offer submitted to the market operator and the actual generation produced.

The effect of a big single-unit generator on a uniform price market model [price-maker model [21], [22]] is outside of the scope of this paper, since only a price-taker model is analyzed by fixing the capacity of the producer without affecting the marginal price. The size of a single unit composed of wind (250 MW) and reversible hydro (116/145 MW) is about 350 MW, which is the average size of generators in European markets. However, it can be noted that the incorporation of a large amount of wind–hydro units would reduce the marginal price as a consequence of a reduced marginal production cost.

In addition, network effects are disregarded in the model proposed; therefore, wind and hydro units should be as close as possible to each other to avoid transmission losses [2], and the network should be meshed. For example, the Spanish Transmission System Operator, Red Eléctrica de España (REE) [23] is making a significant investment to improve the mesh of the network and to incorporate renewable energy into the system.

V. DATA

The models are tested for two case studies. Every case study is composed of a wind farm and a reversible hydro unit with two reservoirs. Data for the hydro unit are given in [11].
Wind speed data have been obtained from two meteorological stations near the wind farm. Wind speed data are used in the expression of the wind turbine, 
\[ P(v) = 0.5 \cdot c_p(v) \cdot \rho \cdot A \cdot v^3, \]
where \( v \) is the wind speed, \( c_p(v) \) is the overall efficiency of the wind turbine as a function of wind speed, \( A \) is the area swept by the wind turbine rotor, and \( \rho \) is the air density. Production costs are \( €16.26/MWh \) for wind generation, \( €10/MWh \) for hydro generation, and \( €3/MWh \) for hydro pumping. The marginal costs are obtained and adjusted from [24]. The confidence level, \( \alpha \), is 0.9 and \( \beta \in (0.1, 0.85) \).

The future water value is calculated as the average price of the periods in each scenario: \( fwp_{1w} \) for the first half of the time horizon and \( fwp_{2w} \) for the second half. \( fwp_{1w} \) is the average price value of the first half of the time horizon per scenario and \( fwp_{2w} \) is the average price value of the second half per scenario. The first value is multiplied by 1.1 and the second by 1.2.

Water inflow scenarios come from a Normal distribution. Market prices [25] and imbalance market prices [23] have been obtained using 24 normal distributions, one per hour for January and February of 2012 and 2013. For each price, the number of scenarios for prices depends on the distribution sample. Wind speed scenarios are obtained using 24 Weibull distributions, one per hour for January and February of 2013.

With respect to risk, a coordinated wind reversible hydro unit changes its behavior with risk aversion, where \( \beta \in (0.1, 0.85) \). This tabulation of \( \beta \) is used to consider risk aversion in the objective function. The strategies defined can be used in any electricity market that includes an imbalance market, although the simulation uses data from the Spanish electricity market. The model is tested in two case studies: 1) Case Study A and 2) Case Study B.

A. Case Study A

This case study evaluates one wind farm and one reversible hydro unit offer for 168 h (1 week). The wind farm has 250 MW of installed capacity. This wind farm has 125 units of 2 MW located in Navarra, Northern Spain. The reversible hydro unit has a capacity of 116.38 MW for generation and 145.37 MW for pumping and its efficiency is 80%.

A scenario tree is used to simulate the two strategies. The tree is composed of: 1) two scenarios of water inflow for the two reservoirs, 2) four scenarios of market prices and positive and negative imbalance market prices, and 3) four scenarios for wind generation and upper and lower limits of the wind farm offer, with a total number of scenarios equal to \( 2 \cdot 4 \cdot 4 = 32 \).

Next, the scenarios for day-ahead market prices, positive and negative imbalance market prices and wind generation are shown in Figs. 3–6 for 168 h.

B. Case Study B

The case study is composed of a wind farm and a reversible hydro unit whose offers are evaluated for 1440 h (2 months). Wind farm capacity is 50 MW and the wind farm is composed of 25 units of 2 MW located in Navarra, Northern Spain.

The reversible hydro unit has a capacity of 28.62 MW for generation and 35.77 MW for pumping with 80% efficiency.
Fig. 6. Maximum, average, and minimum of the scenarios of wind generation for 168 h.

Fig. 7. Scenario tree used in the offering strategies.

The scenario tree is composed of 64 scenarios, as shown in Fig. 7. The number of scenarios is: 1) two scenarios of water inflows; 2) four scenarios of prices; and 3) eight scenarios of wind generation, i.e., \(2 \times 4 \times 8 = 64\).

VI. RESULTS

The results are classified according to the case studies. Case Study A shows a specific scenario and the behavior of the main variables with respect to risk aversion. Case Study B presents the average behavior of the main variables considering risk aversion.

In the next section, the results of both strategies are presented: 1) separate wind reversible hydro offering without a physical connection (SO) and 2) single wind reversible hydro offering with a physical connection (JOPC).

A. Case Study A

The main decision that the generator has to make is the power offered to the market. This offer maximizes profit depending on market prices and reducing imbalances. Therefore, the offering strategies are shown in Fig. 8; the imbalances for both strategies are presented in Fig. 9 for \(\beta = 0.5\) and scenario 15.

Fig. 8 shows that the JOPC strategy offers are lower than the SO strategy offers. Also, Fig. 9 depicts the imbalances for scenario 15 and \(\beta = 0.5\), showing a reduction in the negative imbalances for the JOPC strategy and a slight increase in the positive imbalance.

The risk aversion behavior to make a decision about the quantity offered is presented in Tables I and II, showing the discharge and pumping of reversible hydro power for every \(\beta\) value. The total expected profits are presented in Table III,
as well as CVaR and standard deviation of the total expected profits.

Table II shows that reversible hydro power is essential to increase profits in the JOPC strategy and reduce imbalances. The behavior of the strategies depends on the offer/bid, imbalances, discharge, and pumping. Extra discharge and pumping of reversible hydro power are used to reduce wind imbalances, being those the ones that produce an increase in the JOPC profits. An important issue is that reversible hydro power has limited its generation depending on reserves. Reserves limit generation up to 90% of the upper initial reserve.

As a consequence of this restriction, the energy generated through hydro power is small. Therefore, the capacity to reduce wind uncertainty is limited too.

Tables I and III have to be evaluated simultaneously, because of the dominance of some solutions with respect to the rest of them. The efficient frontier presented in Table III allows us to see which \( \beta \) values are the most profitable. All dominant points present a higher profit and CVaR for the JOPC strategy.

### Case Study B

The expected profit decreases when \( \beta \) increases, but CVaR is higher, as can be seen in Fig. 10.

Also, the expected profit of the coordinated wind reversible hydro strategy (JOPC) is higher than the one of the separate wind and reversible hydro strategy (SO). However, standard deviation versus expected profit decreases when \( \beta \) increases. The higher the risk, the higher the expected profit is.

In Fig. 11, standard deviation versus expected profit is shown. Figs. 12–14 represent the behavior of the offer, positive, and negative imbalances as a function of \( \beta \), respectively.

The coordinated wind reversible hydro (JOPC) offer is lower than the separate wind and reversible hydro offer (SO). It can be observed that with high quantity offers (low risk aversion), profits as well as the negative imbalance are high.

However, the positive imbalance is low because the difference between the capacity of the plants and the offer is low. When risk aversion increases, the opposite effect occurs. The expected profit decreases because the offer is low enough to decrease the standard deviation.

Thus, the negative imbalance is lower than the positive imbalance because the difference between the capacity of the plants and the offer is high.

The models are programmed in MATLAB [26] and GAMS [27] in a HP Z820 Intel Xeon E5-2687W computer, with two processors at 3.10 GHz and 256 GB of RAM. The CPU time per model is approximately 24 h.
VII. CONCLUSION

Two offer models (coordinated wind–hydro and separate wind hydro) are presented to compare their midterm effects, where profits, imbalances, and offers depend on risk aversion. It is noted that the coordinated wind reversible hydro strategy with a physical connection was more profitable and competitive.

In this regard, the main conclusion is as follows.

1) The expected profit of the single wind–reversible hydro offer with physical connection is higher than the separate strategy in a midterm horizon.
2) A single wind–reversible hydro offering with physical connection reduces imbalances in the midterm.
3) A lower risk aversion value maximizes profit, increasing the energy offered and, therefore, reducing the likelihood of positive imbalances while increasing the likelihood of negative imbalances.
4) The standard deviation of the single strategy is lower due to the capacity of absorbing the volatility of wind generation. The single strategy can tolerate more uncertainty due to the storage capacity of the reversible hydro unit.
5) Hydraulic reserve constraints limit the response of reversible hydro power to reduce wind uncertainty.

The specific effects on the system are described as follows.

1) The single unit offering reduces imbalances, as shown in Figs. 13 and 14; hence, the system decreases the spinning reserve used for renewable energy imbalances, especially wind imbalance.
2) The production of both technologies (wind and hydro) that compose the single unit depends only on environmental conditions, although the hydro pumping unit can store the spare energy. Also, in case of low water reserves, the single unit could reduce imbalances by 20%, as shown in Fig. 14.

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renewable energy sources.

Javier Contreras (SM’05–F’15) received the B.S. degree in electrical engineering from the University of Zaragoza, Zaragoza, Spain, in 1989, the M.Sc. degree in electrical engineering from the University of Southern California, Los Angeles, CA, USA, in 1992, and the Ph.D. degree in electrical engineering from the University of California, Berkeley, CA, USA, in 1997.

Currently, he is a Full Professor with the University of Castilla-La Mancha, Ciudad Real, Spain. His research interests include power systems planning, operations and economics, and electricity markets.

José Ignacio Muñoz received the B.S. degree in industrial engineering from the University of Navarra, Navarra, Spain, in 1998 and the M.Sc. degree in industrial and operations engineering and Ph.D. degree in project management from the University of País Vasco, Leioa, Spain, in 2003 and 2009, respectively.

He has been working as a Project Manager in several engineering firms and is, currently, an Assistant Professor with the University of Castilla-La Mancha, Ciudad Real, Spain. His research interests include power systems forecasting, operations and economics, and project management.

João P. S. Catalão (M’04–SM’12) received the M.Sc. degree from the Instituto Superior Técnico (IST), Lisbon, Portugal, in 2003 and the Ph.D. degree in electrical engineering and Habilitation for Full Professor (“Agregação”) from the University of Beira Interior (UBI), Covilhã, Portugal, in 2007 and 2013, respectively.

Currently, he is a Professor with the UBI, Director with the Sustainable Energy Systems Lab, and Researcher with the INESC-ID. He is the Primary Coordinator of the EU-funded FP7 project SINGULAR (“Smart and Sustainable Insular Electricity Grids Under Large-Scale Renewable Integration”), a 5.2 million euro project involving 11 industry partners. He has authored or coauthored more than 350 publications, including 110 journal papers, 220 conference proceedings papers, and 20 book chapters, with an h-index of 24 (according to Google Scholar), having supervised more than 25 postdocs, Ph.D. and M.Sc. students. He is the Editor of the book entitled Electric Power Systems: Advanced Forecasting Techniques and Optimal Generation Scheduling (Boca Raton, FL, USA: CRC Press, 2012), translated into Chinese in January 2014. Currently, he is editing another book entitled Smart and Sustainable Power Systems: Operations, Planning and Economics of Insular Electricity Grids (Boca Raton, FL, USA: CRC Press, 2015). His research interests include power system operations and planning, hydro and thermal scheduling, wind and price forecasting, distributed renewable generation, demand response, and smart grids.

Prof. Catalão is an Editor of the IEEE TRANSACTIONS ON SMART GRID, an Editor of the IEEE TRANSACTIONS ON SUSTAINABLE ENERGY, and an Associate Editor of the IET Renewable Power Generation. He was the Guest Editor-in-Chief for the Special Section on “Real-Time Demand Response” of the IEEE TRANSACTIONS ON SMART GRID, published in December 2012, and he is currently the Guest Editor-in-Chief for the Special Section on “Reserve and Flexibility for Handling Variability and Uncertainty of Renewable Generation” of the IEEE TRANSACTIONS ON SUSTAINABLE ENERGY. He is the recipient of the 2011 Scientific Merit Award UBI-PE/Santander Universities and the 2012 Scientific Award UTL/Santander Totta.